A numerical milestone, no foolin'

Aziz Inan
Guest Commentary

This year's April Fools' Day is one of a kind, and believe it or not, this is not an April Fools' prank! April 1, 2009, when expressed as the full date number 4012009, is a “perfect square date” because it equals square of 203.

A perfect square date is defined as a full date number given by MMDDYYYY (an eight-digit number where the first two digits are the month, the next two are the day and the last four are the year numbers) that is a perfect square. For example, Sept. 1, 2004 is a perfect square date since 9012004 equals square of 3002. Indeed, this date is the first perfect square date of the 21st century, which largely went unnoticed. The second and the third perfect square dates of this century both fall in this year.

The second occurred less than a month ago on March 5, 2009 (since 3052009 equals the square of 1747) and the third one is April Fools' Day.

It is very special indeed for a perfect square April Fools' date to occur in one's lifetime, since the next one will occur 4007 years from now, in 6016 (4016016 equals the square of 2004). Also, before this year's April Fools' Day, the first perfect square April 1st day occurred 1053 years ago in 956 (40956 is square of 634) long before the April Fools' tradition began. (Note that a date in a three-digit year is represented by a seven- instead of an eight-digit number).

Perfect square dates are generally very rare and don't occur every year. In fact, there were only a total of two perfect square dates in the 18th century, two in the 19th century and four in the 20th century.

Perfect square dates can only occur in years ending with double zeros, 1, 4, 6, 9 or 25. Additionally, if the year ends with 00, the preceding digits of the year must be a square number; if it ends with 1, the preceding digits must be divisible by 4; if it ends with 4, the preceding digit must be even; if it ends with 5, the preceding digits must be 0, 2, 06, or 56; if it ends with 6, the preceding digit must be an odd number; and if it ends with 9, the preceding digits must be divisible by 4. Wow!

Among all the centuries involving four-digit years between years 1000 and 9999, the 21st century has the highest number of perfect square dates with a total of twenty-four.

After April Fools' Day, the fourth and the fifth perfect square dates of this century will occur in year 2016, on February 26 and December 22 (since 2262016 and 1222016 are squares of 1504 and 3496). Then, the next five perfect square dates will be in the year 2025, on Jan. 9, March 22, April 18, Sept. 27, and Oct. 27 respectively. After 2025, three more perfect square dates will occur on Jan. 1, Feb. 23, and Dec. 29 in 2036, one on Feb. 4 in 2041, one on Sept. 4 in 2049, one on April 3 in 2064, one on May 2 in 2081, two on Jan. 16 and Feb. 2 in 2084, two on February 15 and May 21 in 2089, two on Jan. 13 and Feb. 6 in 2096, and one on Jan. 23 in 2100.

In this millennium, after the 21st century, there will be eight more perfect square dates in the 22nd century, 14 in the 23rd, nine in the 24th, 19 in the 25th, three in the 26th, six in the 27th, five in the 28th, 10 in the 29th, and only two in the 30th. Year 2500 in the 25th century contains nine perfect square dates and holds the record number of perfect square dates among all the years in this millennium.

There are a total of 68 perfect square dates in the second millennium, 100 in the current (third) millennium, 68 in the fourth, 79 in the fifth, 99 in the sixth, 88 in the seventh, 71 in the eighth, 82 in the ninth, and 81 in the tenth including the ones in the year 10000. Among all the four-digit years between year 1000 and 9999, year 5625 has the highest record with a total of ten perfect square dates followed by year 2500. Also, interestingly enough, there are two consecutive perfect square dates to occur in year 5001 on June 24 and 25, since 6244001 and 6255001 are squares of 2499 and 2501.

The existence of perfect square dates is indeed a fascinating story to share with friends and colleagues especially on this year's April Fools' Day which itself is a perfect square date. It is a story that many people will first listen with a grain of salt, but once they are convinced about the fact that this is a true story and not an April Fools' prank, they will be captivated by it and hopefully remember this special day for the rest of their lives.

Dr. Inan is a professor of electrical engineering at the University of Portland.