From "In Memory of JFK," by A. S. Inan:

1. How do you express JFK's birthday by using three consecutive primes?

Answer: $\underbrace{(23 \times 23)}_{529}$ 1917

2. What is the unique numerical connection between JFK's and RFK's birthdays?

Answer:

JFK's birthday $\boxed{5291}$917

RFK's birthday 1120$\boxed{1925}$

John F. Kennedy
(May 29, 1917-November 22, 1963)

University of Portland
School of Engineering

EE 261-Electrical Circuits-3 cr. hrs.
Fall 2013

Midterm Exam # 3

(Friday, November 22, 2013)
President JFK was killed 50 years ago, on this day.

(Closed Book Exam, Three Formula Sheet are Allowed)
(Total Time: 55 minutes)

Name: SOLUTIONS! ☺

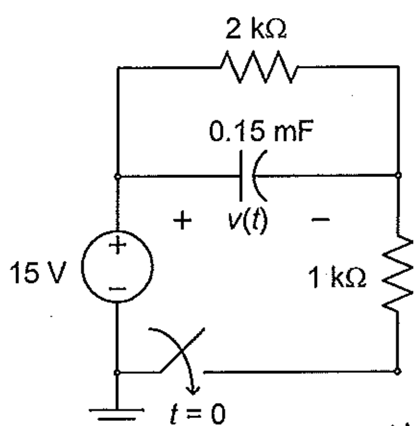
Signature: ______________________ ☺

> ***"An honest mind possesses a kingdom."***
> **Lucius Annaeus Seneca (4B.C.–65A.D.)**
>
> ***"Honest people are the true winners of the universe."***
> **Anonymous**

NOTE: Problem # 4 is take-home, due Monday. On all the problems, please show your work clearly, and provide the appropriate units for your answers. Also mark on the schematic to show any current or voltage that you define in your solution.

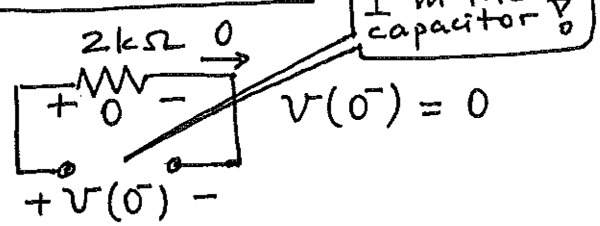
P # 1 (25 pts.)	P # 2 (25 pts.)	P # 3 (25 pts.)	P # 4 (25 pts.)	Total (100 pts.)

1. (15 mins., 25 points) In the electric circuit shown, the switch closes at $t = 0$, after being open for a long time. Find the complete mathematical expression for the voltage $v(t)$ across the 0.15 mF capacitor for $t \geq 0$. (Please show your work clearly and provide brief justifications for the steps you take. Also, don't forget to provide the correct units for your answer.)



$$v(t) = v(0^+)\,e^{-t/\tau} + v(\infty)\left(1 - e^{-t/\tau}\right)$$

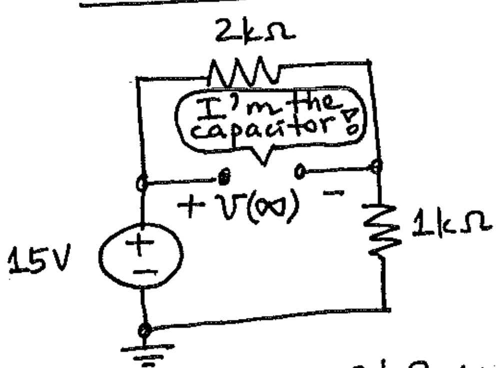
At $t = 0^-$ (SS):



$v(0^-) = 0$

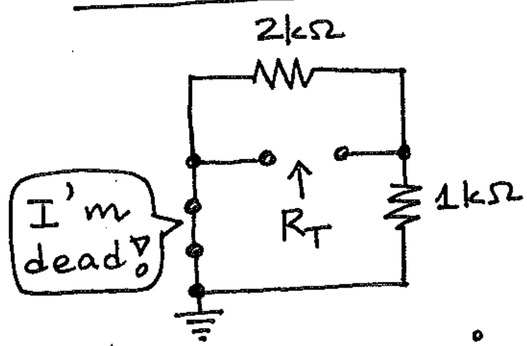
Since capacitor voltage can not jump, $v(0^+) = v(0^-) = 0$

