4/21/2006

University of Portland
School of Engineering

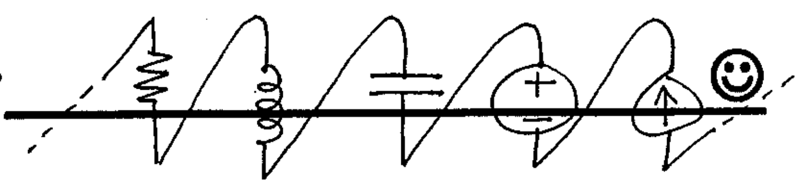
EE 261-Electrical Circuits-3 cr. hrs.
Spring 2006

Midterm Exam # 3

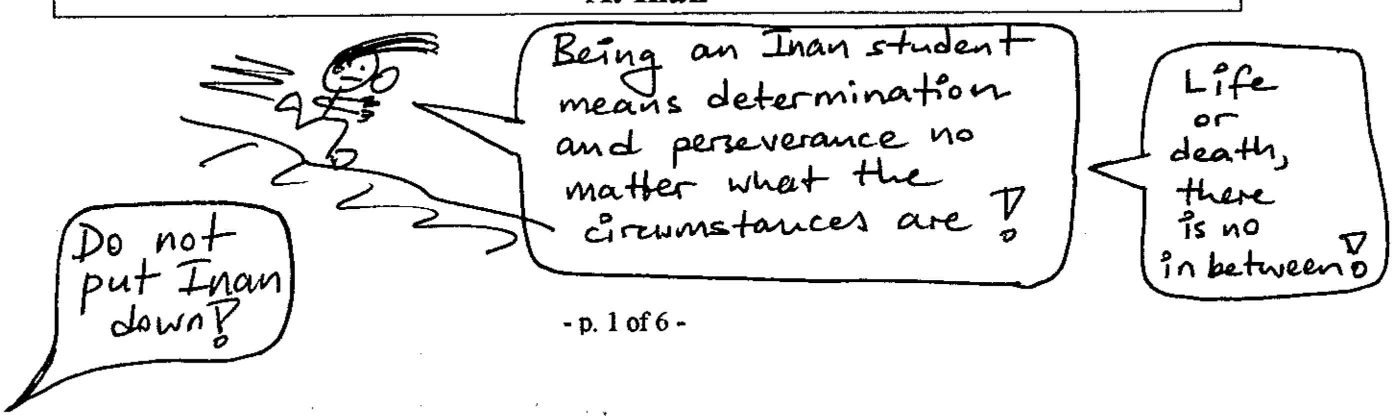
(Friday, April 21, 2006)
(Closed Book Exam, Three Formula Sheets Allowed)
(Total Time: 55 minutes)

Name: SOLUTIONS! ☺

Signature:



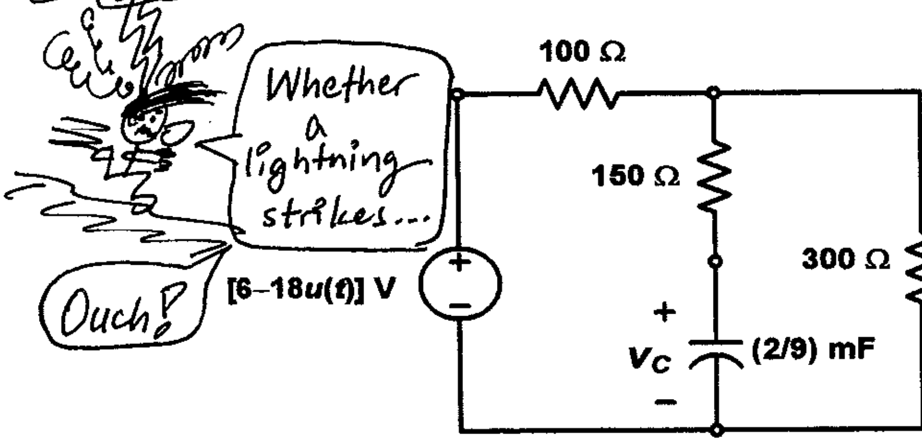
> ***"An honest mind possesses a kingdom."***
> **Lucius Annaeus Seneca (4B.C.–65A.D.)**
>
> ***"Honest people are the true winners of the universe."***
> **Anonymous**
>
> ***"Honesty is not for sale."***
> **A. Inan**



