12/1/2003

University ☺f P☺rtland
Sch☺☺l ☺f Engineering

EE 261-Electrical Circuits-3 cr. hrs.
Fall 2003

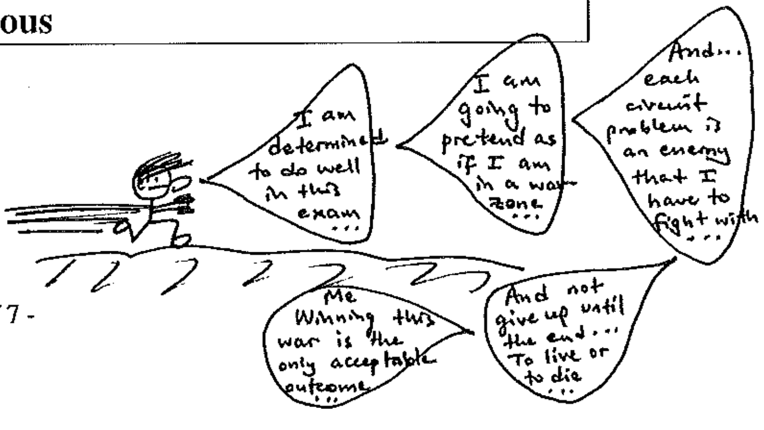
Midterm Exam # 3

(Monday, November 24, 2003, 1:35p.m.)
(Closed Book Exam, Three Formula Sheets are Allowed)
(Total Time: 55 minutes)

Name: S☺LUTI☺NS! ☺

Signature: ☺

> ***"An honest mind possesses a kingdom."***
> **Lucius Annaeus Seneca (4B.C.–65A.D.)**
>
> ***"Honest people are the true winners of the universe."***
> **Anonymous**



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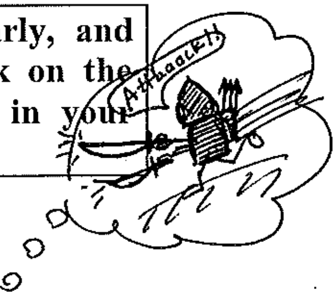
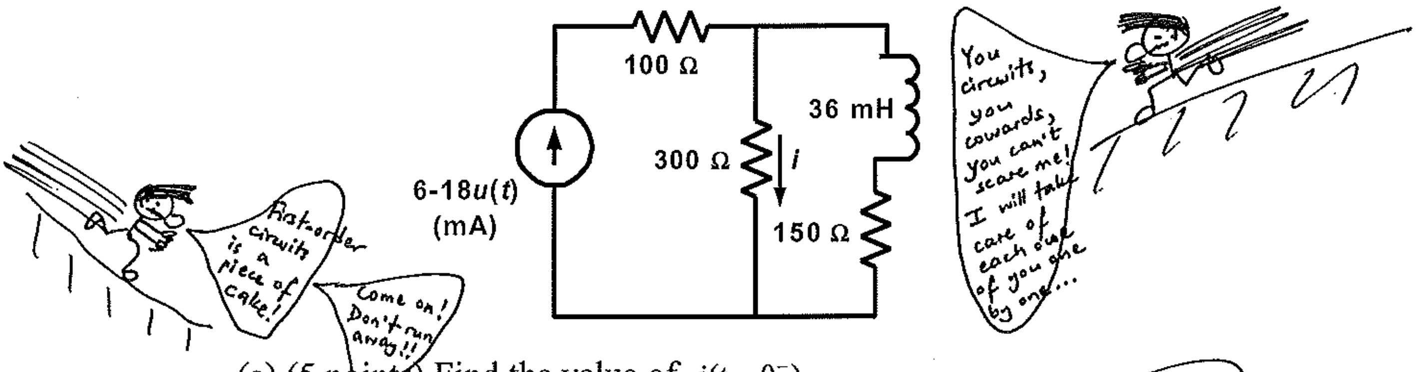
NOTE: On all the problems, please show your work clearly, and provide the appropriate units for your answers. Also mark on the schematic to show any current or voltage that you define in your solution.

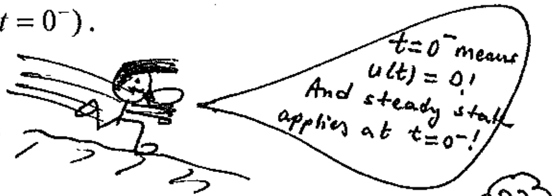


1. (<u>Total: 30 Points</u>) Consider the circuit shown below.