At $t = \infty$ (SS):



$$\text{VDP} \rightarrow v(\infty) = \frac{2k\Omega}{3k\Omega}(15V) = 10V$$

To find $\tau = R_T C$:

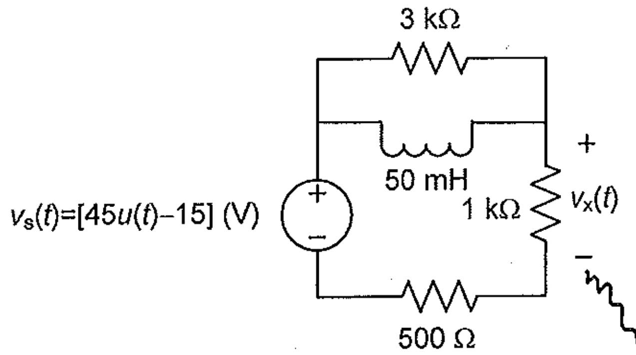


$$R_T = 2k\Omega // 1k\Omega = \frac{(2k\Omega)(1k\Omega)}{(2+1)k\Omega} = \frac{2}{3}k\Omega$$

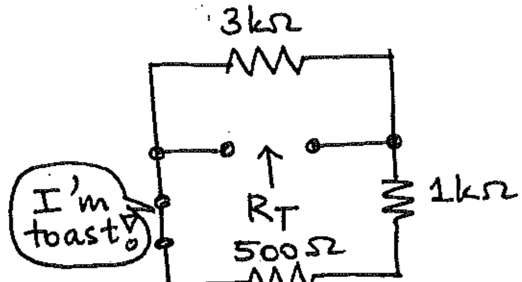
$$\therefore \tau = \left(\tfrac{2}{3}k\Omega\right)(0.15mF) = 0.1s$$

$$\boxed{\therefore v(t) = 10\left(1 - e^{-10t}\right) V, \text{ for } t \geq 0}$$

2. (15 mins., 25 points) In the electric circuit shown, find the complete mathematical expression and sketch the voltage $v_x(t)$ across the 1 kΩ resistor for $t > 0$. (Please show your work step by step.)



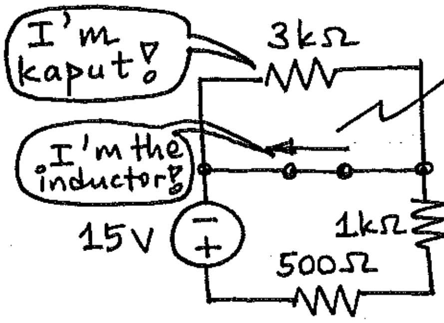
To find $\tau = L/R_T$:



$R_T = 3k\Omega \,//\, 1.5k\Omega$
$= 1k\Omega$

$\tau = \dfrac{50mH}{1k\Omega} = 5\times10^{-5}\,s$

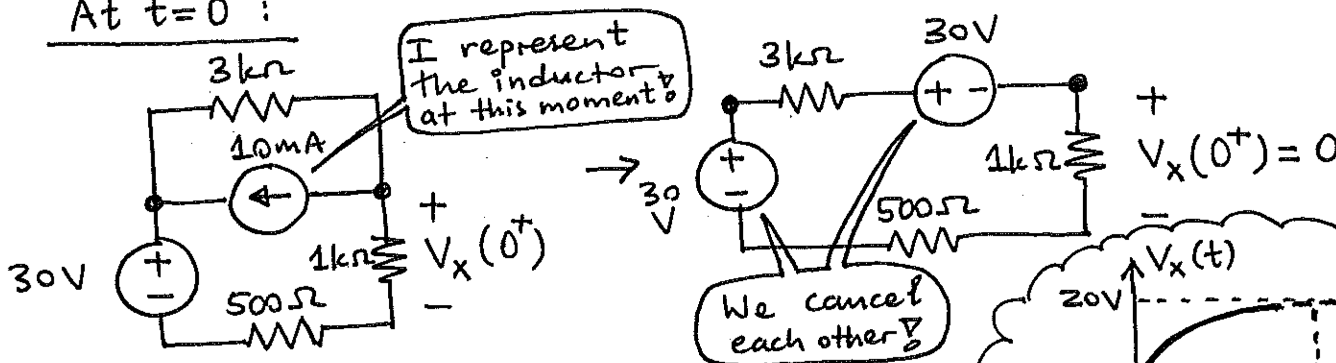
At $t=0^-$ (ss):



