12-2-2011

University of Portland
School of Engineering

EE 261-Electrical Circuits-3 cr. hrs.
Fall 2011

Midterm Exam # 3

(Friday, December 2, 2011)
(Closed Book Exam, Three Formula Sheet are Allowed)
(Total Time: 55 minutes)

Name: SOLUTIONS ☺

Signature: SOLUTIONS ☺

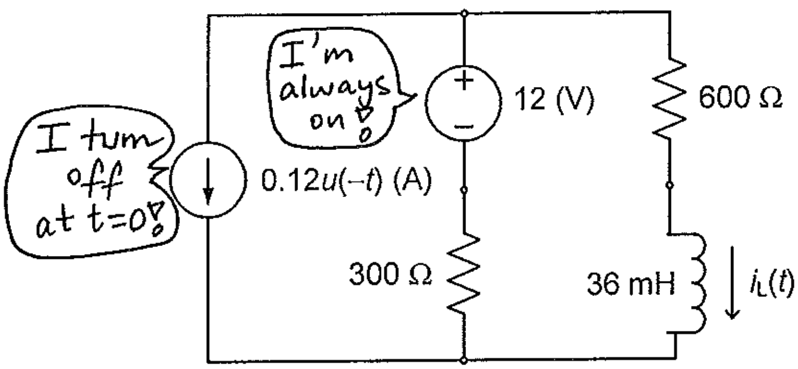
> *"An honest mind possesses a kingdom."*
> **Lucius Annaeus Seneca (4B.C.–65A.D.)**
>
> *"Honest people are the true winners of the universe."*
> **Anonymous**

NOTE: On all the problems, please show your work clearly, and provide the appropriate units for your answers. Also mark on the schematic to show any current or voltage that you define in your solution.

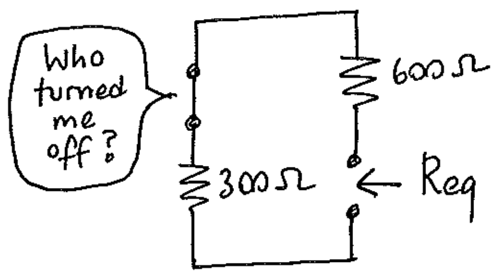
P # 1 (30 pts.)	P # 2 (30 pts.)	P # 3 (40 pts.)	Total (100 pts.)
60	60	80	200 pts!

Inan is scoring his solution with double scores, unfair!

1. (15 mins., 30 points) In the circuit shown, find the complete mathematical expression for the current $i_L(t)$ which flows through the 36 mH inductor for $t \geq 0$. (Please show your work clearly and provide brief justifications for the steps you take. Also, don't forget to provide the correct units for your answer.)



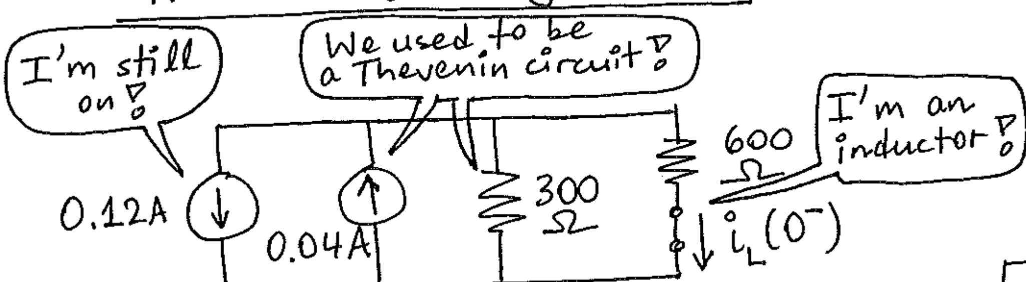
To find the time constant:



