

## **Teaching Electronic Conduction Phenomena to Undergraduate Electrical Engineering Students Using Purdue University's New "Bottom-Up" Approach**

**Dr. Peter M. Osterberg, University of Portland**

Dr. Peter Osterberg is an associate professor in Electrical Engineering at the University of Portland (Portland, OR). He received his BSEE and MSEE degrees from MIT in 1980. He received his Ph.D. degree in electrical engineering from MIT in 1995 in the field of MEMS. He worked in industry at Texas Instruments, GTE, and Digital Equipment Corporation in the field of microelectronics. His research interests are microelectronics, MEMS, and nanoelectronics.

**Dr. Aziz Sukru Inan, University of Portland**

Dr. Aziz Inan is a professor in Electrical Engineering at the University of Portland (Portland, OR), where he has also served as Department Chairman. He received his BSEE degree from San Jose State University in 1979 and MS and Ph.D. degrees in electrical engineering from Stanford University in 1980 and 1983 respectively. His research interests are electromagnetic wave propagation in conducting and inhomogeneous media. He is a member of Tau Beta Pi and senior member of IEEE.

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## Introduction

Historically, undergraduate Electrical Engineering (EE) programs have taught electronic conduction phenomena using a “Top-Down” approach. That is, traditional programs start with large devices (i.e., “Top”) and teach how interesting electronic conduction phenomena change as the size of the device decreases towards the nano-scale (i.e., “Down”). So, for example, if one considers a normal three-dimensional (3-D) macroscopic resistor, as shown in Figure 1 on the left, where diffusive transport due to electron scattering is dominant, students are taught that the resistance is calculated as  $R=L/(\sigma A)$ , where  $L$ ,  $A$ , and  $\sigma$  are the resistor’s length, cross-sectional area, and conductivity, respectively. Therefore, this “Top-Down” approach would predict that as the length is decreased to a 3-D nanoscopic resistor, as shown in Figure 1 on the right, its resistance would approach zero ohms. However, it is now well known that the conductance for nano-scale ballistic conductors is quantized in multiples of  $q^2/h \approx 1/(25\text{k}\Omega)^1$ , where  $q$  is the electron charge ( $1.6 \times 10^{-19}$  C) and  $h$  is Planck’s Constant ( $6.63 \times 10^{-34}$  J-sec). For this reason, the resistance of the simplest 1-D nanoscopic device would approach  $h/q^2 \approx 25\text{k}\Omega$  (not zero ohms).

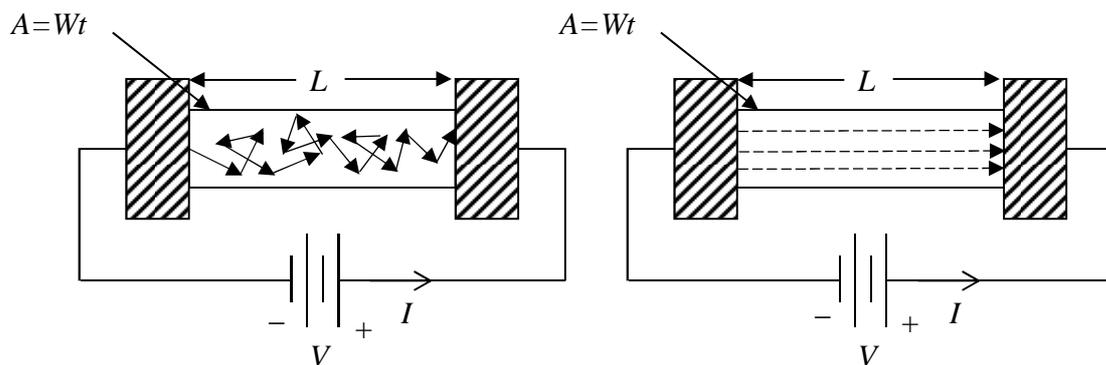


Figure 1. 3-D “macro” resistor (left) with diffusive conduction (i.e., scattering). 3-D nano resistor (right) with ballistic conduction (i.e., no scattering). The two end contacts and the middle conducting channel are shown.

For the past four years, the Department of Electrical Engineering at the University of Portland has been experimenting with teaching the physics of electronic conduction phenomena to EE students in a senior-level advanced electronics elective course using Purdue University’s new “Bottom-Up” Approach. This paper describes the successful teaching content and approach used. Some initial assessment results are also presented. This approach is based entirely on work by Dr. Supriyo Datta from Purdue University<sup>1,2</sup> and the affiliated NCN-sponsored nanohub.org educational websites<sup>3,4</sup>. This “Bottom-Up Approach” first considers the theoretical treatment of electronic conduction in a nano-scale size conductor (i.e., “Bottom”) where ballistic conduction is dominant. This device is known as an “elastic” resistor, meaning that the electrons do not exchange any energy with the conducting channel as they travel through it. Then this

