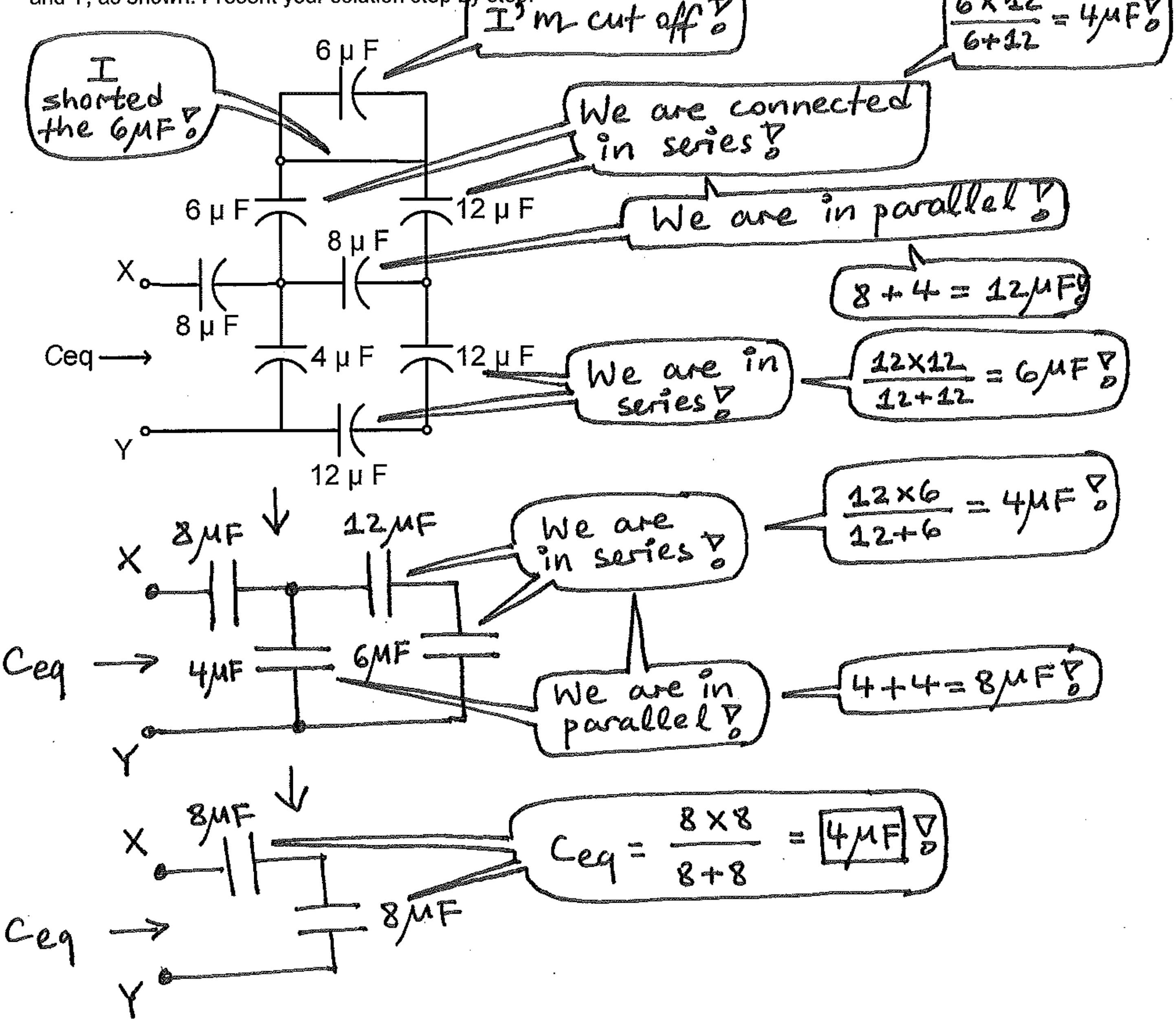
Midterm # 2 (Time: 70 minutes) ***11/16/15*** **Name:** SOLUTIONS !

Do any five out of the six problems given. Indicate which problem you didn't do! Don't try to do all six because you won't get any extra credit.