$\leftarrow R_{eq} = 900\,\Omega$

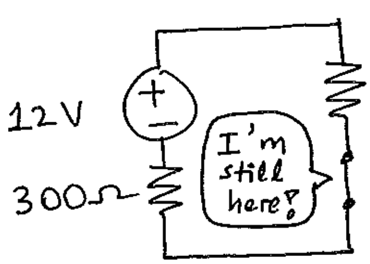
$$\tau = \frac{L}{R_{eq}} = \frac{36\times10^{-3}}{900} = \boxed{4\times10^{-5}\ \text{sec}}$$

At $t=0^-$ (steady state):



Using CDP: $i_L(0^-) = \dfrac{300}{300+600}(0.04-0.12) = \boxed{-\dfrac{80}{3}\ \text{mA}}$

At $t=\infty$ (steady state):



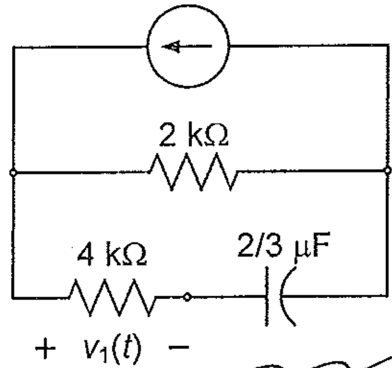
$i_L(\infty) = \dfrac{12\text{V}}{900\,\Omega} = \boxed{\dfrac{40}{3}\ \text{mA}}$

$$\therefore\ i_L(t) = -\frac{80}{3}e^{-25{,}000t} + \frac{40}{3}\left(1-e^{-25{,}000t}\right) = \frac{40}{3} - 40e^{-25{,}000t}\ \text{(mA)}$$

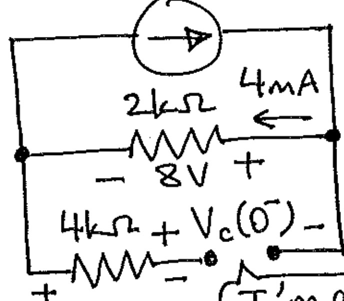
12-2-2011

2. (15 mins., 30 Points) In the circuit shown below, find the complete mathematical expression and sketch the voltage $v_1(t)$ across the 4 kΩ resistor for $t > 0$. (Please show your work step by step.)

$$[10u(t)-4]\ (\text{mA}) = \begin{cases} -4\,\text{mA}, & t<0 \\ 6\,\text{mA}, & t>0 \end{cases}$$



I. At $t=0^-$ (steady state):

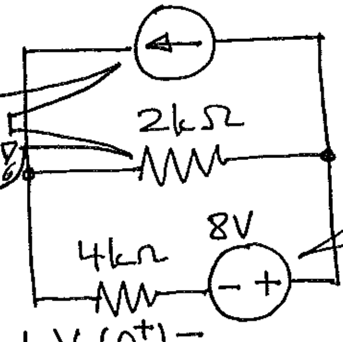


$V_c(0^-) = -8V$

$V_1(0^-) = 0$

I'm acting like an open circuit!

II. At $t=0^+$:

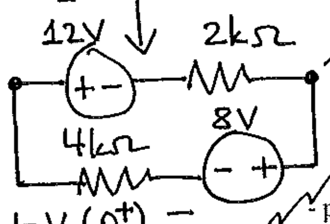


We are a Norton circuit!

Transform us into a Thevenin circuit!

I'm the capacitor!

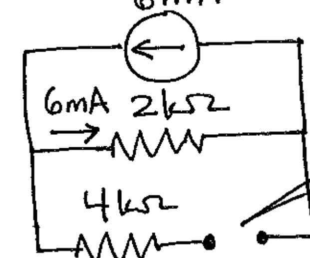
My voltage is set by $V_c(0^+) = V_c(0^-)$!



Note that I jumped!

VDP: $V_1(0^+) = \dfrac{4k\Omega}{6k\Omega}(12+8) = \boxed{\dfrac{40}{3}V}$

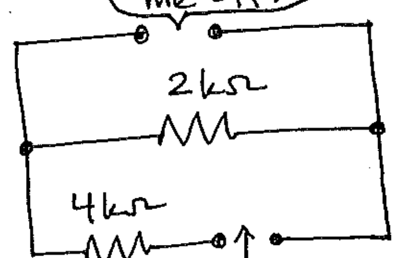
III. At $t=\infty$ (steady state):



$+V_1(\infty) = \boxed{0} -$

I'm open again!