approach works “Up” towards macro-scale conductors where electron scattering conduction is dominant. The advantage of this “Bottom-Up” approach is that Electrical Engineering students can more simply and intuitively understand pure ballistic electronic conduction at the nano-scale, and then work backwards up to larger devices where more complex electron flow phenomena (i.e., diffusive or Boltzmann transport) needs to be applied. Specifically, the course material starts at the “Bottom” (or nano-scale) and considers both the current/voltage ( $I/V$ ) characteristic and the thermoelectronic behavior of a one-dimensional elastic nano-scale conductor with only one energy level. Using simple principles, both the  $I/V$  characteristic plus the thermoelectronic properties of the conductor are easily determined and understood. Also, as the length of the conductor is shrunk, it is noted that a maximum conductance ( $G_{\max}$ ) is reached having a value of  $q^2/h \approx 1/(25k\Omega)$ . This quantity is called the Quantum of Conductance. Additionally, it is easy to teach complex concepts such as simple Ohmic heating (i.e.,  $I^2R$  loss), the Seebeck effect, and the Peltier effect in this “Bottom” regime. The main reason for the simplicity and success of this “Bottom-Up” approach is due to the *clean separation* (or decoupling) between the two physical phenomena within the nano-conductor channel including 1) those involving mechanics versus 2) those involving thermodynamics. In this “Bottom” regime, all the thermodynamic processes (i.e., heating or cooling) occur in the contacts located at either end of the nano-conductor, and none within the nano-conductor channel, itself. Whereas, all the mechanics occur within the nano-conductor channel, and is due to pure ballistic electron transport. Then, the treatment of electronic conduction is extended “Up” towards macro-scale devices where both the mechanics and thermodynamics phenomena are “all mixed up” within the channel resulting in the electron transport physics to become much more complicated. Therefore, the students’ understanding of complex concepts such as simple Ohmic heating, Seebeck effect, and Peltier effect are all easily attained at the nano-scale and then can be extrapolated “Up” to the macro-level. In other words, this “Bottom-Up” approach makes difficult and complex electronic conduction phenomena easily “accessible” to undergraduate EE students at the nano-scale which would otherwise be inaccessible at the conventional macro-scale.

## The Model

In this paper, we will focus on the simplest nano-device which is one-dimensional (1-D) ballistic nano resistor with only one energy level as shown in Figure 3. (Note that the convention of assigning contact 1 with the name “Source”, and contact 2 with the name “Drain”, is used throughout this paper). To analyze this device, we do not simply use Ohm’s Law ( $V=IR$ ) as we normally do with “macro” resistors. Instead, we control the Fermi levels,  $\mu_1$  and  $\mu_2$ , in each contact. The Fermi level (or electrochemical potential),  $\mu$ , is part of the Fermi function equation as shown in Figure 2. The Fermi function determines the “occupation” by electrons within allowed energy levels. Specifically, the Fermi function is the probability that an energy level is filled by an electron, and is dependent on the absolute temperature. The Fermi function has a value of “1” for all energy levels below  $\mu$  (minus a few  $kT$ ), and “0” for all energy levels above  $\mu$  (plus a few  $kT$ ), where  $k$  is Boltzmann’s constant and  $T$  is the absolute temperature. The value of the Fermi function is precisely 1/2 at the Fermi level,  $\mu$ . Note in Figure 2 that at a temperature of zero Kelvin (shown in dotted line), the Fermi function is a simple abrupt step (from 1 to 0) centered around the Fermi level,  $\mu$ . At higher temperatures, the Fermi function transitions from 1 to 0 gradually within a range of a few  $kT$  centered around the Fermi level,  $\mu$ . Figure 2 shows three ways to describe the Fermi function as follows: 1) the Fermi function equation, 2) the

Fermi function graph, and 3) the short-hand notation that is simply showing the position of the Fermi level. This short-hand notation is used throughout this paper.

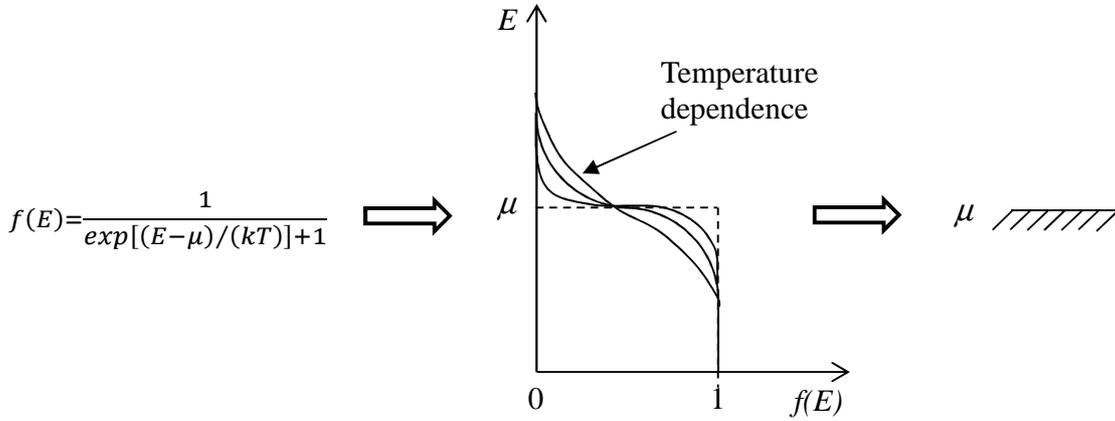


Figure 2. The Fermi function equation, the graph (showing temperature dependence), and the short-hand notation used throughout the paper.

