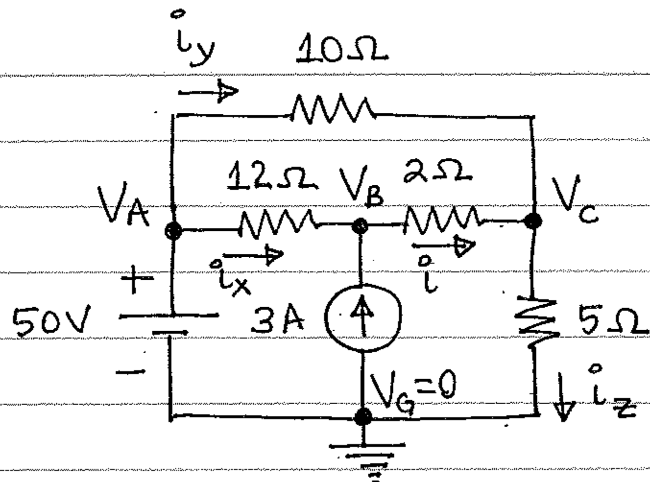
EE 261
FALL 2011
A. INAN

# SOLUTIONS TO HOMEWORK #3

9/28/2011
P#1

1.




$V_A = 50\,V$

Note that the currents $i$, $i_x$, $i_y$ and $i_z$ are arbitrarily assigned. Find $i$.

$$i_x = \frac{V_A - V_B}{12} = \frac{50 - V_B}{12}, \quad i_y = \frac{V_A - V_C}{10} = \frac{50 - V_C}{10}$$

$$i = \frac{V_B - V_C}{2}, \quad i_z = \frac{V_C - \cancel{V_G}^{\,0}}{5} = \frac{V_C}{5}$$

Using KCL at node B:

$$i_x + 3 = i \rightarrow \underset{(1)}{\frac{50 - V_B}{12}} + \underset{(12)}{3} = \underset{(6)}{\frac{V_B - V_C}{2}} \rightarrow \boxed{7V_B - 6V_C = 86}$$

Using KCL at node C:

$$i_y + i = i_z \rightarrow \underset{(1)}{\frac{50 - V_C}{10}} + \underset{(5)}{\frac{V_B - V_C}{2}} = \underset{(2)}{\frac{V_C - \cancel{V_G}^{\,0}}{5}} \rightarrow \boxed{-5V_B + 8V_C = 50}$$

$$7V_B - 6V_C = 86 \xrightarrow{\times 5} \cancel{35V_B} - 30V_C = 430$$

$$-5V_B + 8V_C = 50 \xrightarrow{\times 7} + \cancel{-35V_B} + 56V_C = 350$$

$$26V_C = 780 \rightarrow V_C = 30V$$

$$V_B = 38V$$

$$\therefore\ i = \frac{V_B - V_C}{2} = \frac{38 - 30}{2} = \boxed{4A}$$

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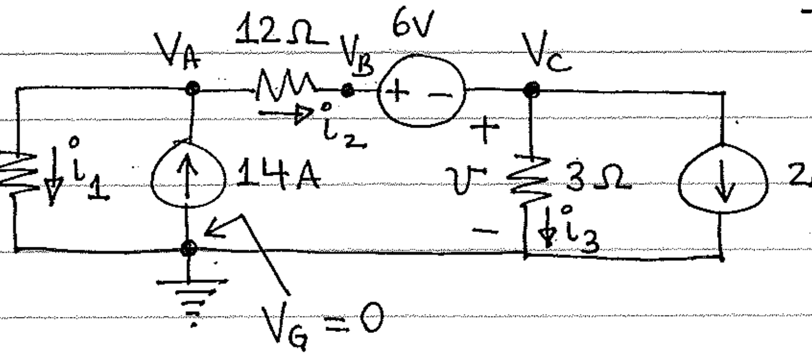
p#2

2.




