12-3-2004

University of Portland
School of Engineering

EE 261-Electrical Circuits-3 cr. hrs.
Fall 2004

SOLUTIONS TO

Midterm Exam # 3

(Wednesday, December 1, 2004)
(Closed Book Exam, Three Formula Sheet are Allowed)
(Total Time: 55 minutes)

Name: Solutions! ☺

Signature: Solutions ☺

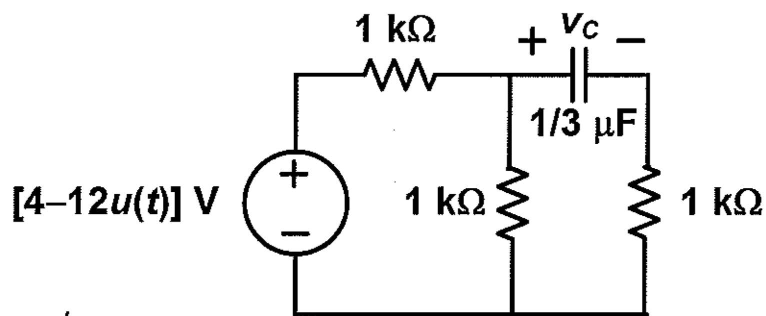
"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

"Honest people are the true winners of the universe."
Anonymous

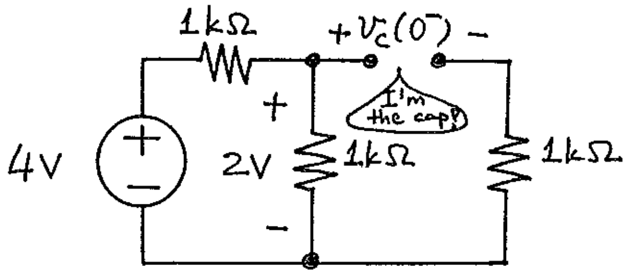
12 - 3 - 2004

NOTE: On all the problems, please show your work clearly, and provide the appropriate units for your answers. Also mark on the schematic to show any current or voltage that you define in your solution.

1. (15 mins., 30 points) In the circuit shown, find the complete mathematical expression for the voltage $v_C(t)$ across the 1/3 μF capacitor for $t \geq 0$. (Please show your work clearly and provide brief justifications for the steps you take. Also, don't forget to provide the correct units for your answers.)



At $t = 0^-$ (steady state):

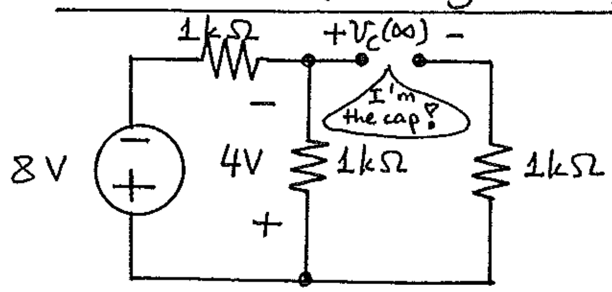


$$v_C(0^-) = 2\text{V}$$

$$\therefore v_C(0^+) = v_C(0^-) = 2\text{V}$$

Initial value!

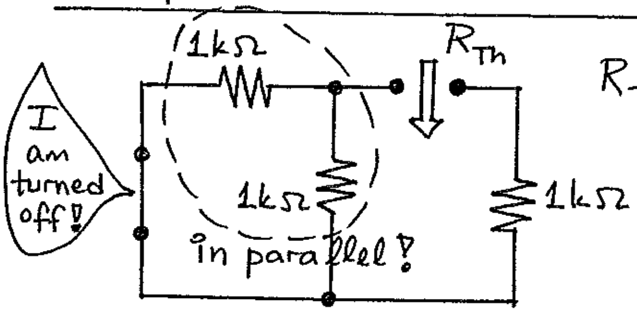
At $t = \infty$ (steady state):



$$v_C(\infty) = -4\text{V}$$

Final value!