Figure 3 shows the simplest nano resistor model used throughout this paper. It is a 1-D elastic nano resistor whose width,  $W$ , and thickness,  $t$ , are both less than or equal to a single deBroglie wavelength and greater than or equal to a half deBroglie wavelength, which yields only a single mode or energy-level for electron transport. The channel length,  $L$ , is much less than its mean-free-path,  $\lambda_{mfp}$ , causing the device to exhibit only ballistic conduction (i.e., no scattering). This device has only one energy level,  $\varepsilon$ , in the channel. Also, shown in Figure 3 are the two contact coupling coefficients (or escape rates),  $\gamma_1/\hbar$  and  $\gamma_2/\hbar$ , where  $\hbar = h/(2\pi)$ . These two escape rates have units of seconds<sup>-1</sup>. This coupling coefficient is a measure of the “goodness” of the contact or how fast an electron at an energy,  $\varepsilon$ , will enter or leave the channel. Therefore, the time it takes for an electron to traverse the channel from source to drain is  $\hbar/\gamma_1 + \hbar/\gamma_2 = 2(\hbar/\gamma)$  seconds, for  $\gamma_1 = \gamma_2 = \gamma$ . As shown in Figure 3, the positions of the two Fermi levels,  $\mu_1$  and  $\mu_2$ , in the two contacts are independently controlled by adjusting each contact’s voltage. For convenience, we hold contact 1 at ground potential, and place a positive voltage,  $V$ , on contact 2. This causes  $\mu_1$  to remain stationary, and  $\mu_2$  to be pushed down by an amount,  $qV$  (in units of electron-volts). As a side note, unlike one might expect, the specific form of this model does not predict that the electron transit time from source to drain is  $t_{transit} = L/v$ , where  $v$  is the velocity of the electron. The particular form of this model is suitable for extremely short ballistic nano resistors, such as electronic conduction through molecules, where the actual electron transit time is  $t_{transit} = 2(\hbar/\gamma)$ . The transit time is very short, and is only due to the contact escape rate of the electron itself (i.e.,  $2(\hbar/\gamma) \gg L/v$ ). Therefore, it is independent of the channel length,  $L$ . Note that the model could be slightly modified to accommodate long ballistic nano resistors, such as carbon nanotubes or silicon nanotubes, where  $t_{transit} = L/v$ , as expected. However, this is beyond the scope of this paper.