IV. To find the time constant:

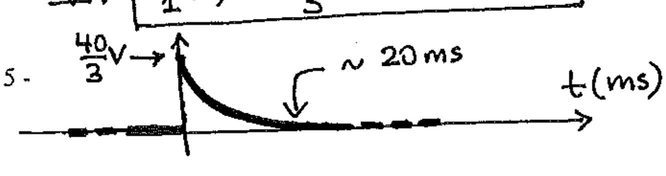


Who shut me off?

$R_{eq} = 6k\Omega$

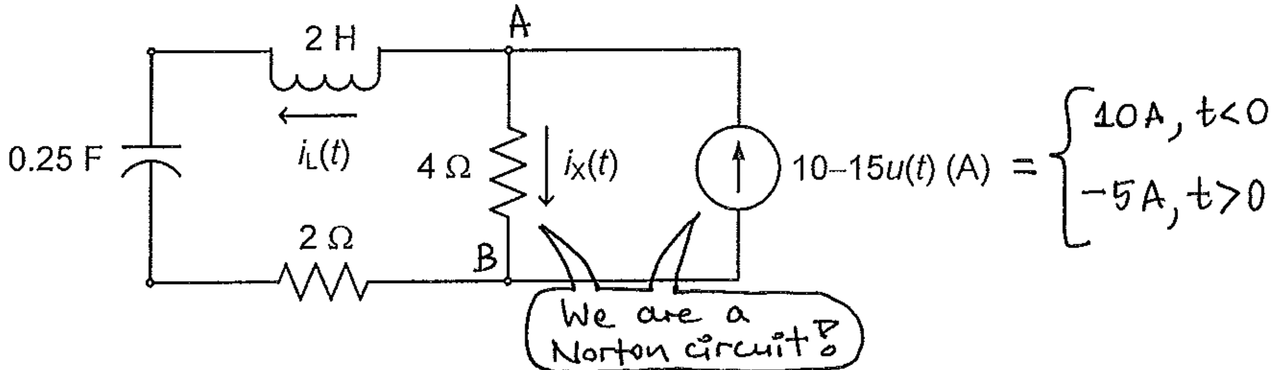
$\therefore \tau = R_{eq}C = (6k)(\tfrac{2}{3}\mu) = \boxed{4\,ms}$

V. $\boxed{V_1(t) = \frac{40}{3}e^{-250t}\ (V)}$

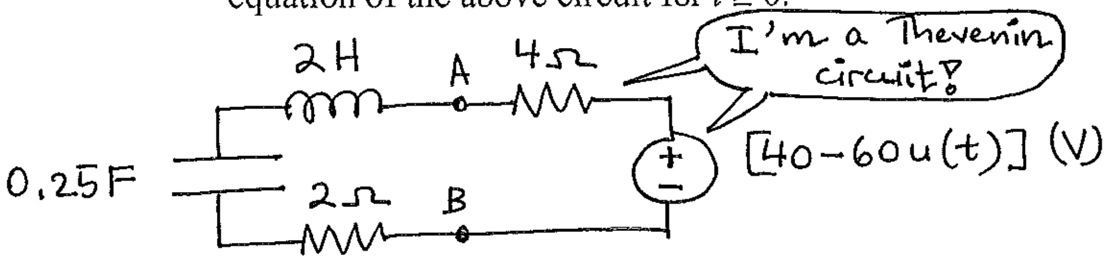


12-2-2011

3. (20 mins., Total: 40 Points) Consider the second-order circuit shown.



(a) (10 points) Solve for the roots (s_1 and s_2) of the characteristic equation of the above circuit for $t \geq 0$.