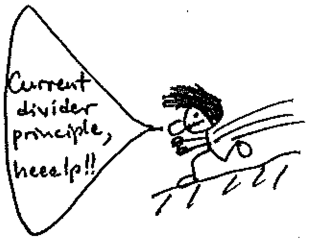
(a) (5 points) Find the value of $i(t = 0^-)$.

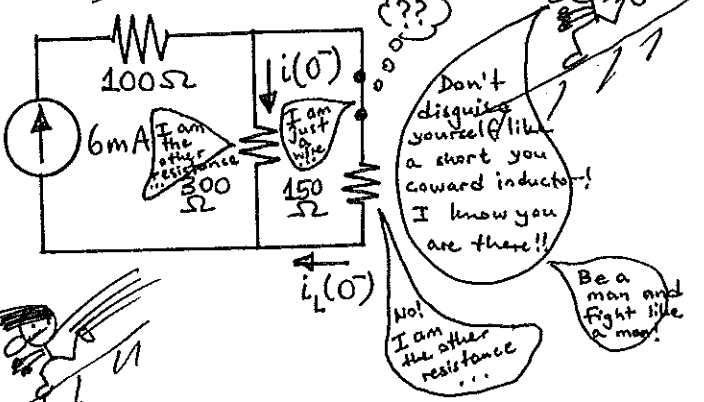
At $t = 0^-$ (steady state):



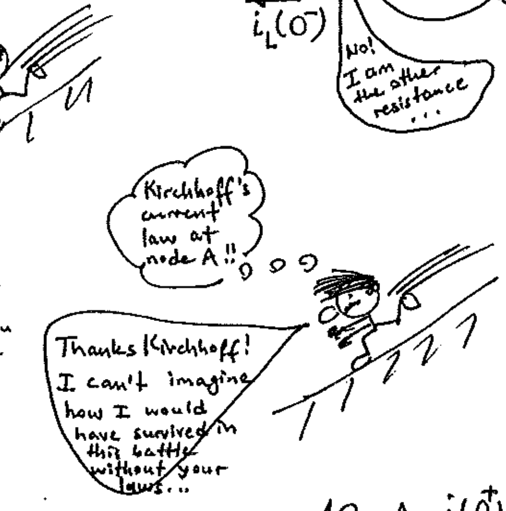
$$i(0^-) = \frac{150}{450}(6\text{mA}) = \boxed{2\text{mA}}$$

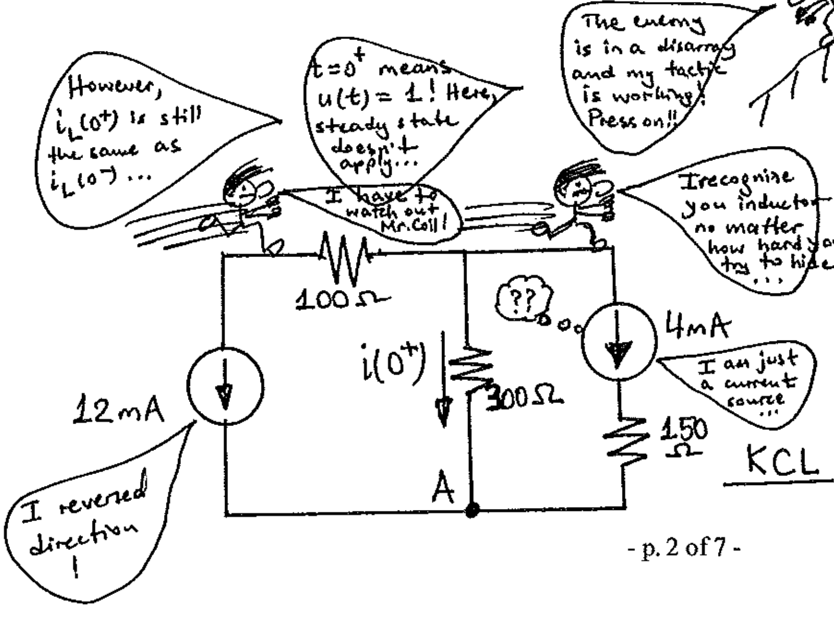
$$i_L(0^-) = \frac{300}{450}(6\text{mA}) = 4\text{mA}$$





(b) (5 points) Find the value of $i(t = 0^+)$.