Problem 1. (20 POINTS) Determine the value of the equivalent capacitance C_{eq} seen between terminals X and Y, as shown. Present your solution step by step.

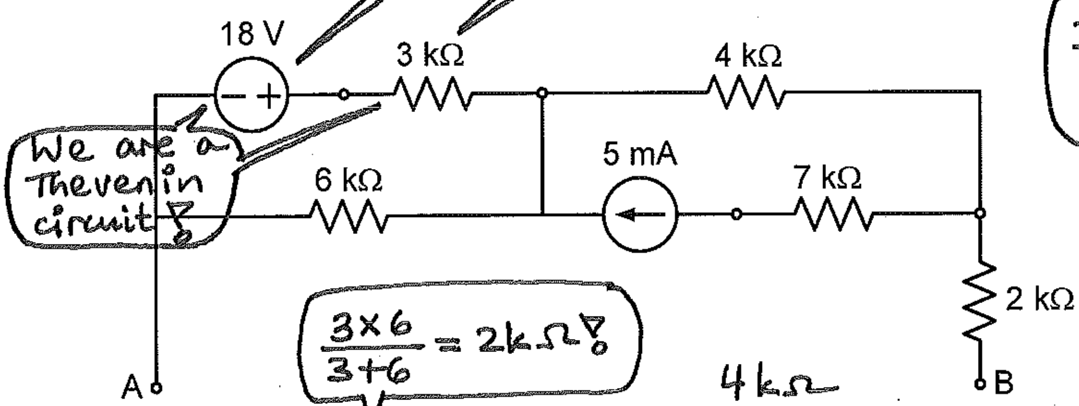


Transform us into a Norton circuit!

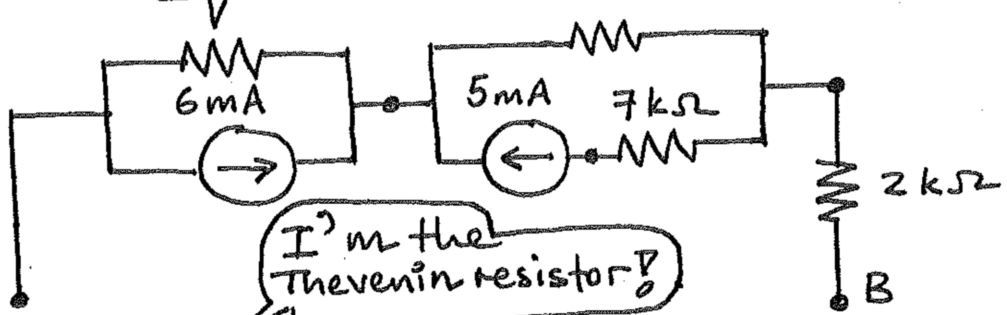
Problem 2 (20 POINTS). Determine the Thevenin equivalent of the circuit shown.

$I_N = \frac{18V}{3k\Omega} = 6mA$!

$R_N = 3k\Omega$!



$\frac{3\times6}{3+6} = 2k\Omega$!

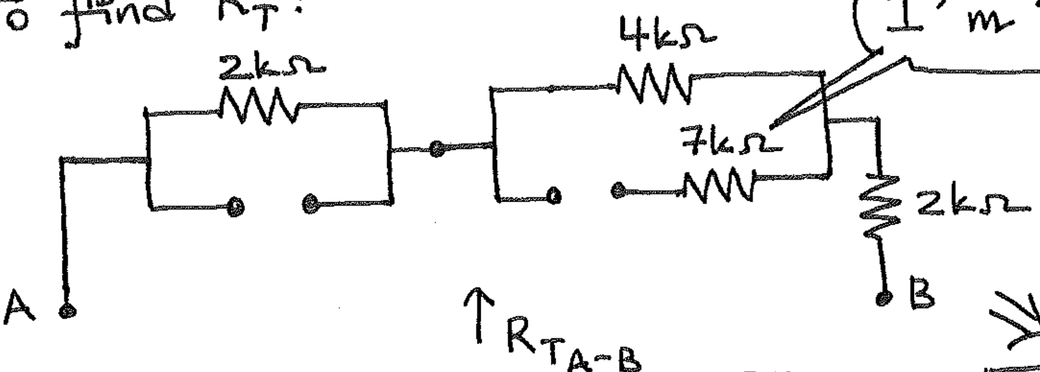


Determine V_T and R_T separately!