Second-order series RLC circuit with characteristic equation given by:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \rightarrow s^2 + 3s + 2 = 0 \rightarrow (s+1)(s+2) = 0$$

∴ Characteristic roots are $\boxed{s_1 = -1, \ s_2 = -2}$

(b) (10 points) Based on the results of part (a), write the general mathematical expression for the inductor current $i_L(t)$ for $t \geq 0$. (At this stage, leave the constant coefficients in your answer as unknown quantities.)

Since characteristic roots are real and distinct, <u>overdamped response</u> given by

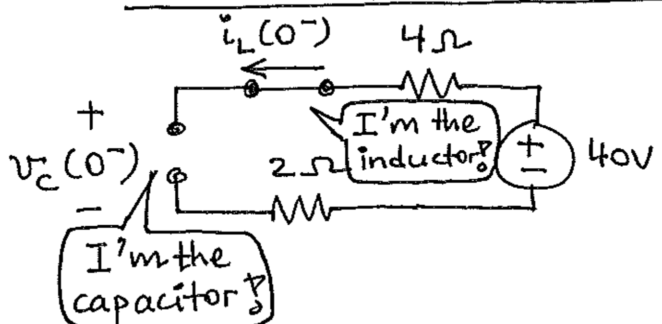
$$\boxed{i_L(t) = A_1 e^{-t} + A_2 e^{-2t} + A_3, \text{ for } t \geq 0}$$

We need to be determined based on initial & final conditions!

(c) (15 points) Find the values of the coefficients of the $i_L(t)$ expression found in part (b) using the initial and final conditions.

To find the initial conditions:

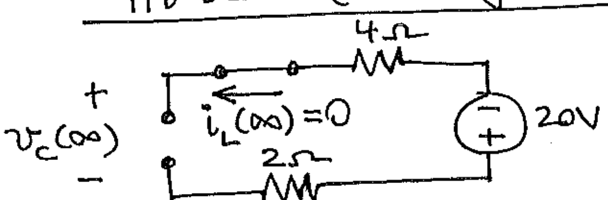
At $t=0^-$ (steady state):



$\rightarrow$ $i_L(0^-) = 0 \rightarrow i_L(0^+) = 0$

$v_C(0^-) = 40V \rightarrow v_C(0^+) = 40V$ } initial conditions!

At $t=\infty$ (steady state):

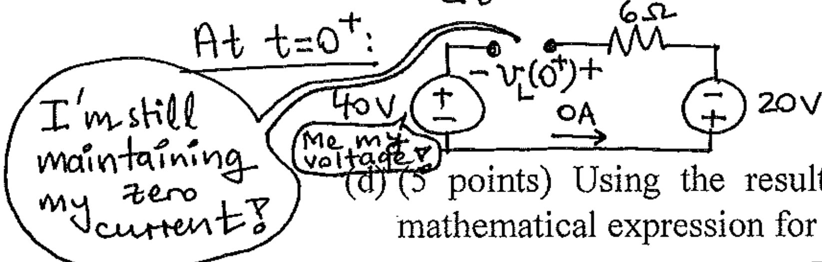


$\rightarrow i_L(\infty) = 0 \rightarrow \boxed{A_3 = 0}$

$i_L(0) = \boxed{A_1 + A_2 = 0}$ Solving

$v_L(t) = L\dfrac{di_L(t)}{dt} = 2\left[-A_1 e^{-t} - 2A_2 e^{-2t}\right]$

At $t=0^+$:



$\rightarrow v_L(0^+) = -60V$ Simultaneously!

$\therefore v_L(0^+) = \boxed{-2A_1 - 4A_2 = -60}$

$\boxed{A_1 = -30 \;\&\; A_2 = 30}$

(d) (5 points) Using the result of part (c), write the complete mathematical expression for the current $i_X(t)$ for $t \geq 0$.

$$\boxed{\therefore\; i_L(t) = -30e^{-t} + 30e^{-2t}, \text{ for } t \geq 0 \quad \text{(A)}}$$

$$\boxed{\therefore\; i_x(t) \overset{KCL}{=} -5 - i_L(t) = 30e^{-t} - 30e^{-2t} - 5 \text{ (A)}, \text{ for } t > 0}$$