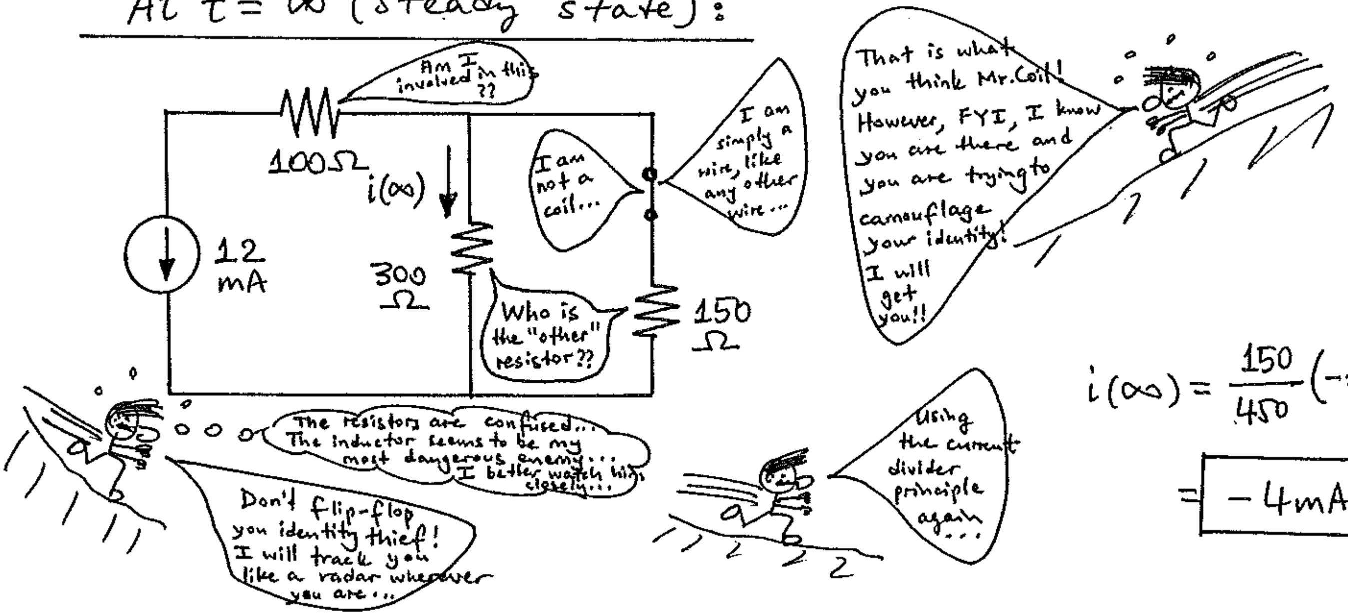
KCL at node A → $12\text{mA} + i(0^+) + 4\text{mA} = 0$

$$\therefore i(0^+) = \boxed{-16\text{mA}}$$

12/1/2003

(c) (5 points) Find the value of $i(t=\infty)$.

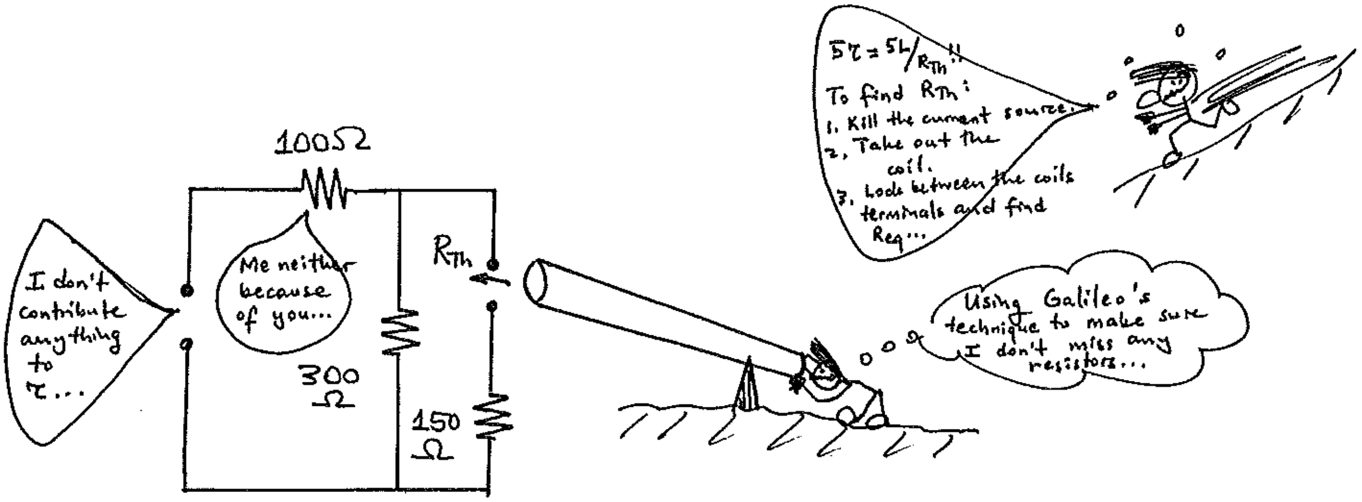
At $t = \infty$ (steady state):