Again, currents $i_1$, $i_2$ and $i_3$ are assigned arbitrarily.

$$i_1 = \frac{V_A - \cancel{V_G}^{\,0}}{6} = \frac{V_A}{6}$$

$$i_2 = \frac{V_A - V_B}{12}$$

$$i_3 = \frac{V_C - \cancel{V_G}^{\,0}}{3} = \frac{V_C}{3}$$

KCL at node A:

$$14 = i_1 + i_2 \rightarrow \underset{(12)}{14} = \underset{(2)}{\frac{V_A}{6}} + \underset{(1)}{\frac{V_A - V_B}{12}} \rightarrow \boxed{3V_A - V_B = 168}$$

KCL at node C:

$$i_2 = i_3 + 2 \rightarrow \underset{(1)}{\frac{V_A - V_B}{12}} = \underset{(4)}{\frac{V_C}{3}} + \underset{(12)}{2} \rightarrow \boxed{V_A - V_B - 4V_C = 24}$$

Also, we know that $V_B - V_C = 6 \rightarrow V_B = 6 + V_C$

Substituting into first and second equations yield:

$$\left.\begin{aligned} 3V_A - V_C &= 174 \\ V_A - 5V_C &= 30 \end{aligned}\right\} \rightarrow \begin{aligned} & \cancel{3V_A} - V_C = 174 \\ - \; & \cancel{3V_A} - 15V_C = 90 \\ \hline & 14V_C = 84 \rightarrow V_C = 6V \end{aligned}$$

$$\therefore\; v = V_C = \boxed{6V}$$

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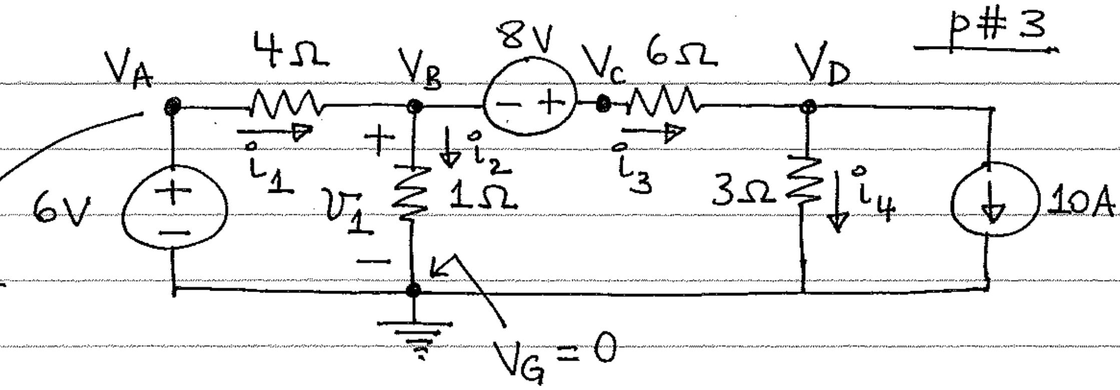
p#3

3.




Note that $V_A = 6V$

Currents $i_1, i_2, i_3$ and $i_4$ assigned.
Their directions are chosen arbitrarily.

$$i_1 = \frac{V_A - V_B}{4} = \frac{6 - V_B}{4}$$

$$i_2 = \frac{V_B - \cancel{V_G}^{\,0}}{1} = V_B$$

$$i_3 = \frac{V_C - V_D}{6}, \quad i_4 = \frac{V_D - \cancel{V_G}^{\,0}}{3} = \frac{V_D}{3}$$

KCL at node B:

$$i_1 = i_2 + i_3 \rightarrow \underset{(3)}{\frac{6 - V_B}{4}} = \underset{(12)}{V_B} + \underset{(2)}{\frac{V_C - V_D}{6}}$$

$$\rightarrow \boxed{15V_B + 2V_C - 2V_D = 18} \quad (\#1)$$

KCL at node D:

$$i_3 = i_4 + 10 \rightarrow \underset{(1)}{\frac{V_C - V_D}{6}} = \underset{(2)}{\frac{V_D - \cancel{V_G}^{\,0}}{3}} + \underset{(6)}{10}$$

$$\rightarrow \boxed{V_C - 3V_D = 60} \quad (\#2)$$

Also, $\boxed{V_C - V_B = 8V} \rightarrow V_C = 8 + V_B$

Substituting into equations #1 & #2 yield:

$$\left.\begin{aligned} 17V_B - 2V_D &= 2 \\ V_B - 3V_D &= 52 \end{aligned}\right\} \rightarrow \begin{aligned} 51V_B - \cancel{6V_D} &= 6 \\ + \quad \mp 2V_B \pm \cancel{6V_D} &= \mp 104 \\ \hline 49V_B = -98 \rightarrow V_B &= -2V \end{aligned}$$

$$v_1 = V_B = \boxed{-2V} \quad \& \quad i_1 = \frac{6 - (-2)}{4} = \boxed{2A}$$