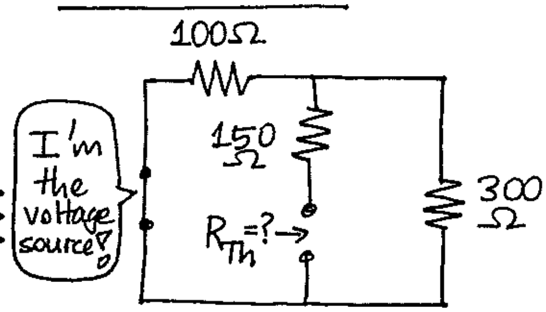
This box will be used by Inan for grading →	Problem # 1
	Problem # 2
	Problem # 3
	Total Score:

NOTE: On all the problems, please show your work clearly, and provide the appropriate units for your answers. Also mark on the schematic to show any current or voltage that you define in your solution.

1. (15 mins., Total: 30 points) **Step excitation of a first-order *RC* circuit.** Consider the first-order *RC* circuit shown.



To find τ:

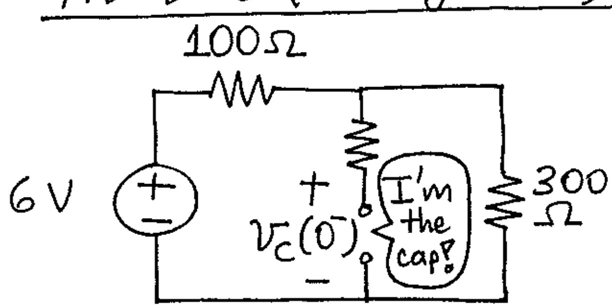


$R_{Th} = 150 + (100 // 300) = 225\Omega$

$\therefore \tau = R_{Th}C = (225)\left(\frac{2}{9}\text{m}\right) = \boxed{50\text{ms}}$

(a) (20 points) Find the complete mathematical expression for the capacitor voltage $v_C(t)$ for $t \geq 0$.

At $t = 0^-$ (steady state):



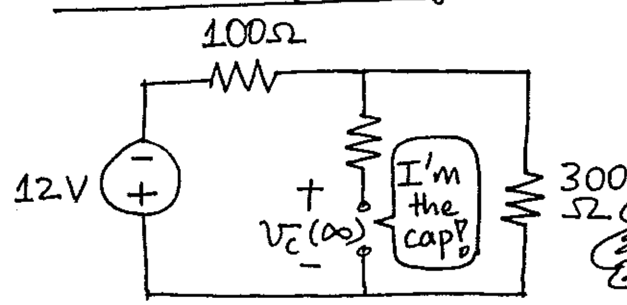
Voltage divider principle:

$v_C(0^-) = \frac{300}{400}(6\text{V})$

$= 4.5\text{V}$

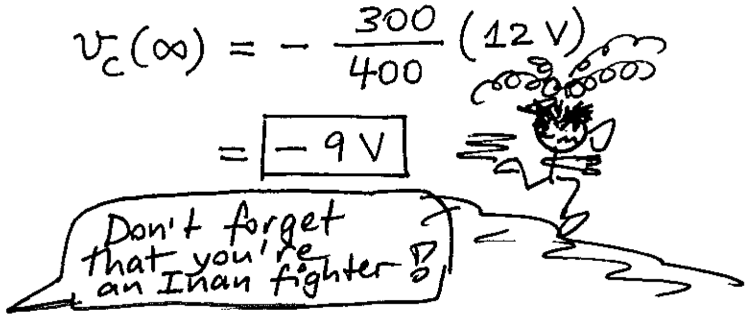
$\therefore v_C(0^+) = v_C(0^-) = \boxed{4.5\text{V}}$

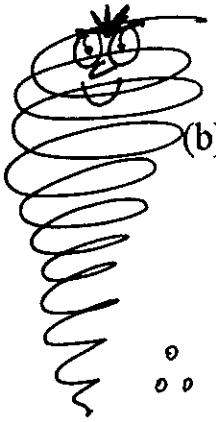
At $t = \infty$ (steady state):