To find the time constant:



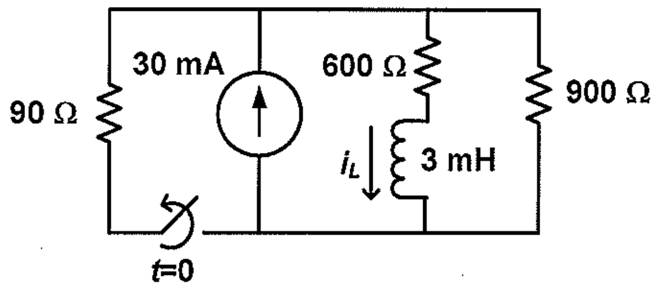
$$R_{Th} = (1k\Omega \,//\, 1k\Omega) + 1\,k\Omega = 1.5\,k\Omega$$

$$\therefore \tau = R_{Th}C = (1.5k)\left(\tfrac{1}{3}\mu\right) = 0.5\text{ms}$$

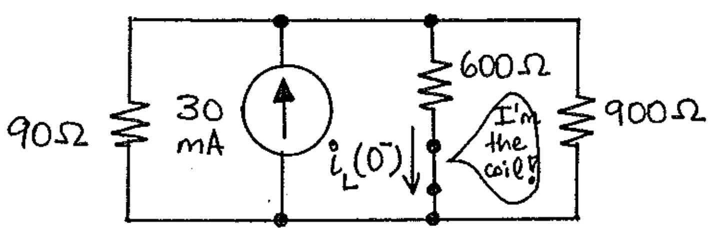
Time constant!

$$\boxed{\begin{aligned} \therefore v_C(t) &= 2e^{-2{,}000t} - 4\left(1 - e^{-2{,}000t}\right) \\ &= -4 + 6e^{-2{,}000t}\ \text{(V)}, \text{ valid for } t \geq 0 \end{aligned}}$$

2. (15 mins., 30 Points) In the circuit shown below, the switch <u>opens</u> at t=0, after being closed for a long time. Find the complete mathematical expression for the current $i_L(t)$ through the 3 mH inductor for $t \geq 0$. (Please show your work step by step. Also, again, provide appropriate units.)



At $t = 0^-$ (steady state):

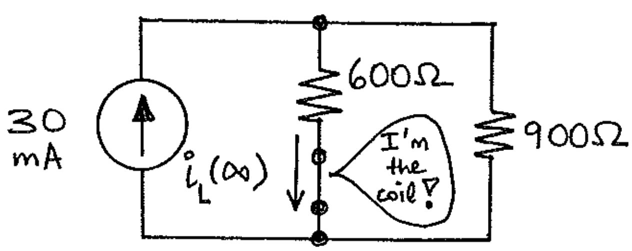


$$\therefore i_L(0^-) = \frac{(90 // 900)}{(90 // 900) + 600}(30\text{mA})$$

$$\cong \frac{81.82}{681.82}(30\text{mA})$$

$$= \underline{\underline{3.6 \text{ mA}}}$$

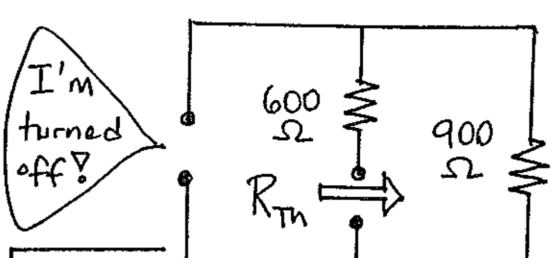
At $t = \infty$ (steady state):



$$\therefore i_L(\infty) = \frac{900}{900 + 600}(30\text{mA})$$

$$= \underline{\underline{18 \text{ mA}}}$$

To find the time constant:



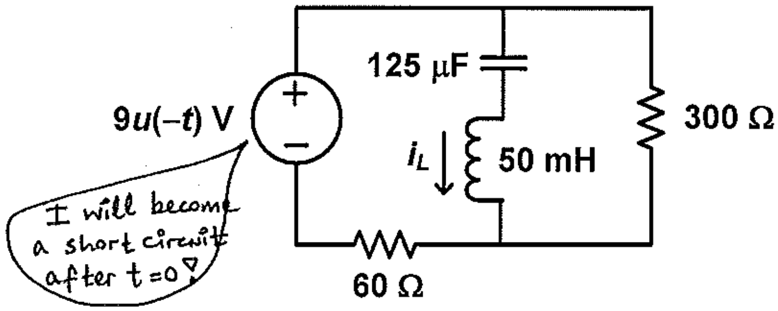
$$\therefore R_{Th} = 600 + 900 = 1{,}500\,\Omega$$

$$\therefore \tau = L/R_{Th} = (3\text{m})/(1.5\text{k}) = \underline{\underline{2\,\mu\text{s}}}$$

$$\boxed{\therefore i_L(t) = 3.6\,e^{-5\times10^5 t} + 18\left(1 - e^{-5\times10^5 t}\right) = 18 - 14.4\,e^{-5\times10^5 t}\ (\text{mA}), \text{ valid for } t \geq 0}$$

