University ☺f P☺rtland
Sch☺☺l ☺f Engineering

EE 261
Fall 2011
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Homework # 8–Arithmetic of Complex Numbers and Phasor Analysis
(Assigned: Monday, November 28, 2011)
(Due: Friday, December 9, 2011, 1:35p.m.)

P 1. Rectangular to polar form. Write the following rectangular-form complex numbers in polar form:

1.a. $V_1 = 5\sqrt{2} + j5\sqrt{2}$ **1.b.** $Z_2 = 120j + 50$

1.c. $Y_3 = j0.02$ **1.d.** $I_4 = -10\sqrt{3} - j10$

1.e. $Z_5 = -j6{,}000 + 8{,}000$ **1.f.** $V_6 = -2.5$

P 2. Polar to rectangular form. Convert the following polar-form complex numbers in rectangular form:

2.a. $I_1 = 10e^{j180^\circ}$ **2.b.** $V_2 = 3\sqrt{2}e^{-j45^\circ}$

2.c. $Z_3 = 200e^{-j90^\circ}$ **2.d.** $Y_4 = \left(e^{-j2\pi/3}\right)/500$

2.e. $V_5 = 2.3e^{j5\pi/6}$ **2.f.** $I_6 = 5.6\times10^{-3}e^{j240^\circ}$

P 3. Phasor representation of sinusoidal signals. Find the phasor-form representation of the following sinusoidal signals. (Note that the capital letters represent the phasor form.)

3.a. $i_1(t) = 2\times10^{-4}\cos\left(2\pi\times10^4 t + 60^\circ\right) \xrightarrow{\text{phasor rep.}} I_1 = ?$

3.b. $v_2(t) = 3.8\cos\left(10^6 t - 25^\circ\right) \rightarrow V_2 = ?$

3.c. $v_a(t) = 1.73\sin\left(10^5 \pi t + \pi/10\right) \rightarrow V_a = ?$

3.d. $i_c(t) = -4.2\sin\left(3\pi\times10^5 t - 3\pi/5\right) \rightarrow I_c = ?$

3.e. $i_s(t) = 0.05\cos\left(6.28\times10^4 t - 135^\circ\right) \rightarrow I_s = ?$

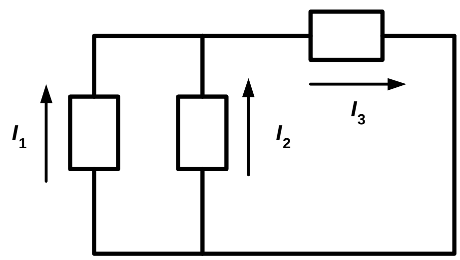
3.f. $v_s(t) = 10\sin\left(8\pi\times10^4 t - 300^\circ\right) \rightarrow V_s = ?$

3.g. $i_T(t) = 0.03\cos\left(10^5 t - 135^\circ\right) - 0.015\sin\left(10^5 t + 60^\circ\right) \rightarrow I_T = ?$

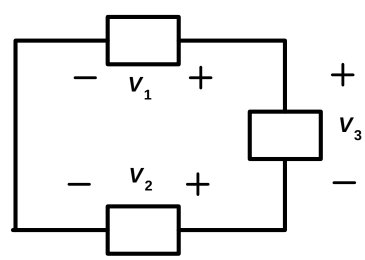
3.h. $v_T(t) = 2.1\sin\left(4.4\pi\times10^4 t - 30^\circ\right) + 4.9\cos\left(4.4\pi\times10^4 t + 30^\circ\right) \rightarrow V_T = ?$

P 4. Basic arithmetic operations of complex numbers. Solve the following circuit problems, simplify each answer and provide it in polar form.