$v_C(\infty) = -\frac{300}{400}(12\text{V})$

$= \boxed{-9\text{V}}$

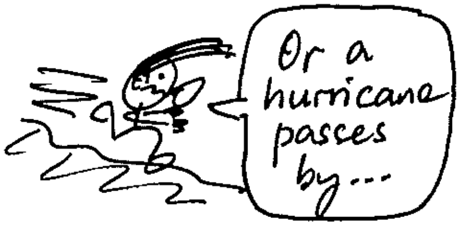


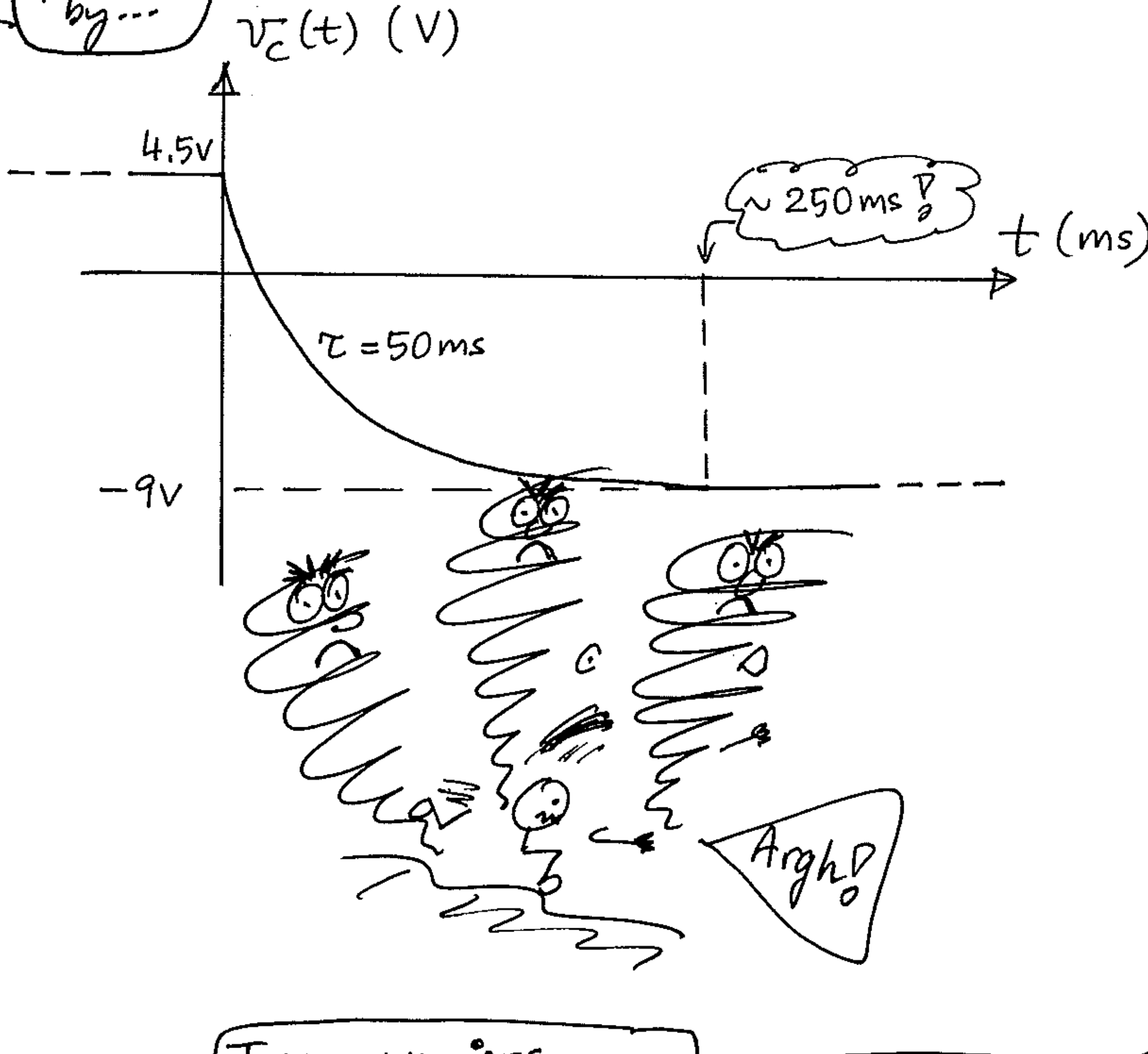


(b) (10 points) Sketch the waveform $v_C(t)$ found in part (a) roughly with respect to time between $t = 0$ and $t = \infty$ and provide all the appropriate values including their units on the sketch.

$$\therefore v_C(t) = v_C(0^+)e^{-t/\tau} + v_C(\infty)(1-e^{-t/\tau})$$

$$= \boxed{4.5e^{-20t} - 9(1-e^{-20t}), \text{ for } t \geq 0}$$





Inan warriors are trained to survive under the toughest conditions!