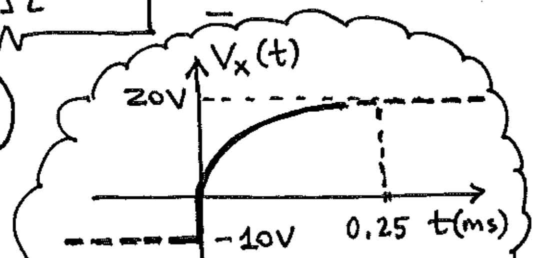
$i_L(0^-) = \dfrac{15V}{1.5k\Omega} = 10mA$

$V_x(0^-) \overset{VDP}{=} -\dfrac{1k\Omega}{1.5k\Omega}(15V) = -10V$

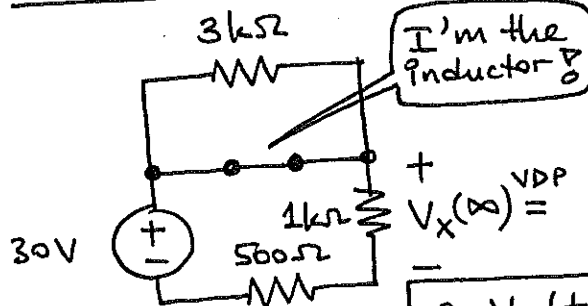
At $t=0^+$:



$V_x(0^+) = 0$



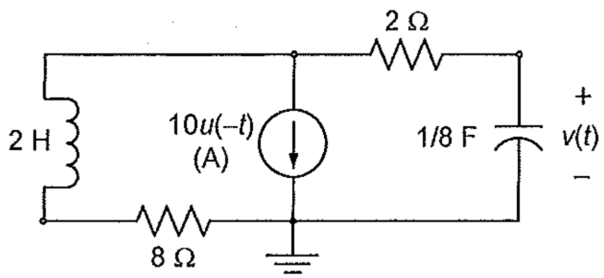
At $t=\infty$ (ss):



$V_x(\infty) \overset{VDP}{=} \dfrac{1k\Omega}{1.5k\Omega}(30V) = 20V$

$$\boxed{\therefore V_x(t) = 20\left(1 - e^{-20{,}000t}\right) V, \text{ for } t>0}$$

3. (20 mins., 25 points) Consider the electric circuit shown. Solve for the complete mathematical expression for the capacitor voltage $v(t)$ for $t \ge 0$. Show your work step by step including justifications.

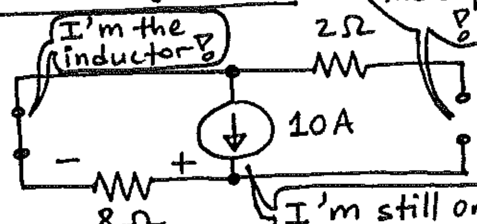


I'm an initial condition!

At $t = 0^-$ (ss):

I represent the capacitor!

Me too!

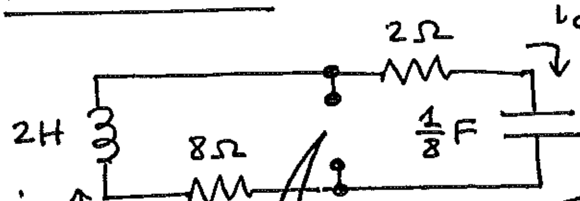


$i_L(0^-) = 10A = i_L(0^+)$

$V(0^-) = -(10A)(8\Omega) = -80V = V(0^+)$

For $t > 0$:

$i_c = i_L$



I'm turned off!

$\rightarrow$ Series RLC circuit:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

I'm the characteristic equation!

$$s^2 + \frac{10}{2}s + \frac{8}{2} = 0$$

$$s^2 + 5s + 4 = 0 \rightarrow s_1, s_2 = -1, -4$$

We are the characteristic roots!

Since real and distinct roots $\rightarrow$ Overdamped response.

$\therefore V(t) = A_1 e^{-t} + A_2 e^{-4t} + A_3$

Since $V(\infty) = A_3$ and $V(\infty) = 0 \rightarrow A_3 = 0$

$V(0^+) = \boxed{A_1 + A_2 = -80}$ Eqn. (I)

$i_c(t) = C\frac{dV(t)}{dt} = \frac{1}{8}\left[-A_1 e^{-t} - 4A_2 e^{-4t}\right] = i_L(t)$

