EE 261
FALL 2011
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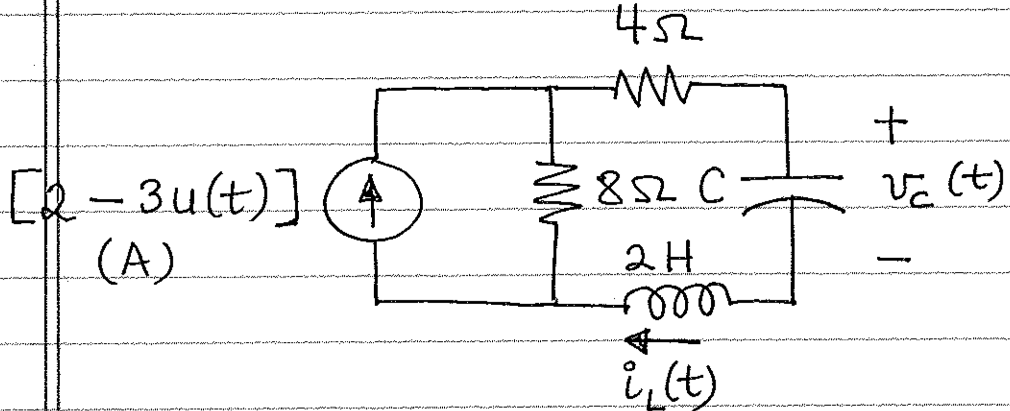
SECOND-ORDER RLC CIRCUIT
STEP EXCITATION PRACTICE PROBLEM

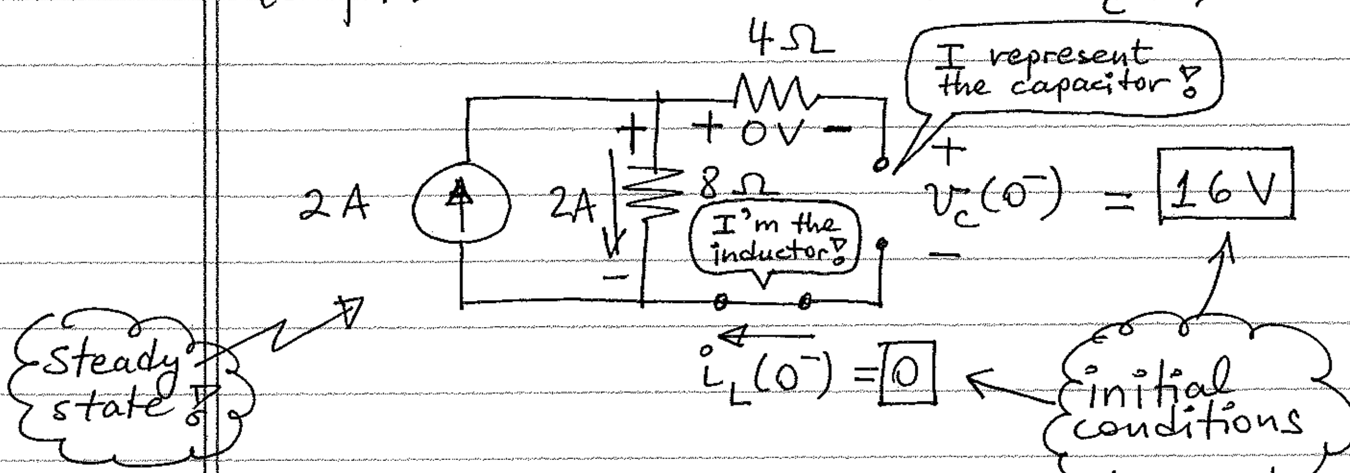
P#1 (five pages)



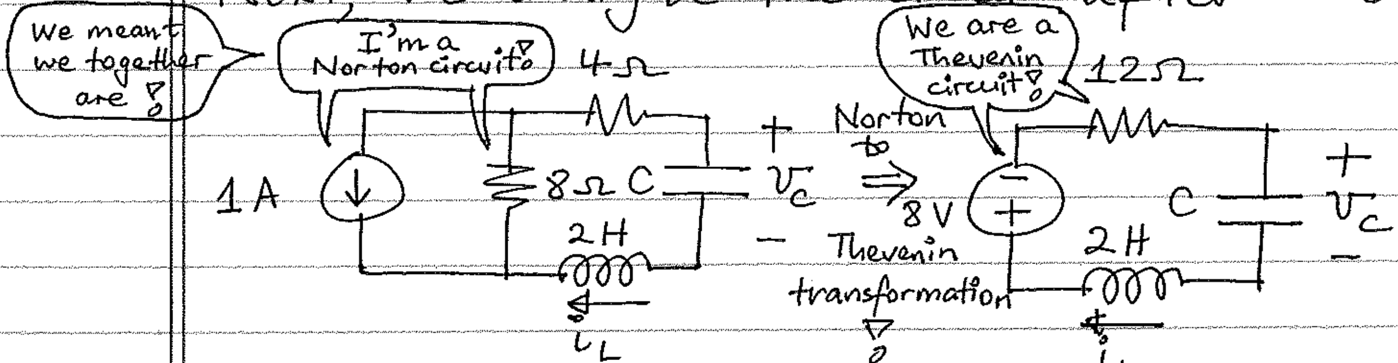
Find $v_C(t)$ for $t \geq 0$ for three cases;