And you are one of them!

Be proud of yourself!

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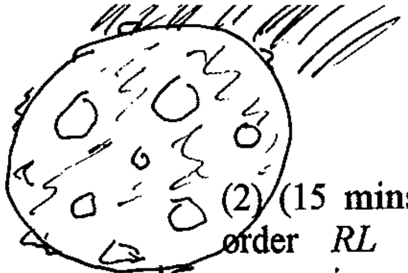
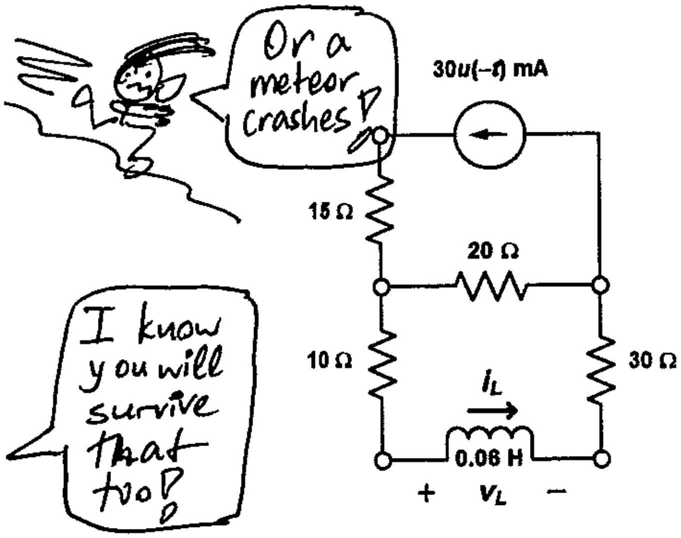
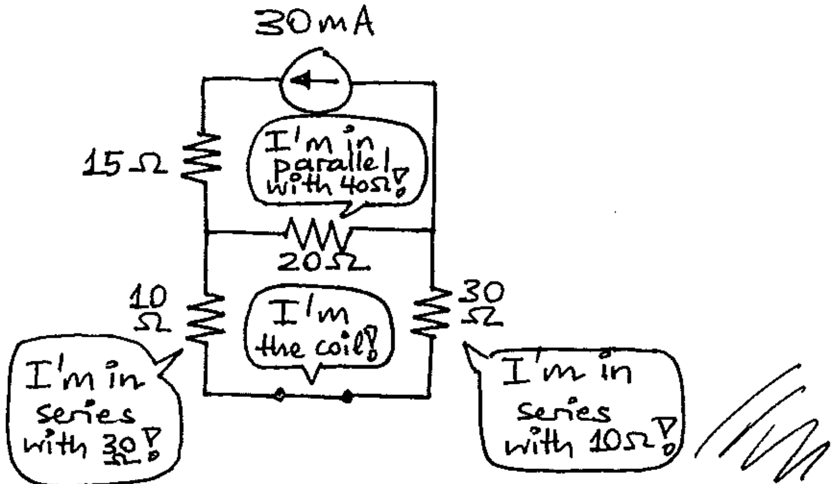


(2) (15 mins., 30 points) **A first-order *RL* circuit.** In the first-order *RL* circuit shown, find the complete mathematical expressions for the inductor current $i_L(t)$ and the inductor voltage $v_L(t)$ for $t \geq 0^-$. Please put each expression in a rectangular box.