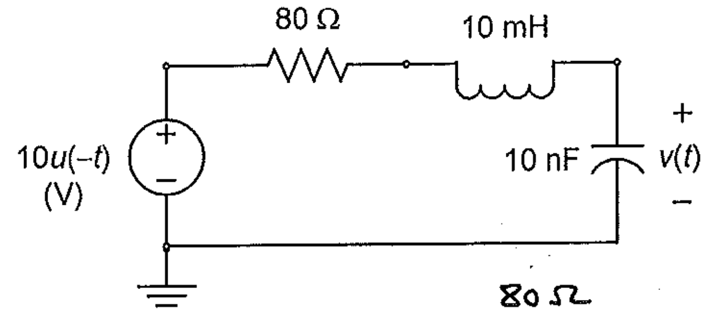
$i_L(0^+) = -\frac{A_1}{8} - \frac{A_2}{2} = 10 \rightarrow \boxed{A_1 + 4A_2 = -80}$ Eqn. (II)

Solving Eqns. (I) and (II) simultaneously: $A_1 = -80$ & $A_2 = 0$.

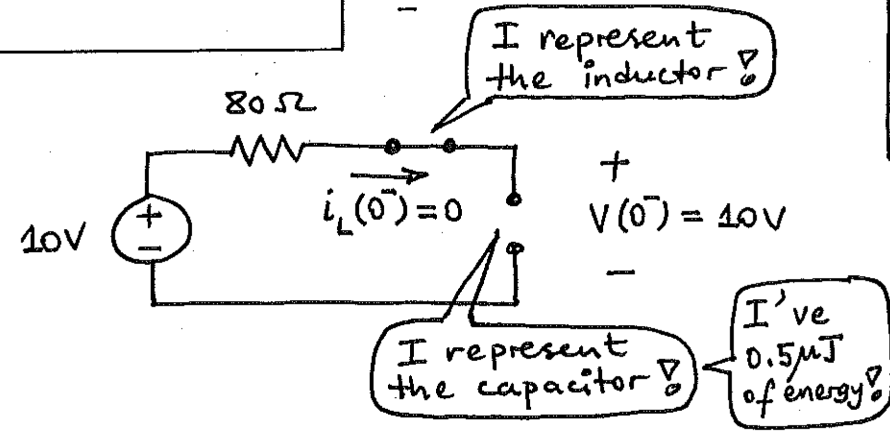
$\boxed{\therefore V(t) = -80e^{-t} \text{ V, for } t \geq 0}$

(4) (Take-home: Total: 25 points)

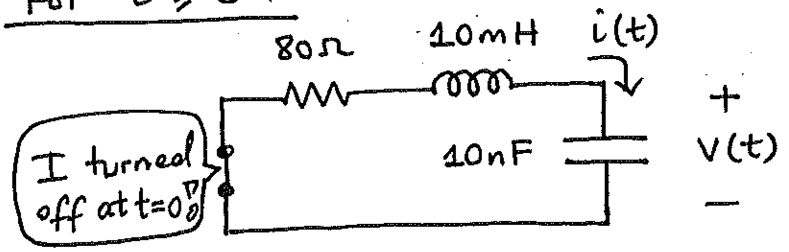
(a) (15 points) For the electric circuit shown, find the complete mathematical expression for voltage $v(t)$ across the 10 nF capacitor for $t \geq 0$. Show your work including justifications step by step.



At $t=0^-$ (ss):



For $t \geq 0$:



Note that

$V(0^+) = V(0^-) = 10V$

$i_L(0^+) = i_L(0^-) = 0$

series RLC circuit → Characteristic equation $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$$\rightarrow s^2 + \frac{80}{0.01}s + \frac{1}{(0.01)(10^{-8})} = 0 \rightarrow s^2 + 8{,}000s + 10^{10} = 0$$

Characteristic roots $\rightarrow s_1, s_2 = \dfrac{-8{,}000 \pm \sqrt{(8{,}000)^2 - 4 \times 10^{10}}}{2}$

$\cong -4000 \pm j\,99{,}920$ (Underdamped response)

$$\therefore v(t) = A_1 e^{-4{,}000t}\cos(99{,}920t) + A_2 e^{-4{,}000t}\sin(99{,}920t) + \cancel{A_3}^{\,0}$$