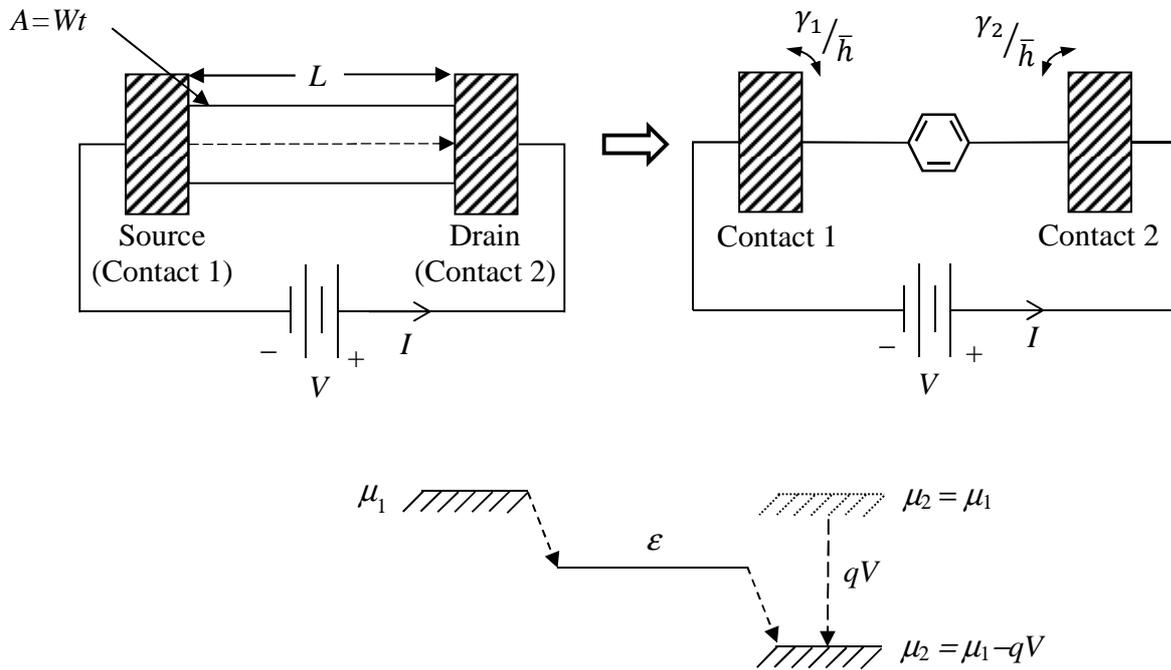


Figure 3. The simplest device: A 1-D, 1-level elastic nano resistor with ballistic transport.  $L \ll \lambda_{mfp}$ ,  $W$  and  $t$  are both  $\geq \lambda_{DB}/2$  and  $\leq \lambda_{DB}$ , creating a single electron mode or energy-level located at  $\epsilon$  within the nano resistor channel.

The current through the nano resistor is<sup>1</sup>:

$$I = \frac{2q}{\hbar} \int dE D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1(E) - f_2(E)]$$

where  $D(E)$  is the Density of States which is simply a unit impulse for our single-level nano resistor. Note that the “factor of 2” in this equation is due to the electron’s two spins. Therefore, for this single-level case, when the applied voltage is high enough such that  $\epsilon$  is “between”  $\mu_1$  and  $\mu_2$ , and assuming  $\gamma_1 = \gamma_2 = \gamma$  the current through the nano resistor is:

$$I = \frac{q\gamma}{\hbar} [f_1(E) - f_2(E)]$$

And, therefore,  $I_{max} = q\gamma/\hbar$ . The assumption that  $\gamma_1 = \gamma_2 = \gamma$  is used for the remainder of this paper. This yields the  $I/V$  characteristic in Figure 4. Notice that the  $I/V$  curve is not a simple step at  $-2\alpha/q$  volts or  $+2\alpha/q$  volts, where  $\alpha = |\mu - \epsilon|$ . Rather it transitions gradually at these two voltages due to 1) the gradual transition of the Fermi function, itself, around the Fermi Level,  $\mu$ , and 2) the energy level at  $\epsilon$  is not actually a sharp impulse, but is “broadened” due to the Heisenberg uncertainty principle. This phenomenon gives rise to the famous Quantum of Conductance ( $q^2/h$ ) which is the maximum slope ( $dI/dV$ ) of the  $I/V$  curve. Further treatment of this behavior is beyond the scope of this paper.

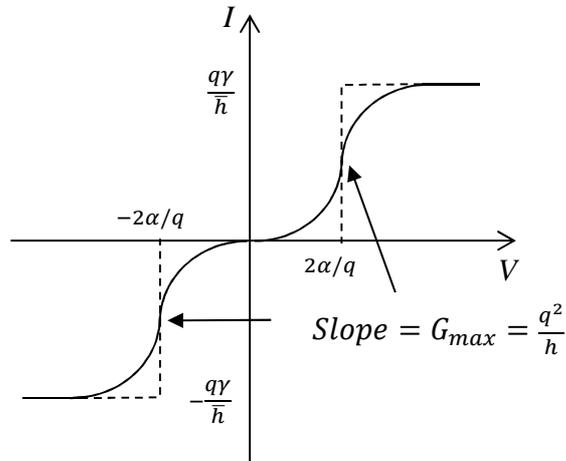


Figure 4.  $I/V$  characteristic. of a 1-D, 1-level elastic nano resistor, where  $\alpha = |\mu - \epsilon|$ .

Figure 5 shows both the n-type and p-type versions of this simple 1-D, single energy level nano resistor. In the n-type case, the single energy level at  $\epsilon$  is located *above* the equilibrium electrochemical potential,  $\mu_1 = \mu_2 = \mu$ , and therefore the single level is normally empty of electrons. Similarly, for the p-type case, the single energy level at  $\epsilon$  is located *below* the equilibrium electrochemical potential,  $\mu_1 = \mu_2 = \mu$  and, therefore, the single level is normally filled with two electrons (one “spin-up” electron and one “spin-down” electron).