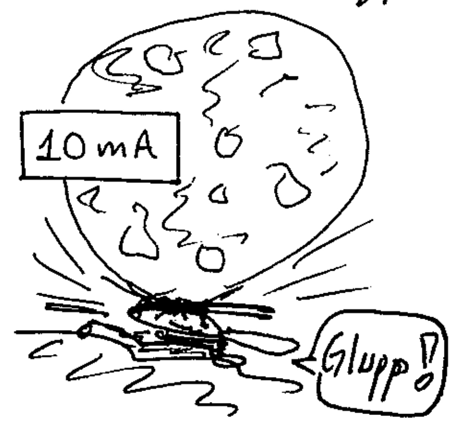


At $t=0^-$ (steady state):

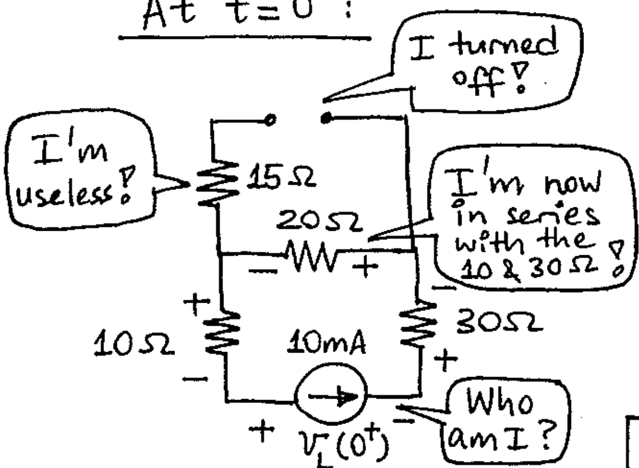


Current divider principle:

$$i_L(0^+) = i_L(0^-) = \frac{20}{20+\underbrace{10+30}_{40}}(30\text{mA}) = \boxed{10\text{ mA}}$$

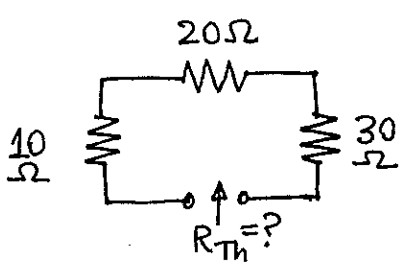


At $t=0^+$:



$$v_L(0^+) = -(60\Omega)(10\text{mA}) = \boxed{-0.6\text{ V}}$$

To find τ:



$R_{Th} = 60\Omega$

$\therefore \tau = \frac{L}{R_{Th}} = \frac{0.06}{60} = 1\text{ ms}$

$$\boxed{\begin{aligned} \therefore\ i_L(t) &= 10e^{-1000t}\ (\text{mA}) \\ v_L(t) &= -0.6e^{-1000t}\ (\text{V}) \end{aligned}} \quad \text{for } t>0$$

Note that $v_L(0^-)=0$. Also note that $i_L(\infty)=0$ & $v_L(\infty)=0$.