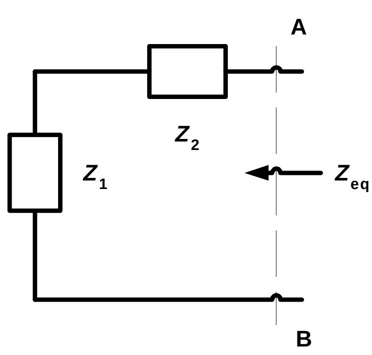
4.a. Kirchhoff's current law (KCL) applied in the phasor domain. Given $I_1 = 10e^{j30^\circ}$, $I_2 = 10e^{-j30^\circ}$, $I_3 = I_1 + I_2 = ?$ (Note that each I_k current is a phasor quantity which represents a real-time **S**inusoidal **S**teady-**S**tate (SSS) current $i_k(t)$ flowing in the circuit shown below. Just like the time-domain currents, the phasor-domain currents must also satisfy KCL.)



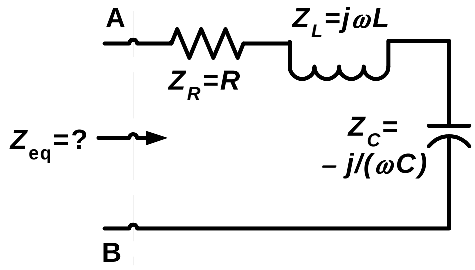
4.b. Kirchhoff's voltage law (KVL) in phasor form. In the phasor-domain circuit shown below, $V_1 = 5\sqrt{2}e^{-j45^\circ}$, $V_2 = 5\sqrt{2}e^{j45^\circ}$, $V_3 = V_1 - V_2 = ?$ (Note that each V_k is a phasor quantity which represents a real-time SSS voltage $v_k(t)$ in the circuit shown below. The phasor-domain voltages must satisfy KVL.)



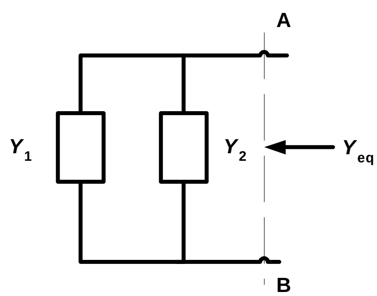
4.c. Equivalent impedance. The circuit shown between terminals A and B is shown in phasor domain. If $Z_1 = (200 - 400j)\Omega$ and $Z_2 = (200 + j100)\Omega$, $Z_3 = Z_1 + Z_2 = ?$ (Note that each Z represent an impedance. Impedances can be combined in series or in parallel just like resistances.)



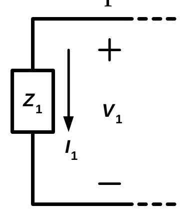
4.d. Equivalent impedance of a series *RLC* circuit. In the series *RLC* circuit shown, the element values are given by $R = 4\,\Omega$, $L = 5\,\text{mH}$, and $C = 1.25\,\text{mF}$ respectively. Find the equivalent impedance of this circuit at three different frequencies: $\omega_1 = 200\,\text{rad/s}$, $\omega_2 = 400\,\text{rad/s}$, and $\omega_3 = 1{,}200\,\text{rad/s}$.



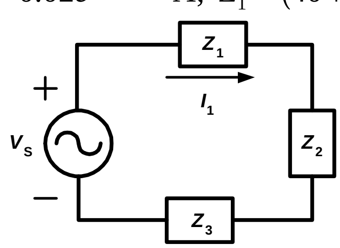
4.e. Equivalent admittance. Admittance of an element represented by Y (in Siemens) is defined as the inverse of the impedance Z (in Ω) of the same element, i.e., $Y = Z^{-1}$. Given $Y_1 = 0.002e^{j\pi/2}$ S, $Y_2 = 0.002\sqrt{2}e^{-j\pi/4}$ S, what is $Y_3 = Y_1 + Y_2 = ?$ (Note that admittances can be combined in series or in parallel just like the same way as conductances.)