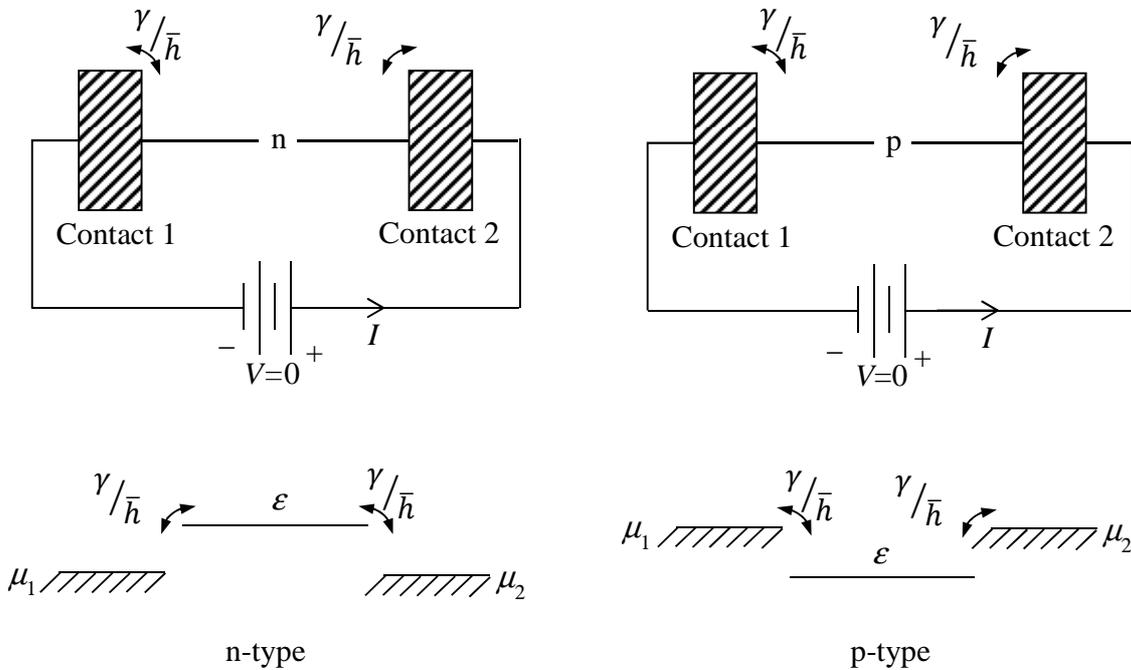


Figure 5. 1-D, 1-level n-type and p-type elastic nano resistors.

### Simple Ohmic Heating (n-type or p-type device)

Using our simple nano resistor model described in the previous sections, we can now easily analyze simple Ohmic heating and understand where the heat goes in a nano resistor. As shown in Figure 6, electrons dump  $\mu_1 - \epsilon$  Joules of heat into contact 1 and  $\epsilon - \mu_2$  Joules of heat into contact 2 which heats both contacts. No heating occurs inside the channel. To undergraduate EE students, this is an astonishing result.

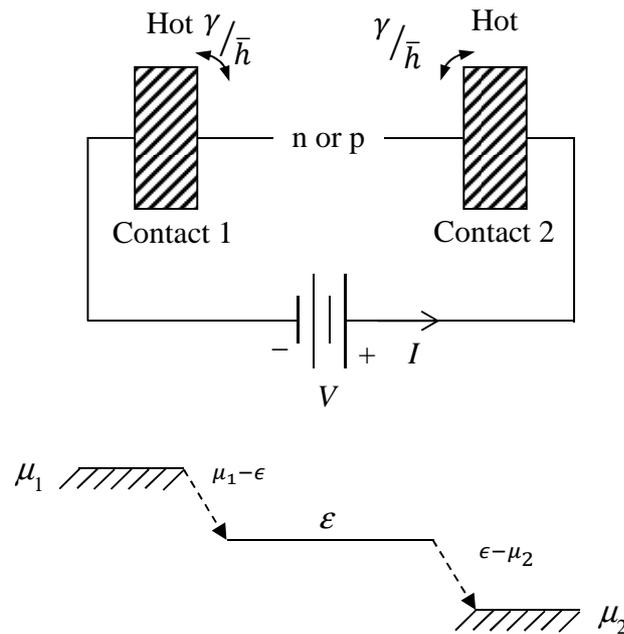


Figure 6. Simple Ohmic heating in 1-D, 1-level elastic nano resistor. Heat is dumped into both contacts 1 and 2 only. No heat is dissipated in the channel.

Furthermore, we note that power is conserved as follows:

$$P = \underbrace{\frac{\mu_1 - \epsilon}{\hbar/\gamma}}_{\text{Power dumped as heat to Contact 1}} + \underbrace{\frac{\epsilon - \mu_2}{\hbar/\gamma}}_{\text{Power dumped as heat to Contact 2}} = \underbrace{\left[ \frac{\mu_1 - \mu_2}{q} \right] \left[ \frac{q\gamma}{\hbar} \right]}_{\text{Power supplied from battery}} = VI$$

### Peltier effect: n-type device

The same simple model allows us to easily analyze the Peltier effect in an n-type nano resistor and again understand where the heating and cooling occurs. As shown in Figure 7, electrons absorb  $\epsilon - \mu_1$  Joules of heat from contact 1 which cools it, and dump  $\epsilon - \mu_2$  Joules of heat into contact 2 to heat it. No heating or cooling occurs inside the channel. Furthermore, note that

more heat is dumped into contact 2 than is absorbed from contact 1. In other words, contact 2 heats up more than contact 1 cools down, satisfying the second law of thermodynamics.