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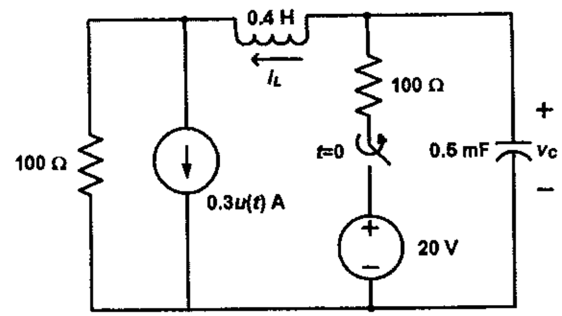
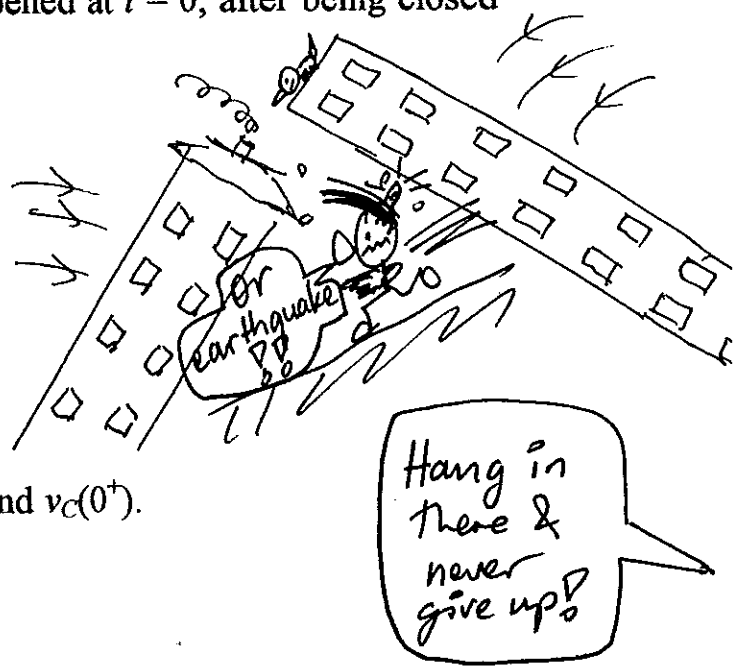
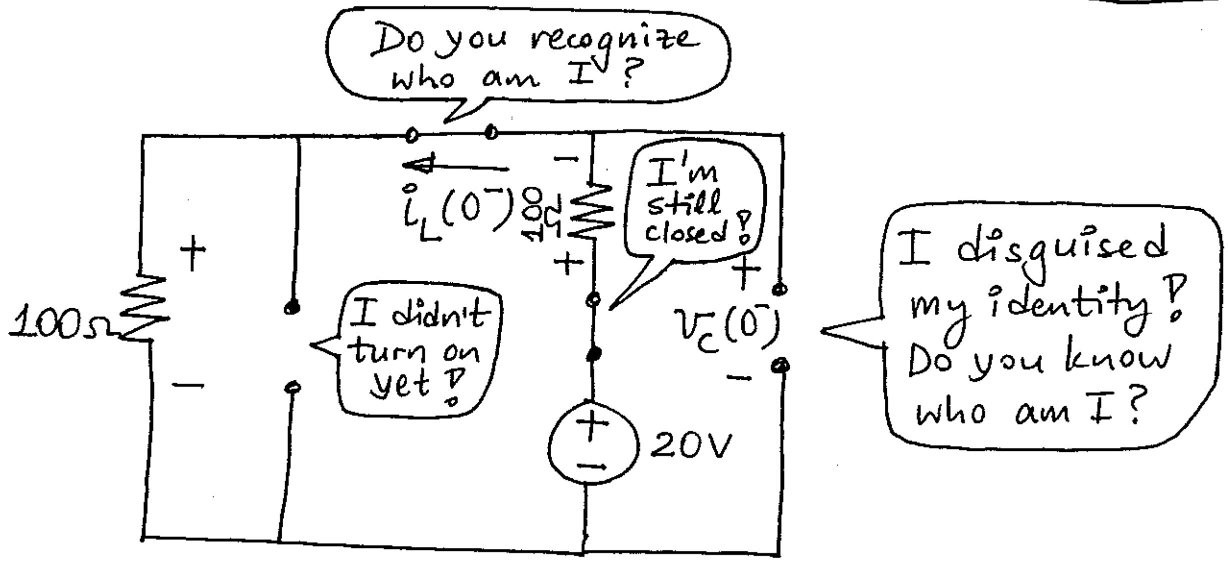
(3) (15 mins., <u>Total:</u> 40 Points) **A second-order switching circuit.** In the second-order circuit shown, the switch is opened at $t = 0$, after being closed for a long time.





(a) (15 points) Find the values of $i_L(0^+)$ and $v_C(0^+)$.

At $t = 0^-$ (steady state):



$$\therefore\; i_L(0^+) = i_L(0^-) = \frac{20\text{ V}}{(100+100)\,\Omega} = \boxed{0.1\text{ A}}$$