$$i(\infty) = \frac{150}{450}(-12mA)$$

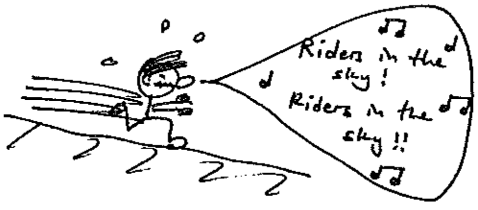
$$= \boxed{-4mA}$$

(d) (5 points) Find the approximate time it will take the circuit to reach steady state.



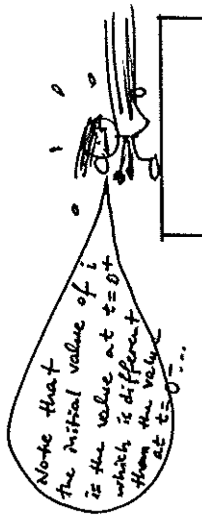
$$\tau = \frac{L}{R_{Th}} = \frac{36\,mH}{(300+150)\,\Omega} = 80\,\mu s$$

$$\therefore\ 5\tau = \boxed{400\,\mu s}$$



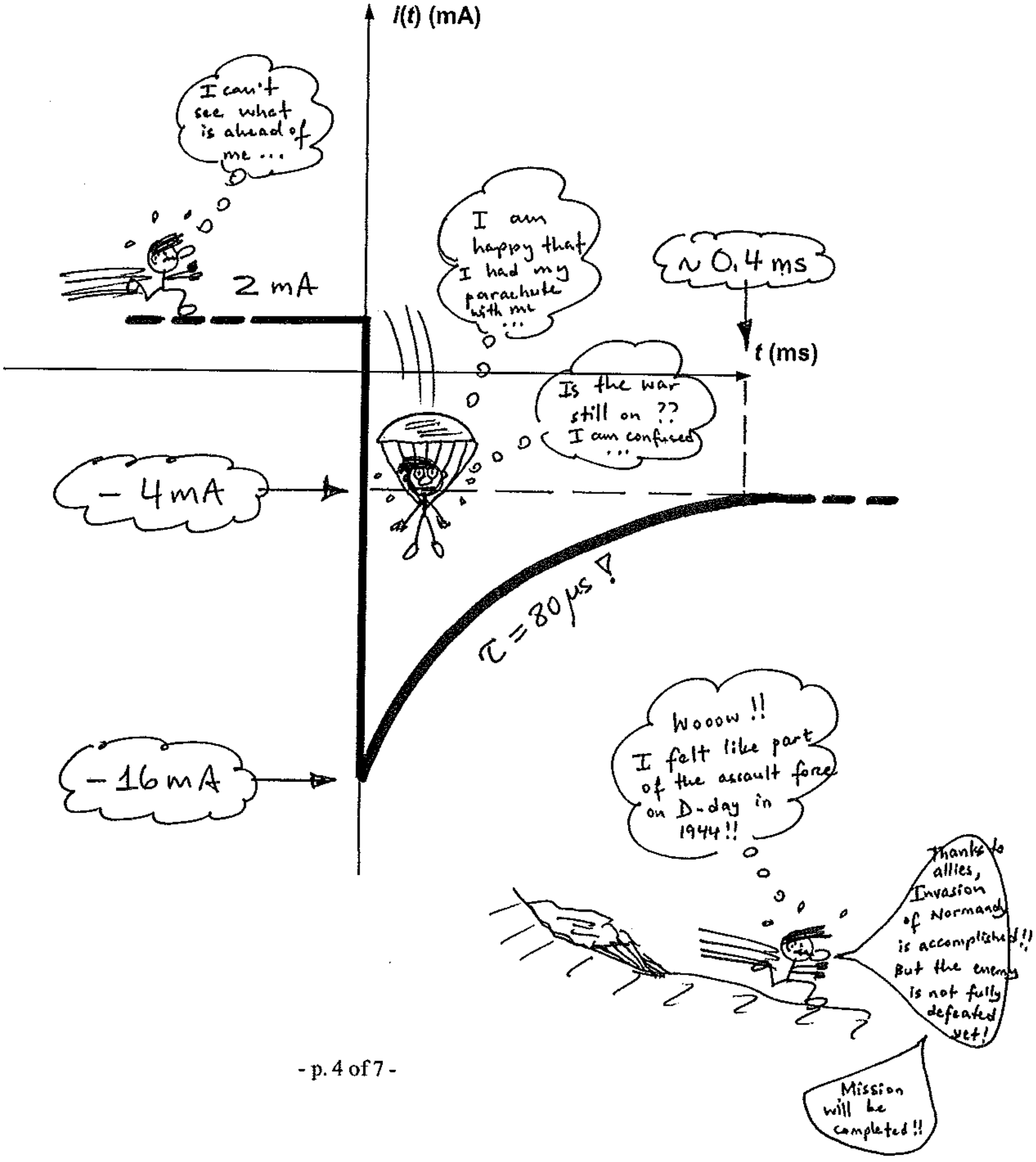
12/1/2003

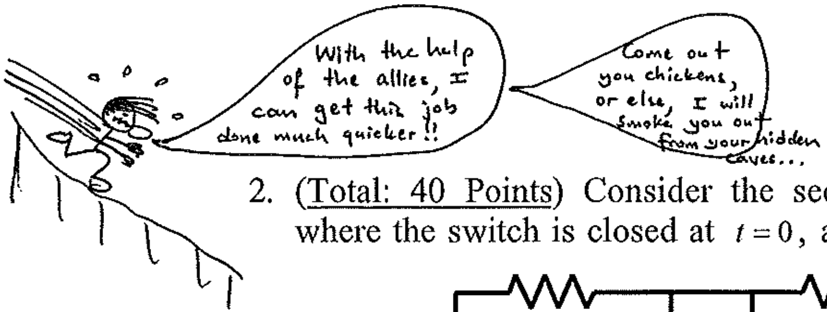
(e) (10 points) Write the complete mathematical expression for $i(t)$ and sketch $i(t)$ over the time interval $-1\,\text{ms} \le t \le 1\,\text{ms}$. Provide all the appropriate time and current values on your sketch.