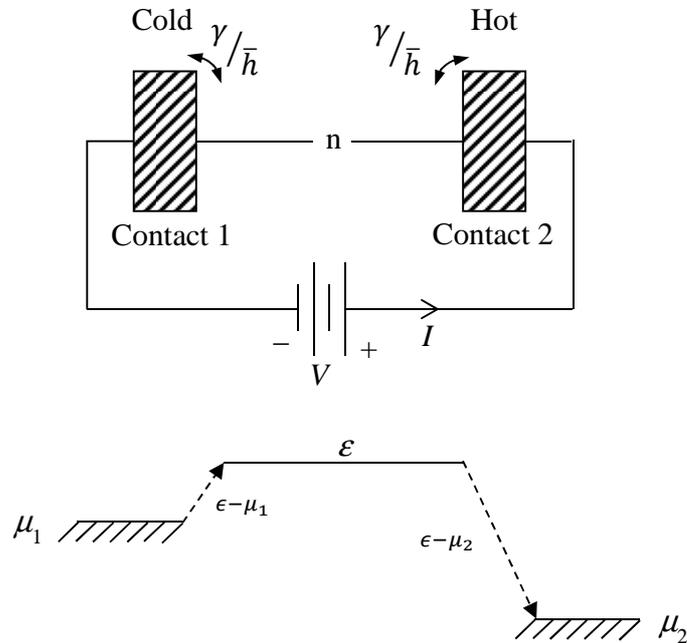


Figure 7. 1-D, 1-level n-type elastic nano resistor used as a Peltier device. Heat is absorbed by electrons flowing out of contact 1 to cool it, and heat is dumped by electrons flowing into contact 2 to heat it. No heating or cooling occurs in the channel.

Again, we note that power is conserved as follows:

$$P = \underbrace{-\frac{\epsilon - \mu_1}{\hbar/\gamma}}_{\text{Power absorbed to cool Contact 1}} + \underbrace{\frac{\epsilon - \mu_2}{\hbar/\gamma}}_{\text{Power dumped as heat to Contact 2}} = \underbrace{\left[\frac{\mu_1 - \mu_2}{q}\right] \left[\frac{q\gamma}{\hbar}\right]}_{\text{Power supplied from battery}} = VI$$

Peltier effect: p-type device

Similarly, the Peltier effect for the p-type nano resistor can be easily analyzed and understood as shown in Figure 8. In this case, electrons dump  $\mu_1 - \epsilon$  Joules of heat into contact 1 which heats it, and absorb  $\mu_2 - \epsilon$  Joules of heat from contact 2 to cool it. Again, no heating or cooling occurs inside the channel, itself. In this case, note that more heat is dumped into contact 1 than is absorbed from contact 2. In other words, contact 1 heats up more than contact 2 cools down, again satisfying the second law of thermodynamics.

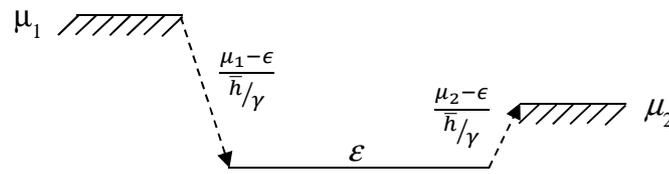
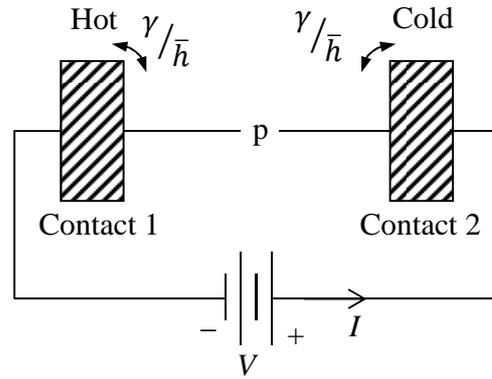


Figure 8. 1-D, 1-level p-type elastic nano resistor used as a Peltier device. Heat is dumped by electrons flowing out of contact 1 to heat it, and heat is absorbed by electrons flowing into contact 2 to cool it. No heating or cooling occurs in the channel.

We note that power is conserved as follows:

$$P = \underbrace{\frac{\mu_1 - \epsilon}{\hbar/\gamma}}_{\text{Power dumped as heat to Contact 1 to cool Contact 2}} + \underbrace{-\frac{\mu_2 - \epsilon}{\hbar/\gamma}}_{\text{Power absorbed}} = \underbrace{\left[\frac{\mu_1 - \mu_2}{q}\right] \left[\frac{q\gamma}{\hbar}\right]}_{\text{Power supplied from battery}} = VI$$

Seebeck effect: n-type device

We can continue using our model to easily analyze and understand the Seebeck effect in a 1-D, single-level n-type elastic nano resistor. As shown in Figure 9, contact 1 is externally cooled causing its Fermi function,  $f_1(E)$ , to be steep. Contact 2 is heated causing its Fermi function,  $f_2(E)$ , to be less steep and smoothed out. Since this is an open-circuit, current must be zero and, therefore,  $f_1(\epsilon)$  must equal  $f_2(\epsilon)$ , causing  $\mu_2$  to be pushed down relative to  $\mu_1$  and creating an open-circuit voltage,  $V_{OC}$ , with the polarity as shown. A simple calculation yields the expression for  $V_{OC}$  to be as follows:

$$V_{OC} = \frac{\mu_1 - \mu_2}{q} = \left[ \frac{\epsilon - \mu_1}{qT_1} \right] [T_2 - T_1] = S\Delta T$$

where  $S$  is the Seebeck coefficient.