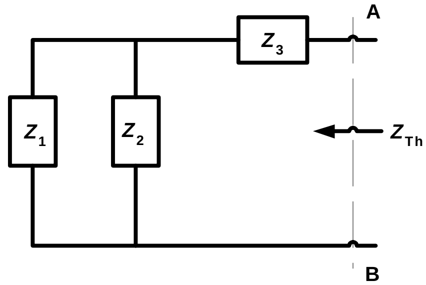
4.f. Ohm's law in phasor form. Given $I_1 = 0.02e^{-j\pi/3}$ A, $Z_1 = 150e^{j\pi/6}\,\Omega$, $V_1 = Z_1 I_1 = ?$ (Note that $V = ZI$ is the phasor-domain equivalent of the time-domain Ohm's law given as $v_R(t) = Ri_R(t)$. Note also that Ohm's law in phasor form is not only limited to resistors but can also be used for inductors and capacitors.)



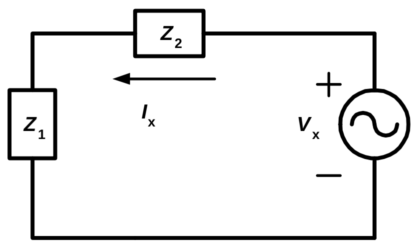
4.g. Phasor-domain solution of sinusoidal steady-state circuits. In the circuit shown, given $V_S = 4e^{j30^\circ}$ V, $I_1 = 0.02e^{-j23.13^\circ}$ A, $Z_1 = (40 + j80)\,\Omega$, $Z_2 = (30 - j20)\,\Omega$, $Z_3 = ?$



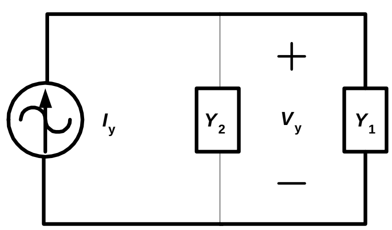
4.h. Thevenin impedance. For the phasor-domain circuit shown, given the three impedance values to be $Z_1 = 50\,\Omega,\ Z_2 = j50\,\Omega,\ Z_3 = -j50\,\Omega,\ Z_{\text{Th}} = Z_3 + \dfrac{Z_1 Z_2}{Z_1 + Z_2} = ?$



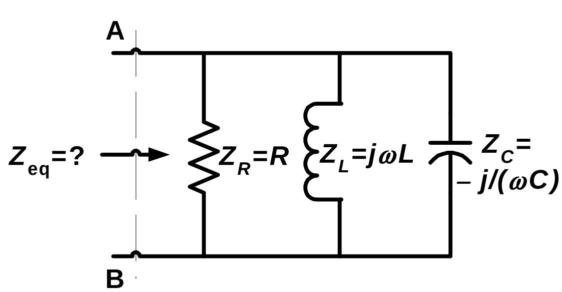
4.i. KVL and Ohm's law. In the phasor-domain circuit shown below, given $Z_1 = (300 + j300)\,\Omega,\ Z_2 = 300\sqrt{2}e^{-j\pi/4}\ \Omega,\ I_x = 0.04e^{j\pi/3}\ \text{A},\ V_x = (Z_1 + Z_2)I_x = ?$



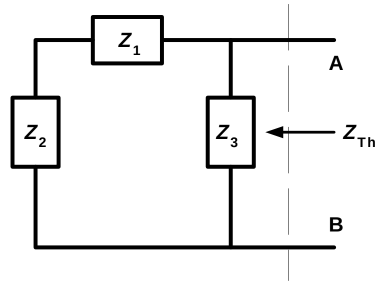
4.j. KCL and Ohm's law (optional). In the following phasor-domain circuit shown, $Y_1 = 0.02e^{-j\pi/2}\ \text{S},\ Y_2 = 0.01\sqrt{3}e^{j\pi/3}\ \text{S},\ V_y = e^{j60^\circ}\ \text{V},\ I_y = (Y_1 + Y_2)V_y = ?$