(a) $C = 0.1\,F$ (b) $C = 0.05\,F$ (c) $C = 1/18\,F$

Solution: We start with the $t=0^-$ circuit to find the initial conditions $v_C(0^-)$ and $i_L(0^-)$:



Next, we analyze the circuit after $t=0$:



Since this is a second-order series RLC circuit, its characteristic equation is given by

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \quad \leftarrow \text{(From table)}$$

(a) Substituting $R = 12\,\Omega$, $L = 2\,H$ and $C = 0.1\,F$:

$$s^2 + 6s + 5 = 0 \rightarrow s_1, s_2 = -1, -5$$

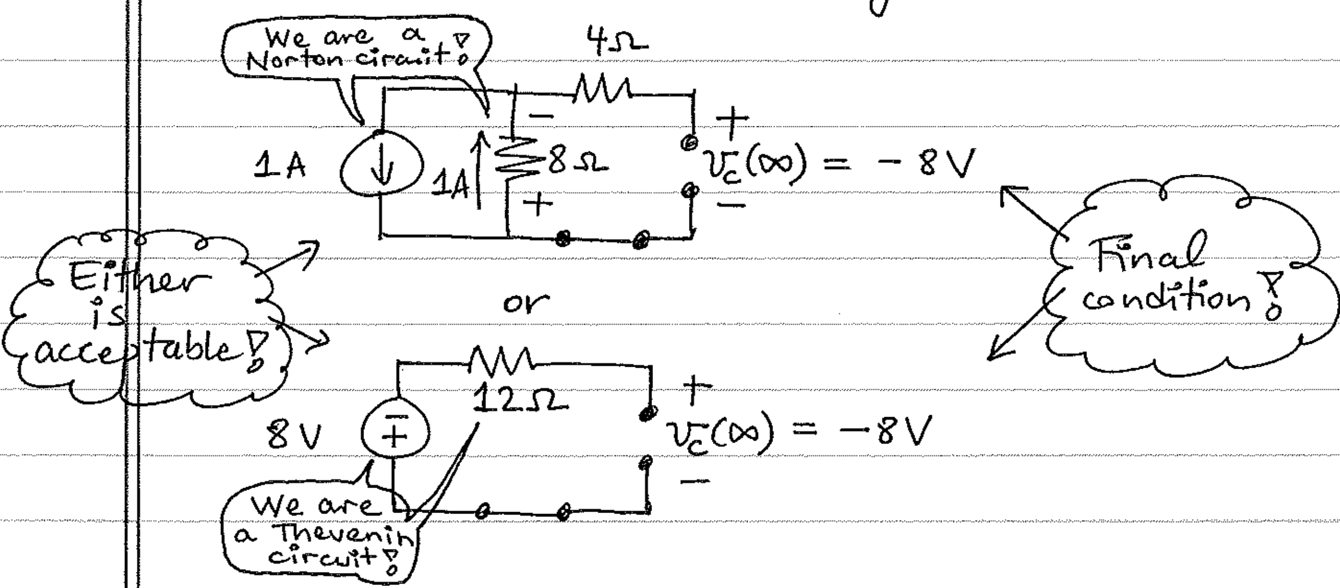
Therefore the solution is an <u>overdamped</u> response given by

$$v_c(t) = A_1 e^{-t} + A_2 e^{-5t} + A_3 \text{ , for } t \geq 0$$

Since $v_c(0^+) = v_c(0^-) = 16\,V$, we can write

$$v_c(0^+) = \boxed{A_1 + A_2 + A_3 = 16} \quad \leftarrow (\text{I})$$

Note that at $t = \infty$ (steady state holds), we have



So, $v_c(t\to\infty) = A_3 = \boxed{-8V} \leftarrow (\text{II})$

Next, we find $i_L(t) = i_c(t) = C\dfrac{dv_c(t)}{dt}$:

$$i_L(t) = 0.1\left[-A_1 e^{-t} - 5A_2 e^{-5t}\right]$$

Since $i_L(0^+) = i_L(0^-) = 0$, we can write

$$i_L(0^+) = -0.1A_2 - 0.5A_2 = 0 \rightarrow \boxed{A_1 = -5A_2} \leftarrow (\text{III})$$