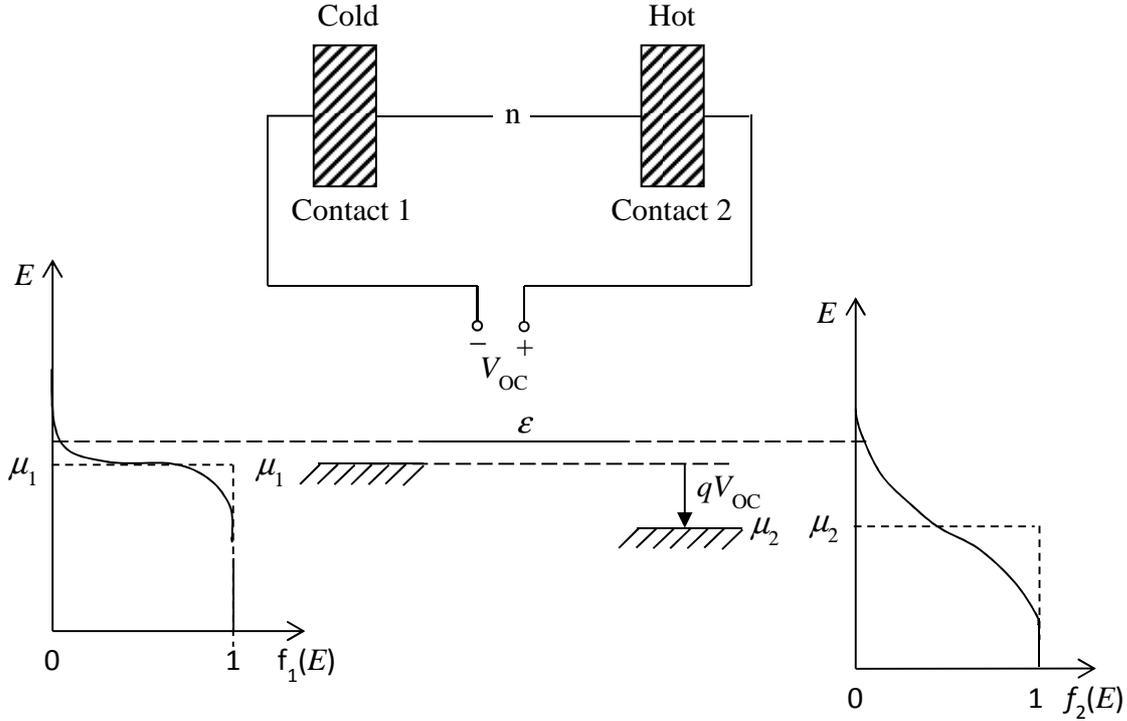


Figure 9. 1-D, 1-level elastic n-type nano resistor used as Seebeck device.

Seebeck effect: p-type device

Similarly, we can continue using our model to easily analyze and understand the Seebeck effect in a 1-D, single-level p-type elastic nano resistor. As shown in Figure 10, contact 1 is externally cooled causing its Fermi function,  $f_1(E)$ , to be steep. Contact 2 is heated causing its Fermi function,  $f_2(E)$ , to be less steep and smoothed out. Since this is an open-circuit, current must be zero and, therefore,  $f_1(\epsilon)$  must equal  $f_2(\epsilon)$ , causing  $\mu_2$  to now be pushed up relative to  $\mu_1$  and creating an open-circuit voltage,  $V_{OC}$ , with the polarity as shown. Similar to the previous n-type case, a simple calculation yields the expression for  $V_{OC}$  to be as follows:

$$V_{OC} = \frac{\mu_1 - \mu_2}{q} = \left[ \frac{\epsilon - \mu_1}{qT_1} \right] [T_2 - T_1] = S\Delta T$$

where  $S$  is the Seebeck coefficient.