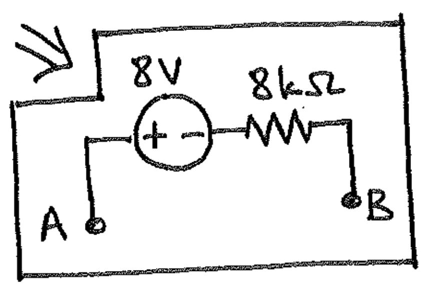
I'm the Thevenin resistor!

To find R_T:

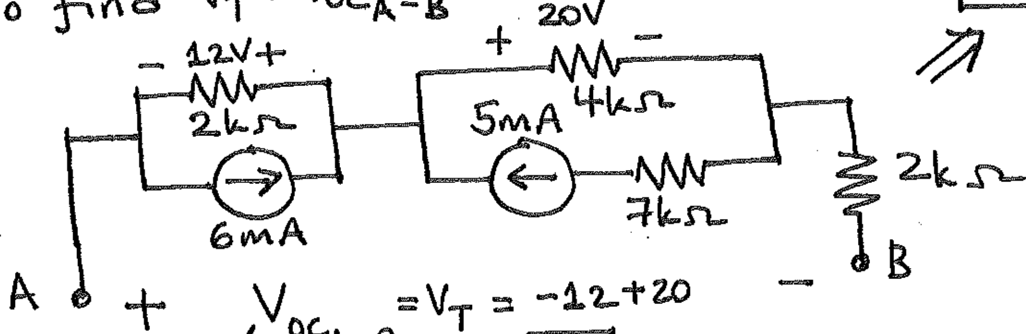
I'm ineffective!



$R_{T_{A-B}} = 2+4+2 = \boxed{8k\Omega}$



To find $V_T = V_{OC_{A-B}}$:

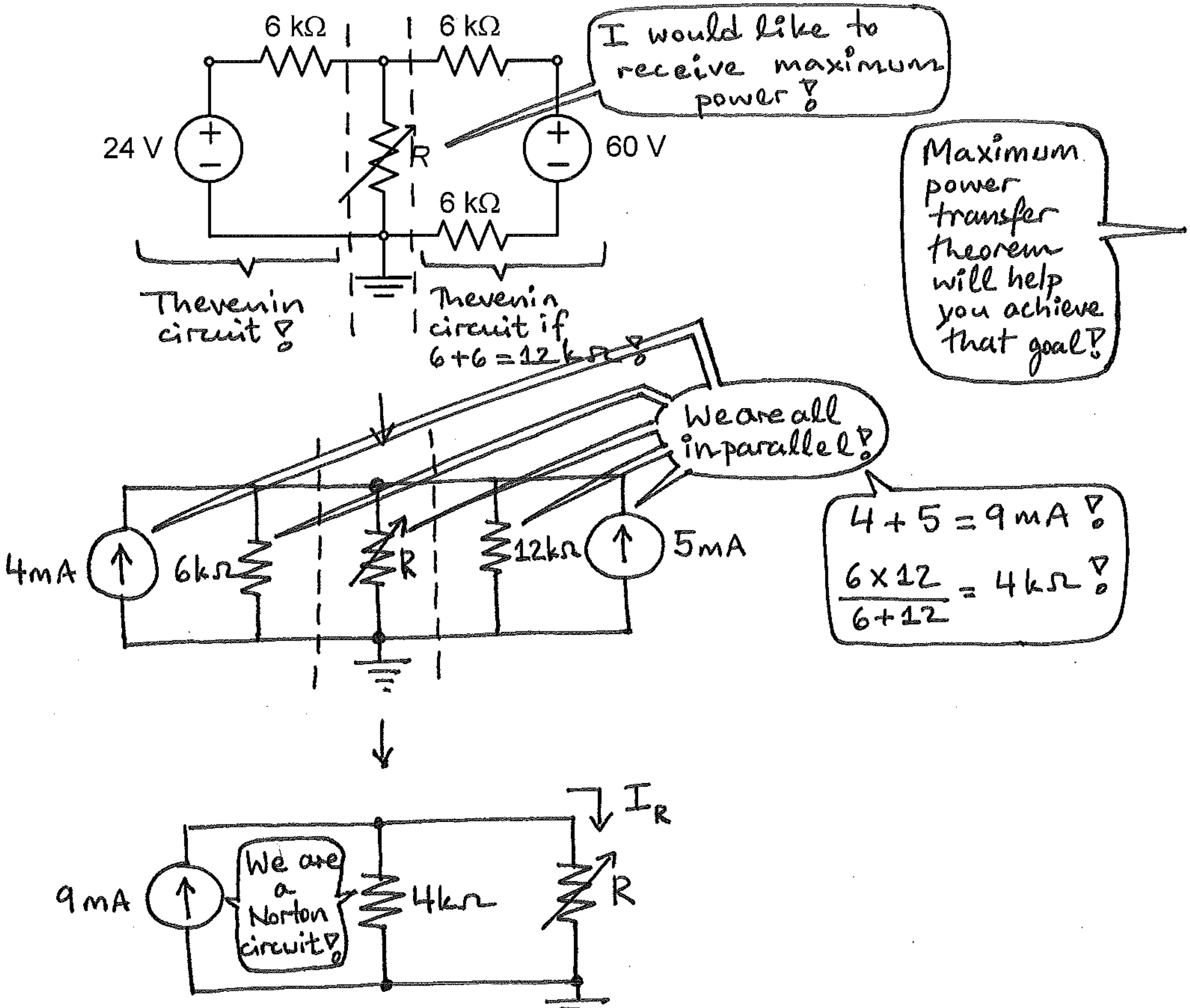


$V_{OC_{A-B}} = V_T = -12+20$

$= \boxed{8V}$

I'm the open-circuit voltage across A-B!

Problem 3 (20 POINTS). Consider the circuit shown below. Determine the value of the unknown resistance R such that it will receive maximum power from the circuit. What is the maximum power received by the R resistor you have chosen? Show your work step by step. Note that you need to provide two answers for this problem.



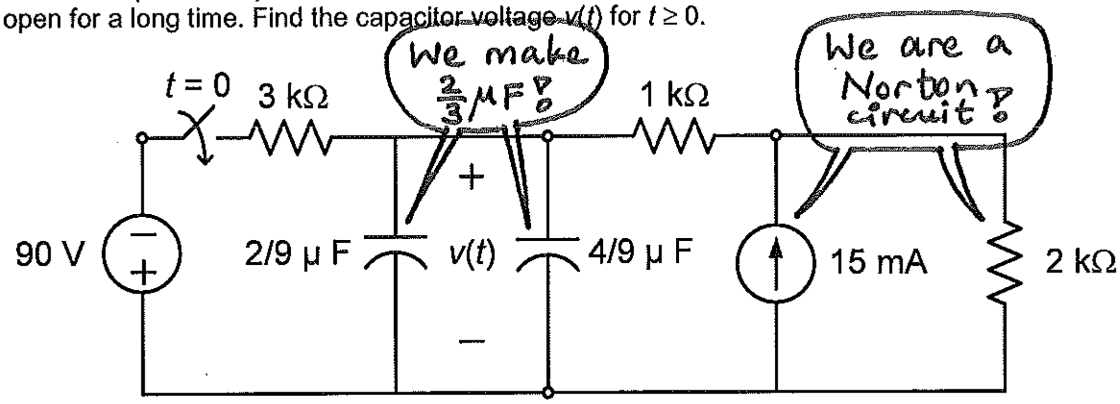
Based on Maximum Power Transfer Theorem:

$$R = R_N = \boxed{4k\Omega}$$

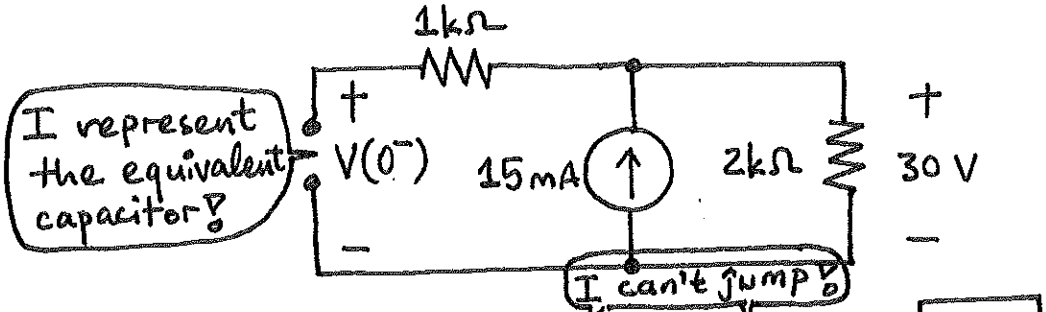
Then, $I_R = \frac{9}{2} = 4.5mA$

$\therefore P_{max} = I_R^2 R = (4.5m)^2(4k) = \boxed{81\ mW}$

Problem 4 (20 POINTS). Consider the circuit shown below where the switch closes at $t = 0$, after being open for a long time. Find the capacitor voltage $v(t)$ for $t \geq 0$.

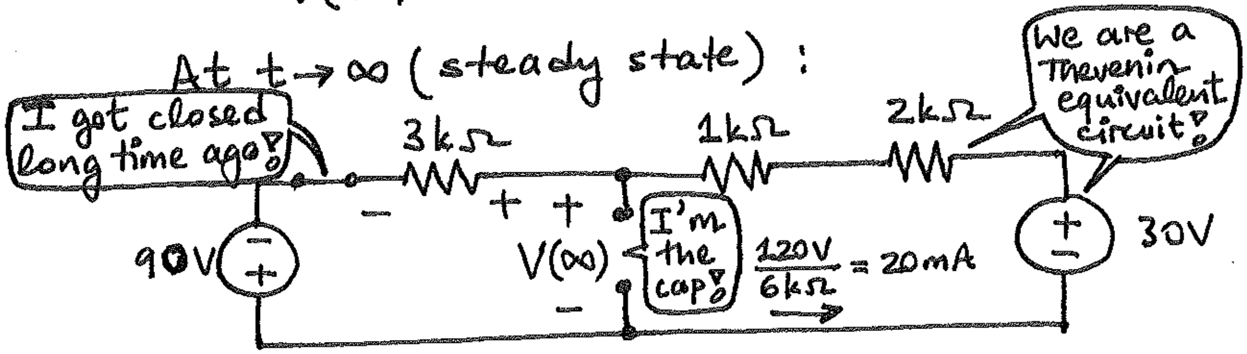


At $t = 0^-$ (steady-state condition applies):



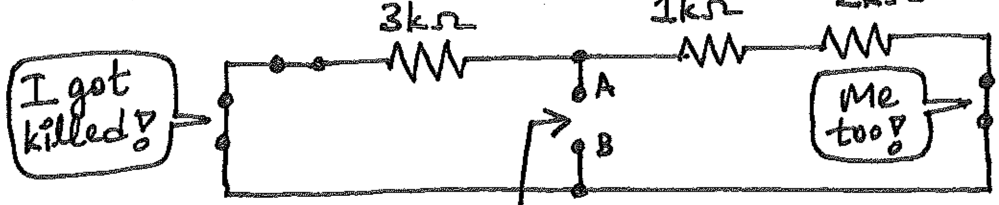
$$V(0^-) = 30\text{V} \longrightarrow V(0^+) = V(0^-) = \boxed{30\text{ V}}$$

At $t \to \infty$ (steady state):



$$\text{KVL} \rightarrow V(\infty) = (3k\Omega)(20\text{mA}) - 90 = \boxed{-30\text{ V}}$$

To determine $\tau = R_T C$:

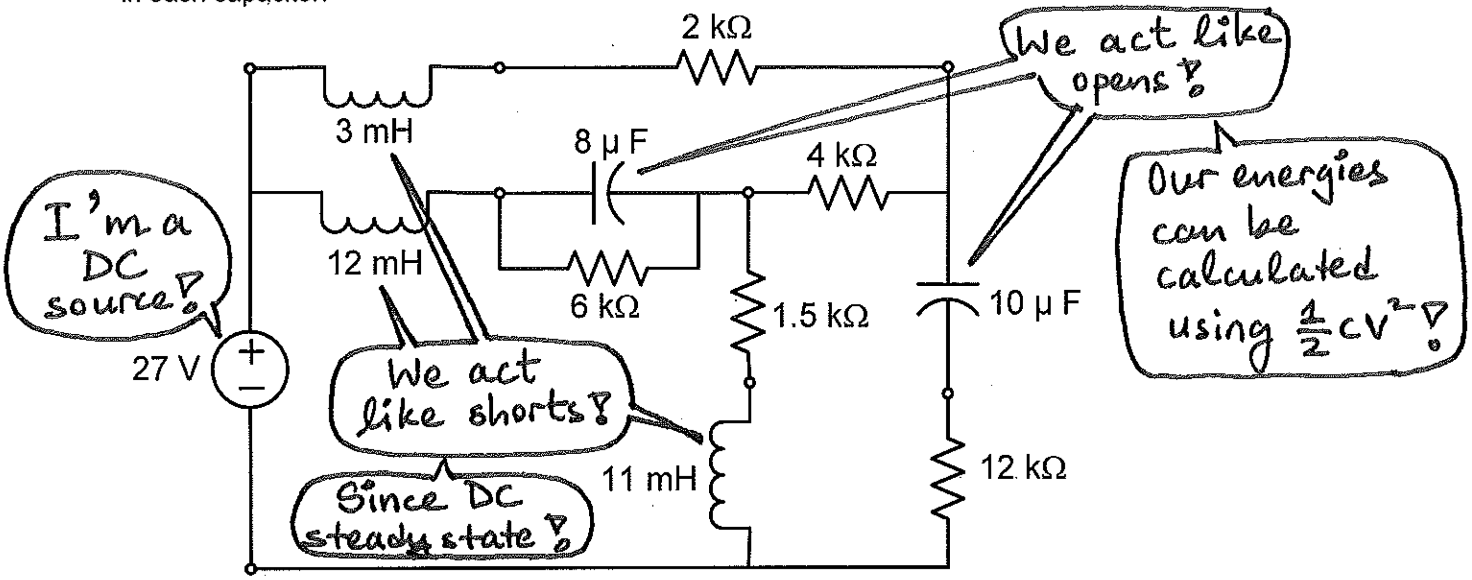


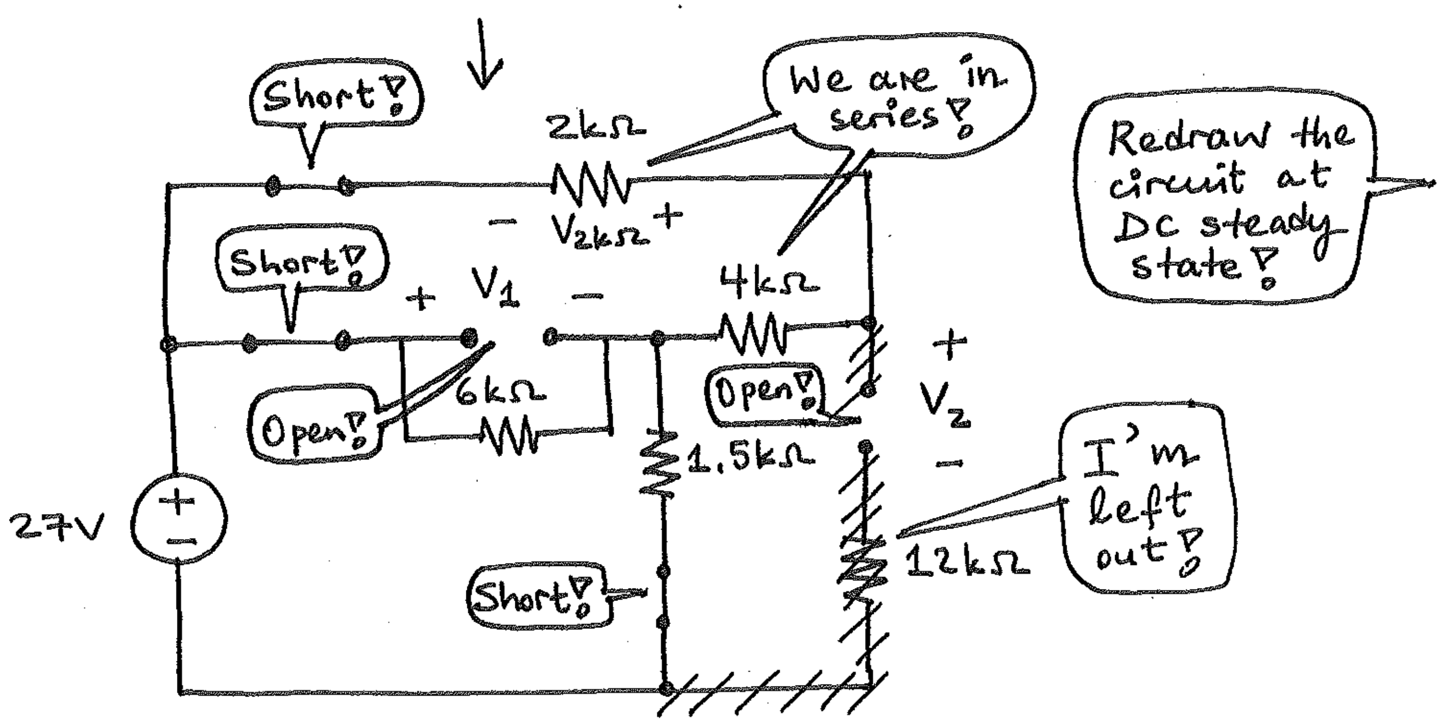
$$R_{T_{A\text{-}B}} = 3k\Omega // 3k\Omega = 1.5k\Omega$$

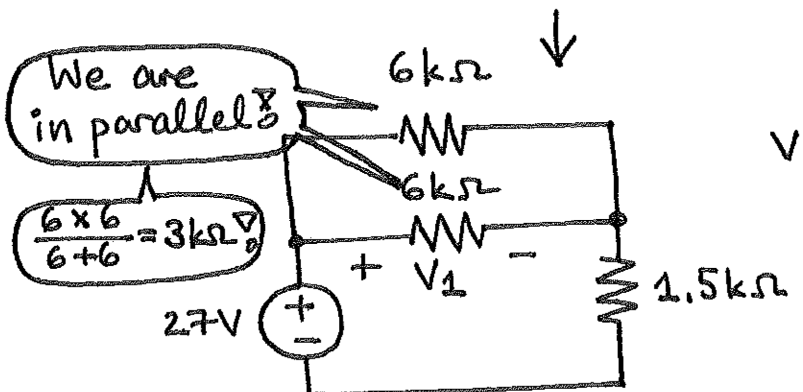
$$\tau = (1.5k\Omega)\left(\tfrac{2}{3}\mu F\right) = \boxed{1\text{ ms}}$$

$$\boxed{\therefore\ V(t) = 30e^{-1000t} - 30\left(1 - e^{-1000t}\right) = 60e^{-1000t} - 30\text{ V},\ \text{for } t \geq 0}$$

Problem 5. (20 POINTS) Assuming the circuit shown below is at steady state, determine the energy stored in each capacitor.