Substituting (II) and (III) into (I) yields:

$$-5A_2 + A_2 - 8 = 16 \rightarrow A_2 = \boxed{-6} \rightarrow A_1 = -5A_2 = \boxed{30}$$

Therefore, the capacitor voltage expression is given by

$$\boxed{v_c(t) = 30e^{-t} - 6e^{-5t} - 8 \quad (V), \text{ for } t \geq 0}$$

(b) For $C = 0.05\,F$, the characteristic equation is given by

$$s^2 + 6s + 10 = 0 \rightarrow s_1, s_2 = -3 \pm \hat{j}$$

Therefore the solution is an <u>underdamped</u> response given by

$$v_c(t) = A_1 e^{-3t}\cos t + A_2 e^{-3t}\sin t + A_3$$

The initial & final conditions are still the same, so, following the same steps:

$$v_C(0^+) = \boxed{A_1 + A_3 = 16} \leftarrow (\text{I})$$

$$v_C(t\to\infty) = A_3 = \boxed{-8\text{ V}} \leftarrow (\text{II})$$

$$i_L(t=0^+) = 0.05\Big[-3A_1 e^{-3t}\cos t - A_1 e^{-3t}\sin t - 3A_2 e^{-3t}\sin t + A_2 e^{-3t}\cos t\Big]_{t=0^+}$$

$$= -0.15A_1 + 0.05A_2 = 0 \to \boxed{A_2 = 3A_1} \leftarrow (\text{III})$$

Substituting (II) into (I) yields:

$$A_1 = 16 - A_3 = 16 + 8 = \boxed{24}$$

Then, using (III) $\to A_2 = \boxed{72}$

$$\boxed{\therefore\ v_C(t) = 24e^{-3t}\cos t + 72e^{-3t}\sin t - 8\ (\text{V}),\ \text{for } t \geq 0}$$

(c) $s^2 + 6s + 9 = 0 \to s_1 = s_2 = -3$

Therefore, the solution is critically damped response given by

$$v_C(t) = (A_1 + A_2 t)e^{-3t} + A_3$$

$$v_C(0^+) = \boxed{A_1 + A_3 = 16} \leftarrow (\text{I})$$

$$v_C(t\to\infty) = A_3 = \boxed{-8\text{V}} \leftarrow (\text{II})$$

$$i_L(t=0^+) = \frac{1}{18}\left[A_2 e^{-3t} - 3(A_1 + A_2 t)e^{-3t}\right]_{t=0^+}$$

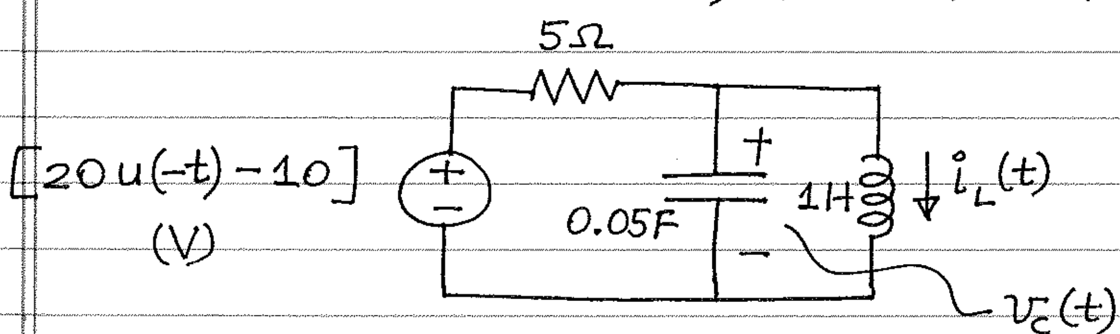
$$= \frac{1}{18}\left[A_2 - 3A_1\right] = 0 \leftarrow \text{(III)}$$

Solving, we find out that $A_1 = \boxed{24}$ & $A_2 = \boxed{72}$

$$\boxed{\therefore\ v_c(t) = 24(1+3t)e^{-3t} - 8 \ \text{(V)}, \text{ for } t \geq 0}$$

Practice Problem:

For the circuit shown, find $i_L(t)$ for $t \geq 0$.



Using $i_L(t)$ expression, find $v_c(t)$ for $t \geq 0$.

Partial answer:

$$i_L(t) = 4e^{-2t}\cos(4t) + 2e^{-2t}\sin(4t) - 2 \ \text{(A)}, \text{ for } t \geq 0$$

— THE END —