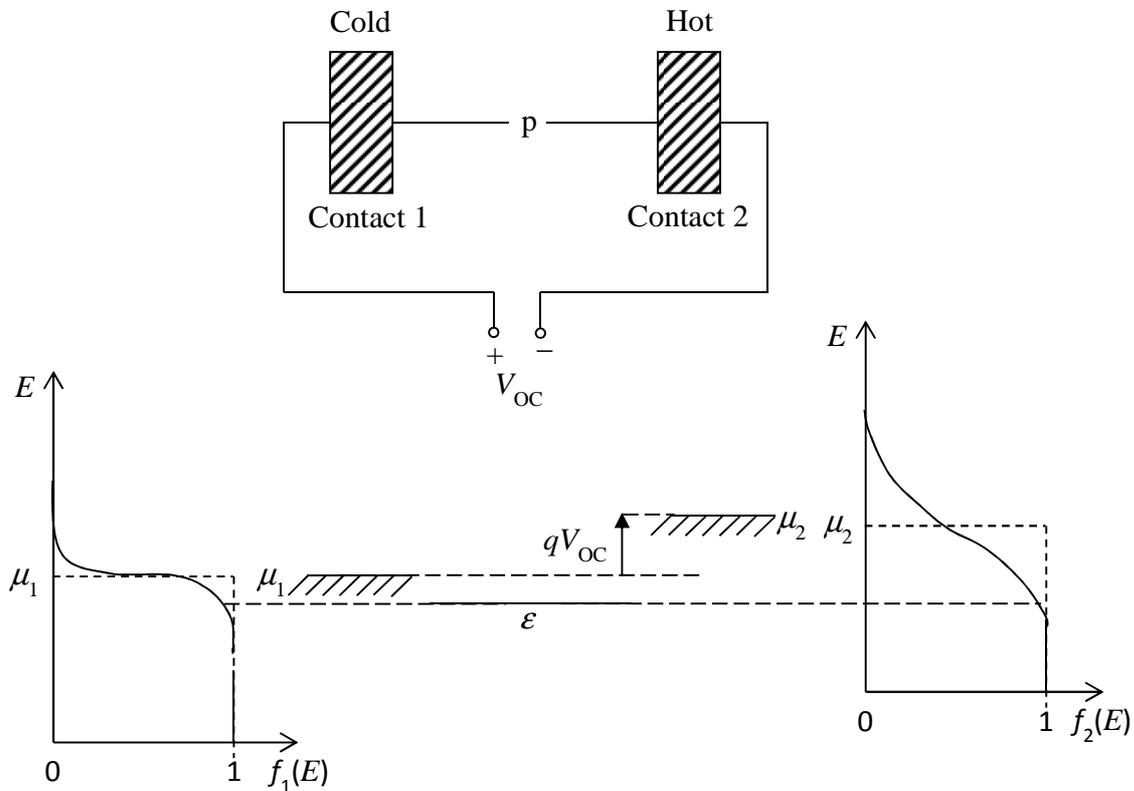


Figure 10. 1-D, 1-level elastic p-type nano resistor used as Seebeck device.

### Assessment

The initial assessment of this new approach at the University of Portland has been accomplished through two final exam questions, and student evaluations in our senior-level EE advanced analog electronics elective course in the fall 2015 semester. The two final exam questions covered the use of the 1-D, single energy level elastic nano resistor in order to analyze the Peltier effect and the Seebeck effect. The results were as follows. Eighteen students took the exam and achieved an average score of 89% on the first question (Peltier effect question), and 86% on the second question (Seebeck effect question). These excellent evaluations along with very positive student comments reveal that the students' understanding, interest, and enthusiasm for nanoelectronics and electronic conduction phenomena was greatly enhanced, making this "Bottom-Up" approach very effective in improving EE undergraduate students' fundamental knowledge of electronic conduction phenomena. Based on these initial assessment results, it is concluded that incorporating Purdue University's new "Bottom-Up" approach in our EE undergraduate curriculum is successful, and we plan to continue using it. The authors will continue to assess the effectiveness of this new approach in our senior-level EE analog electronics elective course each future fall semester, going forward.

## Conclusion

In this paper, we have shown how the University of Portland has successfully applied the “Bottom-Up” approach (using the simple 1-D “elastic” nano resistor) developed by Dr. Supriyo Datta at Purdue University<sup>1,2</sup> and the affiliated NCN-sponsored nanohub.org websites<sup>3,4</sup> to teach fundamental electronic conduction phenomenon, including simple Ohmic heating, the Peltier effect, and the Seebeck effect, to undergraduate EE students. This approach successfully made this complex theory easily accessible and understandable by first focusing the analysis at the nano-scale (“Bottom”) where there is an inherent *clean separation* between mechanics (occurring only inside the channel) and thermodynamics (occurring only at the contacts), and then extrapolating the results “Up” to the macro-scale where the mechanics and thermodynamics become “all mixed-up” and complicated within the conductor channel. The authors find this new approach to be elegant in its simplicity and profound in its richness of content. Based on our success, we encourage other undergraduate Electrical Engineering programs to consider incorporating this “Bottom-Up” approach as an introduction for teaching electronic conduction phenomena.

- [1] S. Datta, “Quantum Transport: Atom to Transistor”, Cambridge University Press (2005).
- [2] S. Datta, “Lessons from Nanoelectronics: A New Perspective in Transport”, World Scientific (2012).
- [3] <http://nanohub.org/resources/5346#series>, “ECE 495N: Fundamentals of Nanoelectronics”, Network for Computational Nanotechnology (NCN) Nanohub-U on-line course taught by Dr. S. Datta from Purdue University.
- [4] <http://nanohub.org/courses/FoN1>, “Fundamentals of Nanoelectronics, Part 1: Basic Concepts”, Network for Computational Nanotechnology (NCN) Nanohub-U on-line course taught by Dr. S. Datta from Purdue University.