Note that the initial value of i is the value at t=0+ which is different than the value at t=0−...

$$\boxed{\begin{aligned} i(t) &= -16e^{-12{,}500t} - 4(1-e^{-12{,}500t}) \\ &= -4 - 12e^{-12{,}500t}, \quad \text{for } t > 0 \end{aligned}}$$





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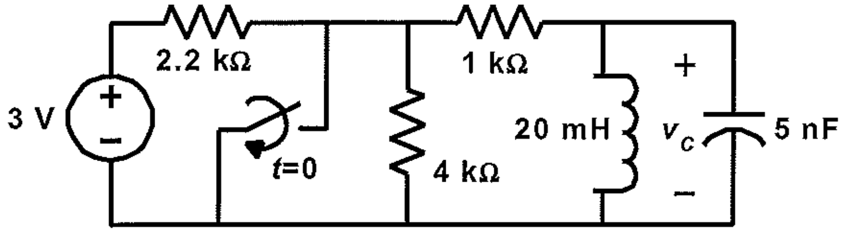
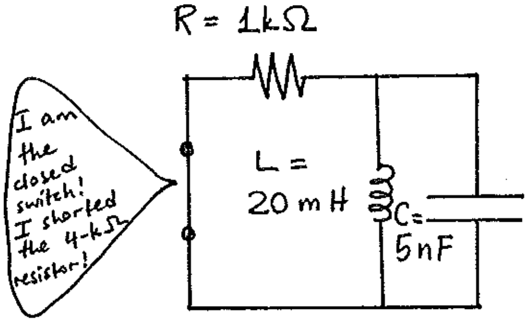
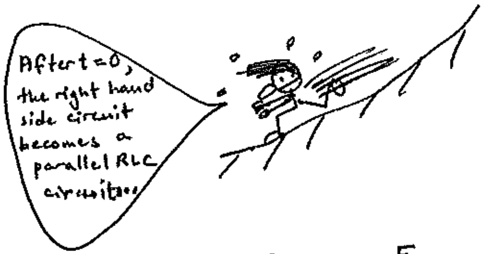
2. (Total: 40 Points) Consider the second-order circuit shown below where the switch is closed at $t=0$, after being open for a long time.