$v(0^+) = \boxed{A_1 = 10}$

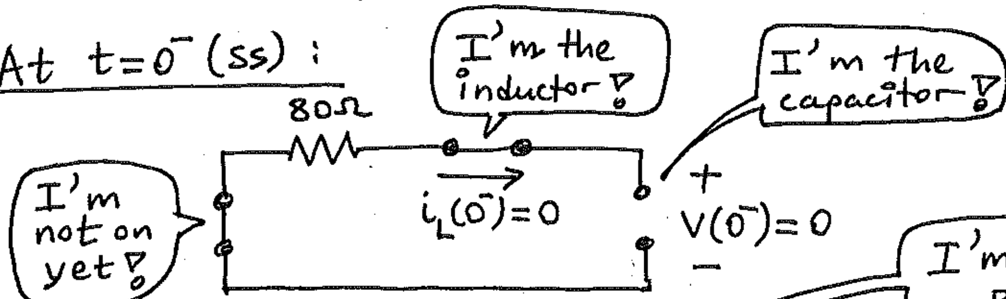
$$i(t)\Big|_{t=0^+} = C\frac{dV(t)}{dt}\Big|_{t=0^+} = 10^{-8}\left[-4000A_1 + 99{,}920A_2\right] = i_L(0^+) = 0$$

$$A_2 = \frac{4000A_1}{99{,}920} \cong \boxed{0.4003}$$

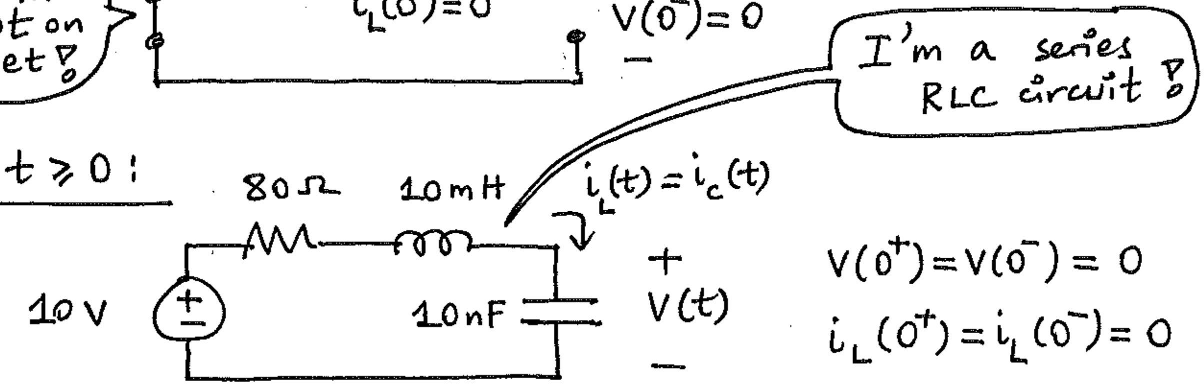
$$\boxed{\therefore V(t) \cong 10e^{-4{,}000t}\cos(99{,}920t) + 0.4e^{-4{,}000t}\sin(99{,}920t)}$$

(b) (10 points) Repeat part (a) if the source voltage is changed from $10u(-t)$ to $10u(t)$.

At $t=0^-$ (ss):



For $t \geq 0$:



$V(0^+)=V(0^-)=0$

$i_L(0^+)=i_L(0^-)=0$

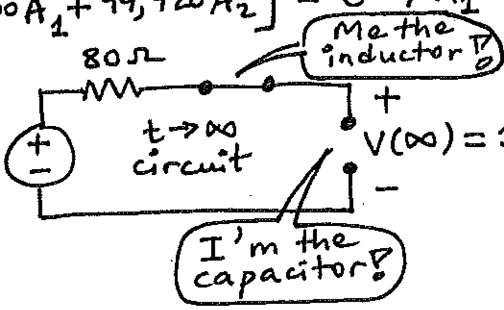
Same characteristic roots found in part (a) apply.

$$\therefore\ V(t) = A_1 e^{-4{,}000t}\cos(99{,}920t) + A_2 e^{-4{,}000t}\sin(99{,}920t) + A_3$$

$$V(0^+) = A_1 + A_3 = 0$$

$$i_L(0^+) = C\left.\frac{dV(t)}{dt}\right|_{t=0^+} = 10^{-8}\left[-4{,}000A_1 + 99{,}920A_2\right] = 0 \rightarrow A_1 \cong 24.98A_2$$

$V(\infty) = A_3 = 10\text{ V}$ since



$$\rightarrow A_1 = -A_3 = -10\text{V}$$

$$\rightarrow A_2 \cong \frac{A_1}{24.98} \cong -0.4003$$

$$\boxed{\therefore\ V(t) \cong -10e^{-4{,}000t}\cos(99{,}920t) - 0.4003e^{-4{,}000t}\sin(99{,}920t) + 10\ \text{V},\ \text{for } t \geq 0}$$