4.d. Equivalent impedance of a parallel *RLC* circuit (optional). In the parallel *RLC* circuit shown, the element values are given by $R = 8\,\Omega,\ L = 2\ \text{mH},$ and $C = 5\ \mu\text{F}$ respectively. Find the equivalent impedance of this circuit at three different frequencies: $\omega_1 = 5{,}000\ \text{rad/s}$, $\omega_2 = 10{,}000\ \text{rad/s}$, and $\omega_3 = 20{,}000\ \text{rad/s}$.



4.k. Thevenin impedance (optional). For the impedance circuit shown below, if $Z_1 = (2 - j2)\,\Omega$, $Z_2 = (j2 + 2)\,\Omega$, $Z_3 = 2 - j6\,\Omega$, what is the Thevenin impedance $Z_{\rm Th}$?



P 5. Addition/subtraction of sinusoidal signals. Four sinusoidal-voltage signals are given by $v_1(t) = 10\sin(10^5 t + \pi/2)$, $v_2(t) = 10\cos(10^5 t - 2\pi/3)$, $v_3(t) = 10\sin(10^5 t + \pi/6)$, and $v_4(t) = 10\cos(10^5 t - \pi)$ respectively. Find each of the following signals below and express them in terms of a single sinusoidal waveform:

5.a. $v_5(t) = v_1(t) + v_2(t) = ?$ (See the figure on the left below.)

5.b. $v_6(t) = v_2(t) - v_3(t) = ?$ (See the figure on the right below.)

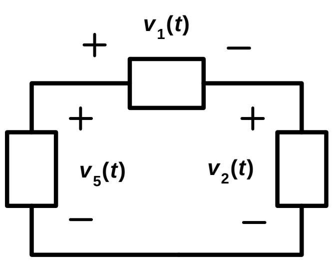
5.c. $v_7(t) = v_2(t) + v_3(t) - v_4(t) = ?$ (See the figure in the middle below.)

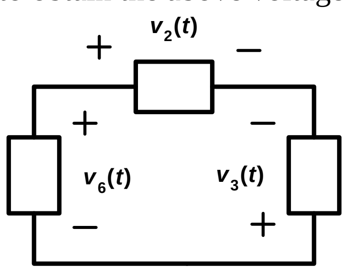
5.d. (Optional.) $v_8(t) = v_1(t) - v_2(t) + v_3(t) = ?$

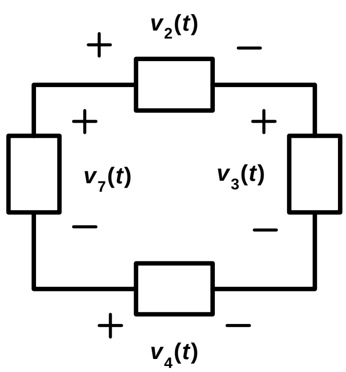
5.e. (Optional.) $v_9(t) = v_2(t) - v_3(t) + v_4(t) = ?$

5.f. (Optional.) $v_{10}(t) = v_1(t) - v_3(t) - v_4(t) = ?$

(Suggestion: Use the phasor-domain approach to obtain the above voltages.)







Please use the following guidelines for your homework solutions:

1) On the first sheet, at the top, indicate that this is EE 261/Fall 2011/HW#8 Solutions and provide your name somewhere on that sheet where the grader can easily see it.
2) Solve each problem on a separate sheet unless there is a solution which is very short.
3) Do not use the back of the sheets unless you have to.
4) Staple your solutions in the above order before you turn them in.

Please turn in your homework on time. The solutions for each homework assignment will be provided as a separate handout on the due date.

Another important reminder:

EE 261–Final Exam is scheduled for Wednesday, December 14, 2011, 13:30-15:00!

It is a 90 minute closed book exam. Formula sheets are allowed.