(a) (15 points) Find the roots of the characteristic equation of the circuit and determine the type (over-damped, critically-damped, or under-damped?) of the natural response $v_C(t)$ across the 5-nF capacitor for $t \geq 0$.





Characteristic equation is

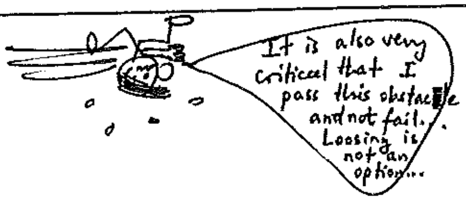
$$s^2+\frac{1}{RC}s+\frac{1}{LC}=0 \rightarrow s^2+2\times10^5 s+10^{10}=0$$

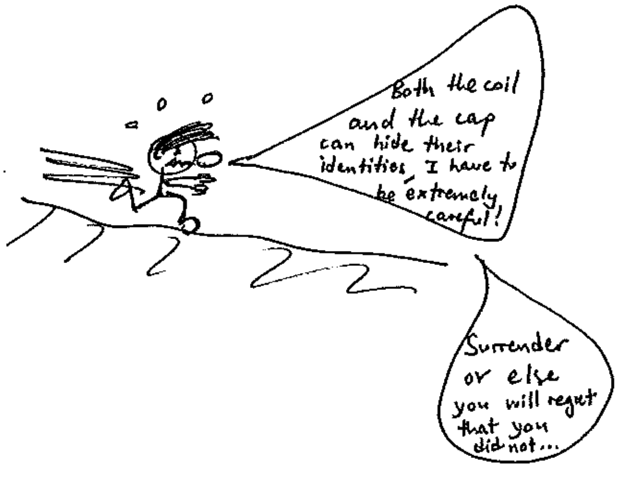
$$\rightarrow (s+10^5)^2=0 \rightarrow \boxed{\therefore s_1, s_2 = -10^5}$$

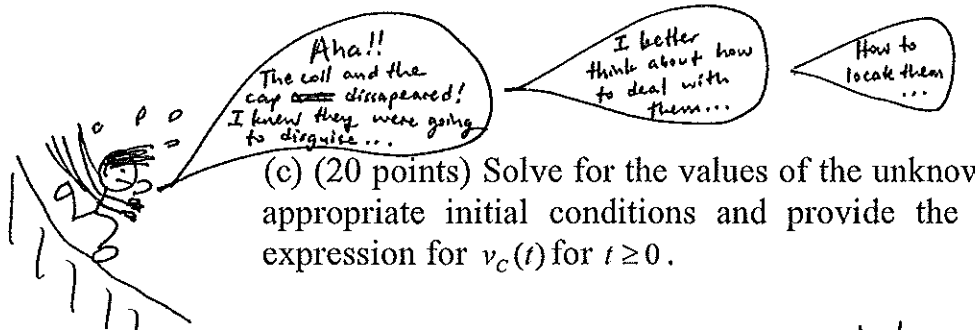
$\boxed{\therefore \text{Critically-damped response.}}$

(b) (5 points) Based on part (a) results, write the general mathematical expression for the voltage $v_C(t)$ valid for $t \geq 0$.

