Salute to a Once-A-Century Repunit Palindrome Day—11-11-11!
(By Aziz S. Inan, Ph.D., Electrical Engineering, University of Portland, in Portland, Oregon)
(11-11-11)

November 11th is an annual United States holiday called Veterans Day to remember and honor military veterans. It's also observed as Remembrance Day in Commonwealth countries to remember the members of their armed forces who died in line of duty since World War I. This day commemorates the end of World War I, at the 11th hour of the 11th day of the 11th month of 1918!

November 11 also happens to be my mom's birthday. She will turn 83 this year where 8 + 3 = 11! (As an aside, she survived a major earthquake when she was 11.)

November 11 this year (2011) to occur this coming Friday also has a special numerical property. If expressed as 11-11-11 (or simply, 111111), it is a very special once-a-century repunit* palindrome day! In recognition of this calendar day, I investigated its numerical properties further as well as other such palindrome dates and here is what I found out:

- Note that calendar day 111111 repeats once every century on November 11th in both date formats (month-day-year and day-month-year).
- Last time 111111 occurred was in 1911 on Saturday, November 11, 1911. As far as which day of the week 111111 occur, from one century to the next, 111111 shifts backwards by either one (when the last year of the previous century is a leap year) or two days (when it's not). This year (2011), 111111 will occur on Friday since year 2000 was a leap year!
- If 111111 is expressed in terms of its prime factors as 111111 = 37 x 7 x 3 x 11 x 13, the leftmost and the rightmost three numbers put side-by-side without the multiplication signs each constitute a palindrome number (3773 and 31113). In addition, 111111 equals the product of two palindrome numbers written as 111111 = 111 x 1001 and this expression can be presented in a palindrome form as:

- This year marks the 93rd anniversary of both Veterans and Remembrance Days where 93 = 31 x 3 where prime factors 31 and 3 put side-by-side without the multiplication sign forms 313, a palindrome number!
- 111111 can be obtained from its largest prime factor 37 as follows: First, subtract 37 from its reverse which yields 36. (Note that 36 equal the square of the sum of the digits of 111111. Also, in binary number system, 111111 correspond to 63, which is the reverse of 36!) Then, split 36 into three consecutive numbers 11, 12 and 13 which add up to 36. Then, multiply numbers 11, 13, 37 and the reverse of 12! What comes out? Isn't this fun? (By the way, prime factor 37 is also special because the leftmost and rightmost two digits of 1918 when World War I ended add up to 37! Also, the sum of the prime factors of 1918 (which are 2, 7 and 137) is 146 which equal twice the reverse of 37.)
- In a non-leap year such as 2011, November 11th is the 315th day of the year where the digits of 315 given by 1, 3 and 5 are three consecutive odd numbers. In addition, 315 = 5 x 7 x 9 where 5, 7 and 9 are also three consecutive odd numbers. Also, reverse of 315 is 513 which equal 513 = 3

x 3 x 3 x 19 = 3 x 171 = 9 x 57 where its prime factors 3 and 19 add up to 22, a palindrome number. (Note that 22 = 11 + 11 which leads to 1111 without the in-between plus sign and 1111 corresponds to November 11!) Also, 9 + 57 = 66!

- Assigning letters in the English alphabet numbers as A = 1, B = 2,…, Z = 26, the letters of "FRIDAY" and "ELEVEN" each add up to 63 which equals binary 111111! Also, 63 can be expressed as 3 x 7 x 3, a palindrome expression! Note also that 5 x 63 = 315 (i.e., the 315th day)!

- Also, the full date number of this Friday is 11-11-2011, or simply 11112011. If this number is split in the middle into numbers 1111 and 2011, these two numbers add up to 3122 = 2 x 7 x 223 where the sum of these three prime numbers equals 232, a palindrome number! Amazingly, the full date number of last week's one-of-a-kind palindrome day November 2nd, 2011 expressed as 11022011 equals 7^2 x 11^3 x 13^2 where the three superscript numbers put side-by-side constitutes the very same palindrome number, 232!

- Also, reverse of 11112011 is 11021111 which equal 1753 x 6287 where these two prime factors add up to 8040 = 2 x 2 x 2 x 3 x 5 x 67 where the sum of these prime factors equals 77! (Also, interestingly enough, by removing the prime factors 3 and 5 in the above product and replacing 2 x 2 x 2 x 67 by 2 x 2267 yields 4534, which corresponds to the difference of the prime factors 1753 and 6287!) Isn't that something?

- There are 11 such palindrome days in this month including 11-11-11. The other ten are 11-1-11 (which is a repunit day), 11-2-11 (also a special one-of-a-kind palindrome day when written in full date format as 11-02-2011), 11-3-11, 11-4-11, 11-5-11, 11-6-11, 11-7-11, 11-8-11, 11-9-11, and 11-22-11. (Note that 11-1-11 can also be interpreted as 1-11-11 representing January 11, 2011. Also, a four-digit repunit day that occurred at the beginning of this year was January 1, 2011 represented by 1-1-11. In addition, January 10, 2011 written in full date format as 1-10-2011 was a palindrome day.) Among them, 112211 is an interesting one because it equals 101 x 11 x 101, a palindrome expression! Also, 11211 = 111 x 101, 11611 = 17 x 683 where 683 – 17 = 666, and 11811 = 3 x 31 x 127 where 3 + 31 + 127 = 161.

- After this Friday, the next time 11-11-11 will occur 100 years from now in a palindrome (22nd) century in 2111, on Wednesday (since 2100 is not a leap year). Interestingly enough, in 2111, 11-11-11 and eight-digit palindrome day 11-12-2111 will be two concurrent palindrome days! Also, the full date number of 11-11-11 in 2111 will be 11-11-2111 involving seven 1's (in fact, one could ignore the thousand digit of the year number and express this date as 11-11-111)!

- 11-11-11 in the 23rd century will also be interesting because its full date number is 11-11-2211 where the leftmost two digits of the year number is 22 = 11 + 11. How about writing this full date number as 11-11-(11+11)11? ☺

- Another interesting full palindrome day to occur in the 23rd century 11 days after November 11 is 11-22-2211 or simply 11222211 which equals 101 times this Friday's date, that is, 101 x 111111!

Happy 93rd Veterans and Remembrance Days, and happy 83rd birthday, Mom!

*In recreational mathematics, a repunit is a number like 11, 111, 1111 that contains only the digit 1. The term stands for repeated unit and was coined in 1966 by Albert H. Beiler.