$$\text{VDP: } V_1 = \frac{3k\Omega}{4.5k\Omega}(27V)$$

$$= 18V$$

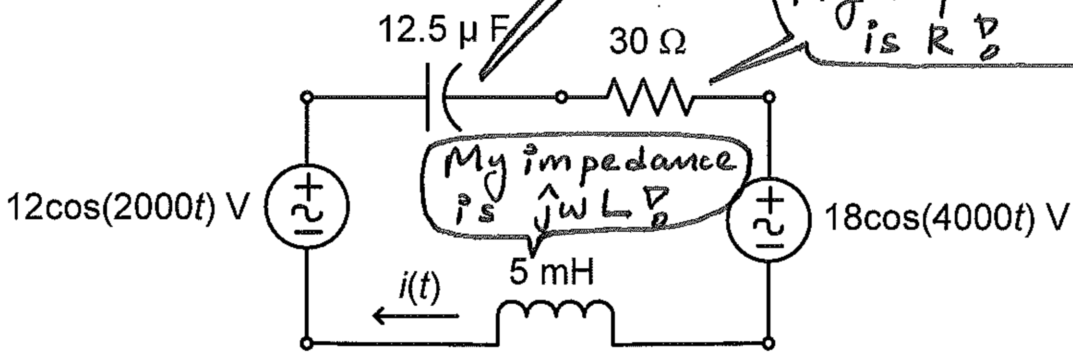
$$V_2 = V_{2k\Omega} + 27 = -6+27 = 21V$$

$$\therefore W_{8\mu F} = \frac{1}{2}(8\mu)(18)^2 = \boxed{1.296\,mJ} \;\&\; W_{10\mu F} = \frac{1}{2}(10\mu)(21)^2 = \boxed{2.205\,mJ}$$

My impedance is $-\hat{j}/(\omega C)$!

Problem 6. (20 POINTS) The circuit shown below is at steady state. Determine the full mathematical expression for the current $i(t)$. Show your work step by step.

My impedance is R!



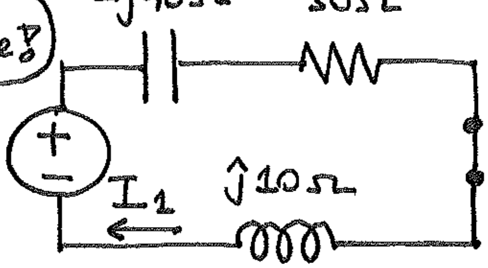
Since AC steady state, transform the circuit into phasor domain!

Superposition principle: (In phasor domain)

I'm a phasor voltage!

$V_{s1} = 12e^{\hat{j}0}$ V

$(\omega_1 = 2000$ rad/s$)$



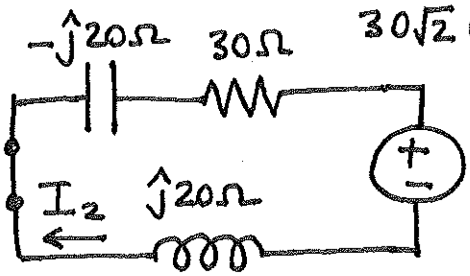
I'm eliminated!

$$\left.\begin{array}{l}\text{KVL}\\ \text{KCL}\\ \text{Ohm}\end{array}\right\} \rightarrow I_1 = \frac{V_{s1}}{Z_C + Z_R + Z_L} = \frac{12e^{\hat{j}0}}{-\hat{j}40 + 30 + \hat{j}10} = \frac{12}{30 - \hat{j}30}$$

I'm a phasor current!

$$= \frac{12}{30\sqrt{2}\, e^{-\hat{j}45°}} = \frac{\sqrt{2}}{5} e^{\hat{j}45°} \text{ A}$$

I'm dead!



I'm phasor!

$V_{s2} = 18e^{\hat{j}0}$ V

$(\omega_2 = 4000$ rad/s$)$

$$\left.\begin{array}{l}\text{KVL}\\ \text{KCL}\\ \text{Ohm}\end{array}\right\} \rightarrow I_2 = \frac{-V_{s2}}{Z_C + Z_R + Z_L} = \frac{-18e^{\hat{j}0}}{\cancel{-\hat{j}20} + 30 + \cancel{\hat{j}20}} = -0.6e^{\hat{j}0} \text{ A}$$

We are phasors!

$$\therefore\ I = I_1 + I_2 = \frac{\sqrt{2}}{5} e^{\hat{j}45°} - 0.6e^{\hat{j}0}$$

You can't subtract us because we have different frequencies!

~0.2828...

$$\boxed{\therefore\ i(t) = \frac{\sqrt{2}}{5}\cos(2000t + 45°) - 0.6\cos(4000t) \text{ A}}$$