$$\boxed{v_C(t)=(K_1+K_2t)\,e^{-10^5t} \quad \text{valid for } t \geqslant 0.}$$



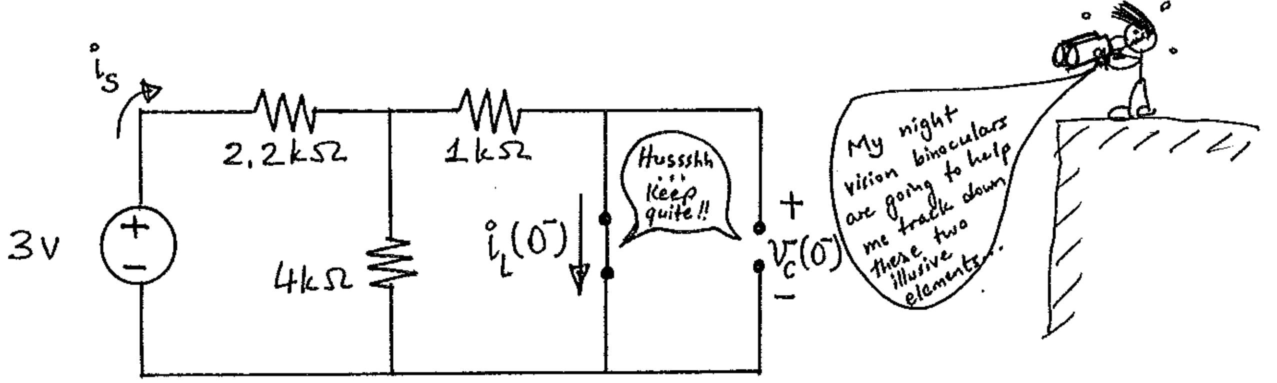




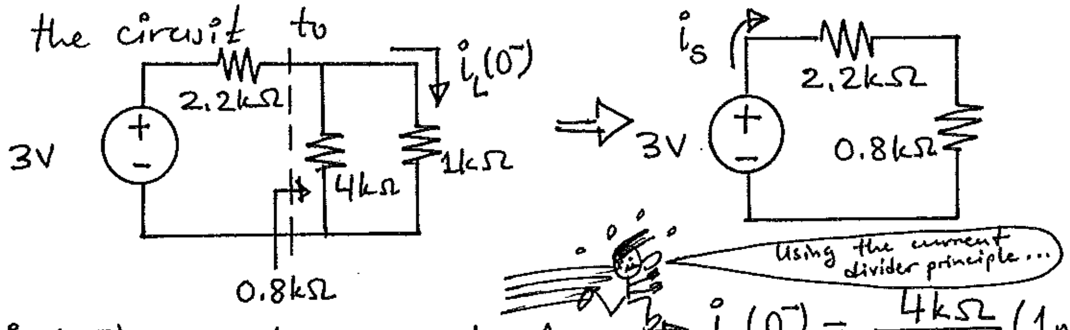
12/1/2003

(c) (20 points) Solve for the values of the unknown coefficients using the appropriate initial conditions and provide the complete mathematical expression for $v_C(t)$ for $t \geq 0$.

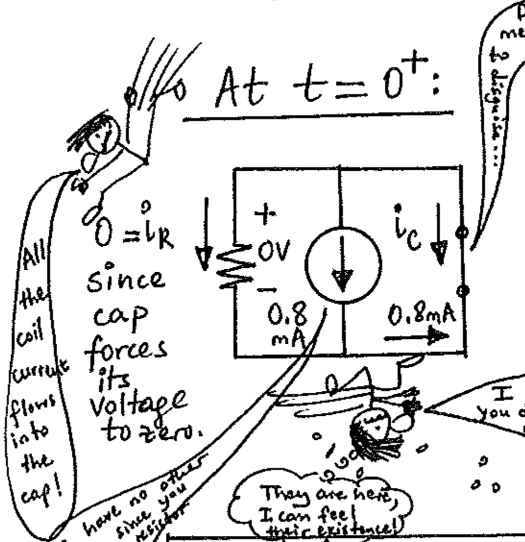
To find the initial condition, we go back in time to the $t = 0^-$ circuit (steady state applies):



Obviously, $v_c(0^-) = \underline{\underline{0}}$. To obtain $i_L(0^-)$, we simplify the circuit to



$i_s(0^-) = 3V/3k\Omega = 1mA \longrightarrow i_L(0^-) = \frac{4k\Omega}{5k\Omega}(1mA) = \underline{\underline{0.8mA}}$



$0 = i_R$ since cap forces its voltage to zero.