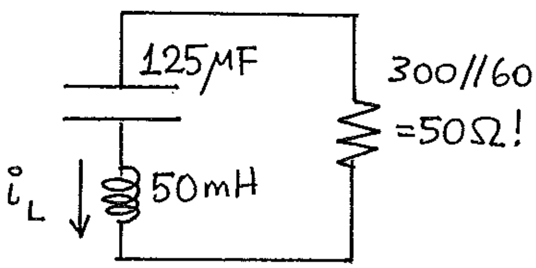
12-3-2004

3. (20 mins., <u>Total: 40 Points</u>) Consider the circuit shown.



(a) (15 points) Solve for the roots (s_1 and s_2) of the characteristic equation of the above circuit for $t \geq 0$.

After $t=0$:



The characteristic equation is given by

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \rightarrow s^2 + \frac{50}{50m} + \frac{1}{(50m)(125\mu)} = 0$$

$$\rightarrow \boxed{s^2 + 1{,}000\,s + 160{,}000 = 0}$$

∴ The characteristic roots are

$$\boxed{s_1, s_2 = -200, -800}$$

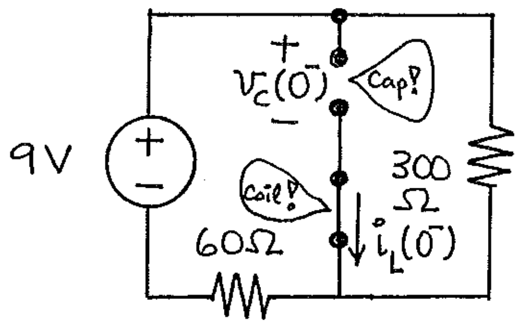
(b) (10 points) Based on the results of part (a), write the general mathematical expression for the inductor current $i_L(t)$ for $t \geq 0$. (At this stage, you leave the coefficients in your expression as unknown variables.)

Since s_1 and s_2 roots are both real & $s_1 \neq s_2$, this second-order circuit will have an overdamped response. Therefore, the inductor current can be written as

$$\boxed{i_L(t) = A_1 e^{-200t} + A_2 e^{-800t}}$$

(c) (15 points) Find the values of the coefficients of the $i_L(t)$ expression found in part (b) using the initial conditions.

To obtain the initial conditions $v_C(0^+)$ & $i_L(0^+)$ (which we need to solve the coefficients A_1 & A_2), we focus our attention to the circuit at $t=0^-$:



$$i_L(0^-) = 0$$

$$v_C(0^-) = \frac{300}{300+60}(9V) = 7.5V$$

$$\therefore \; v_C(0^+) = v_C(0^-) = 7.5V \quad \& \quad i_L(0^+) = i_L(0^-) = 0.$$

Next, using these initial conditions, we have

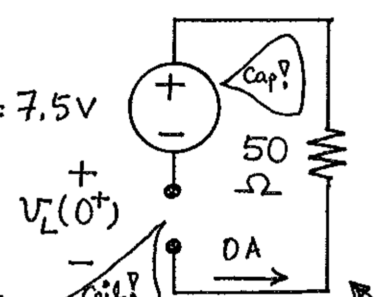
$$i_L(0) = A_1 \cancelto{1}{e^{-200t}} + A_2 \cancelto{1}{e^{-800t}} = \boxed{A_1 + A_2 = 0} \leftarrow (I)$$

$$v_L(t) = L\frac{di_L(t)}{dt} = (50m)\left[-200A_1 e^{-200t} - 800A_2 e^{-800t}\right]$$

$$v_L(0^+) = \boxed{-200A_1 - 800A_2 = -7.5}$$

↑ (II)

$v_C(0^+) = 7.5V$



$t=0^+$ circuit

Solving I & II simultaneously yields

$$A_1 = -0.25 \quad \& \quad A_2 = 0.25$$

$$\boxed{\therefore \; i_L(t) = -0.25e^{-200t} + 0.25e^{-800t} \quad \text{(A)} \quad \text{valid for } t \geq 0}$$