$$v_C(0^+) = v_C(0^-) = (100\,\Omega)(0.1\text{ A}) = \boxed{10\text{ V}}$$

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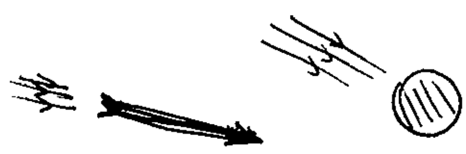
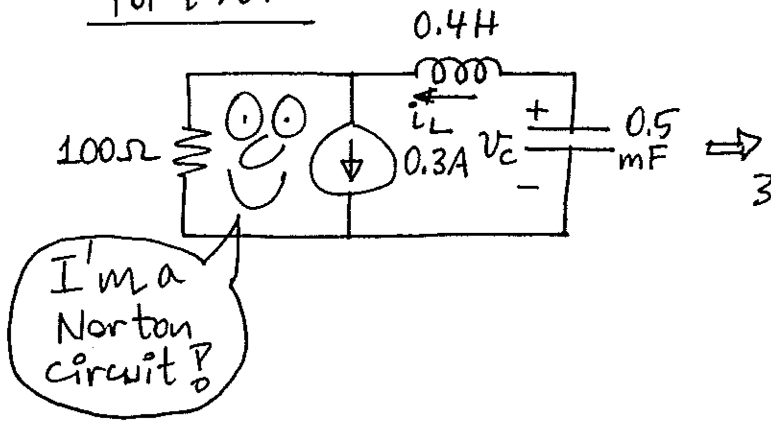
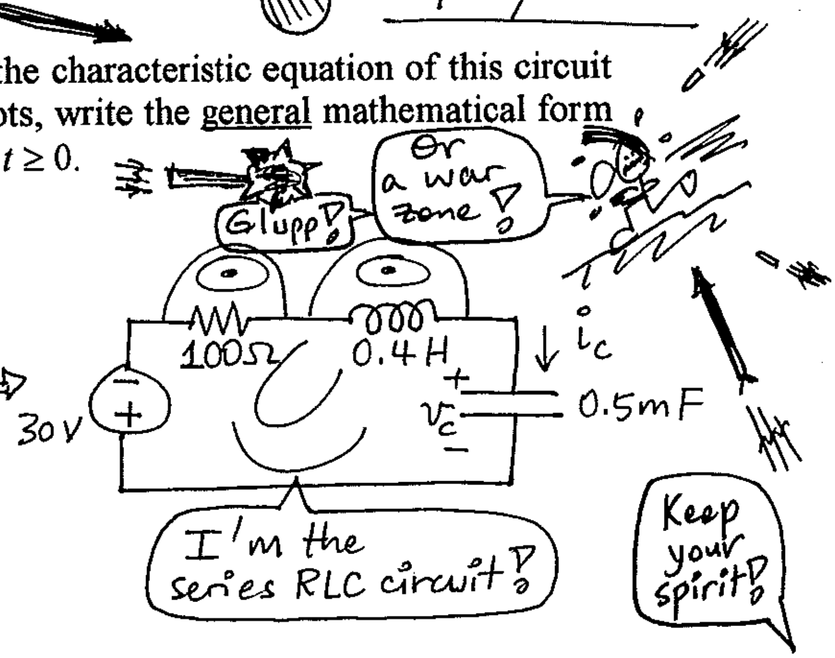


(b) (15 points) Find the roots of the characteristic equation of this circuit after $t = 0$ and based on these roots, write the general mathematical form for the capacitor voltage $v_C(t)$ for $t \geq 0$.

For $t > 0$:





The characteristic equation is given by

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + \frac{100}{0.4}s + \frac{1}{(0.4)(0.5m)} = s^2 + 250s + 5000 = 0$$

The characteristic roots are $s_1, s_2 = -125 \pm \sqrt{(125)^2 - 5000} \cong -21.9$ & -228.1

$$\therefore v_C(t) = A_1 e^{-21.9t} + A_2 e^{-228.1t} + A_3, \text{ valid for } t \geq 0$$

(c) (10 points) Find the complete mathematical expression for the capacitor voltage $v_C(t)$ after $t = 0$. (This is the part where you find the coefficients of $v_C(t)$.)

$$v_C(0) = A_1 + A_2 + A_3 = 10 \text{ V}$$

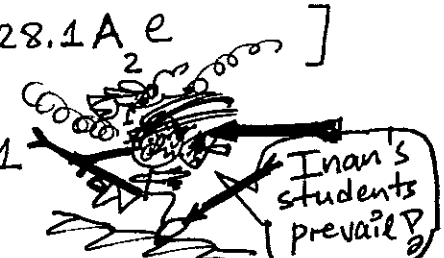
$$v_C(\infty) = A_3 = -30 \text{ V}$$

$$\rightarrow A_1 + A_2 = 40 \text{ V} \quad \text{— (I)}$$

$$i_L(t) = -i_C(t) = -C\left[-21.9A_1 e^{-21.9t} - 228.1A_2 e^{-228.1t}\right]$$

$$i_L(0) = (0.5m)\left[21.9A_1 + 228.1A_2\right] = 0.1$$

$$\rightarrow 21.9A_1 + 228.1A_2 = 200 \quad \text{— (II)}$$



Solving equations (I) & (II) simultaneously yields

$A_1 \cong 43.3$

$A_2 \cong -3.28$

$$\therefore v_C(t) \cong 43.3e^{-21.9t} - 3.28e^{-228.1t} - 30, \text{ for } t \geq 0$$