At $t = 0^+$:

$v_C(0^+) = v_C(0^-) = 0.$

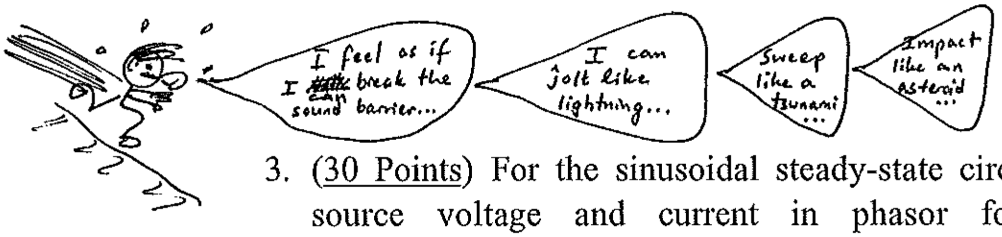
$i_L(0^+) = i_L(0^-) = 0.8mA.$

$\therefore v_C(0) = \boxed{K_1 = 0}$

$$\text{Also, } i_c(t)\Big|_{0^+} = C\frac{dv_c(t)}{dt}\Big|_{0^+} = (5n)\left[\cancelto{0}{-10^5 K_1} + K_2\right] = -0.8\text{ mA}$$

$\therefore K_2 = \frac{-0.8m}{5n} = \boxed{-160{,}000}$

$$\boxed{\therefore v_C(t) = -160{,}000\, e^{-10^5 t} \text{ valid for } t \geq 0.}$$



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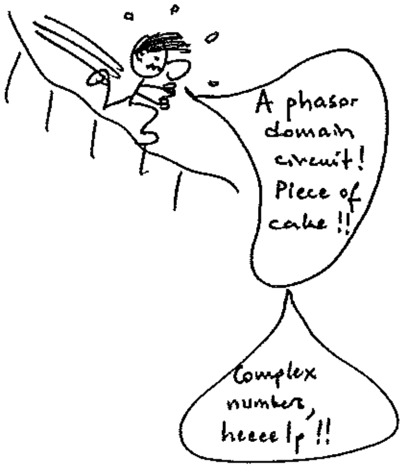
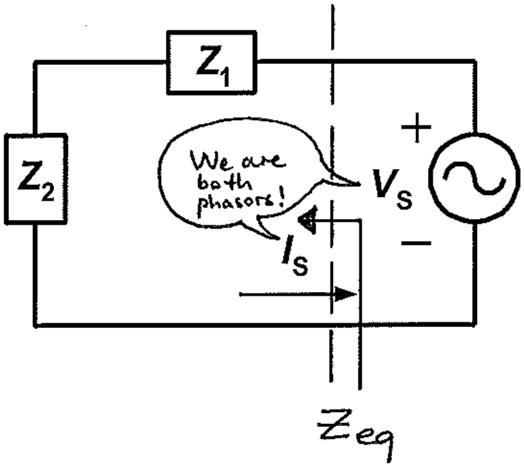
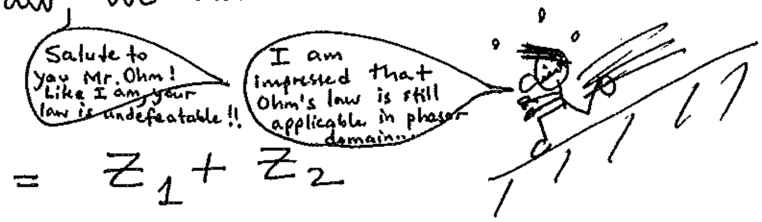
3. (30 Points) For the sinusoidal steady-state circuit shown, given the source voltage and current in phasor form as $V_S = 4.5e^{j0}$ V, $I_S = 0.03e^{-j53.13^\circ}$ A, and given the impedance $Z_1 = (50 - j50)\Omega$, find the value of the impedance Z_2.





Using Ohm's law, we have



$$\frac{V_s}{I_s} = Z_{eq} = Z_1 + Z_2$$

Substituting the known values,

$$\frac{4.5\,e^{\hat{j}0}}{0.03\,e^{-\hat{j}53.13^\circ}} = \underbrace{150\,e^{\hat{j}53.13^\circ}}_{90+\hat{j}120} = 90+\hat{j}120 = 50-\hat{j}50+Z_2$$

∴ Solving for Z_2, we find

$$\boxed{Z_2 = 40 + \hat{j}\,170\,\Omega}$$