4/22/2007

University of Portland
School of Engineering

EE 261-Electrical Circuits-3 cr. hrs.
Spring 2007

Midterm Exam # 3

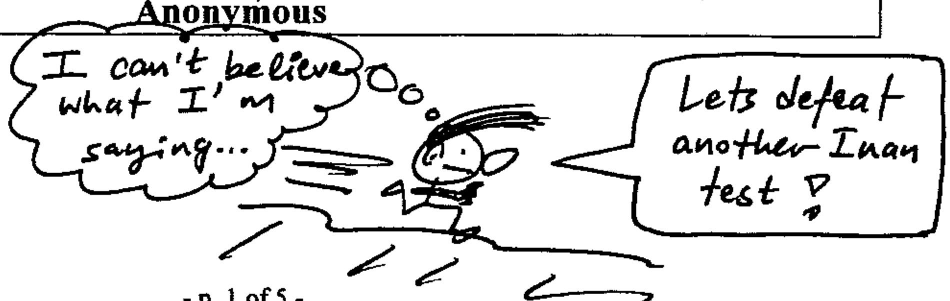
(Friday, April 20, 2007)
(Closed Book Exam, Three Formula Sheet are Allowed)
(Total Time: 55 minutes)

Name: SOLUTIONS ! ☺

Signature: Solutions ! ☺

"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

"Honest people are the true winners of the universe."
Anonymous



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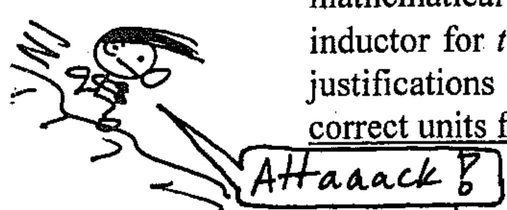
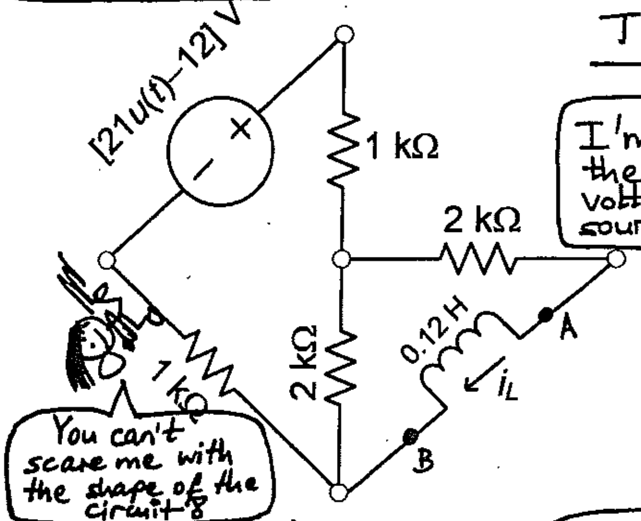
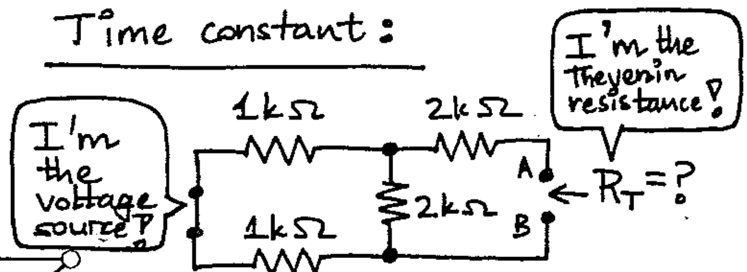
> **NOTE:** On all the problems, please show your work clearly, and provide the appropriate units for your answers. Also mark on the schematic to show any current or voltage that you define in your solution.

1. (15 mins., 30 points) In the circuit shown, find the complete mathematical expression for the current $i_L(t)$ through the 0.12 H inductor for $t \geq 0$. (Please show your work clearly and provide brief justifications for the steps you take. <u>Also, don't forget to provide the correct units for your answers.</u>)





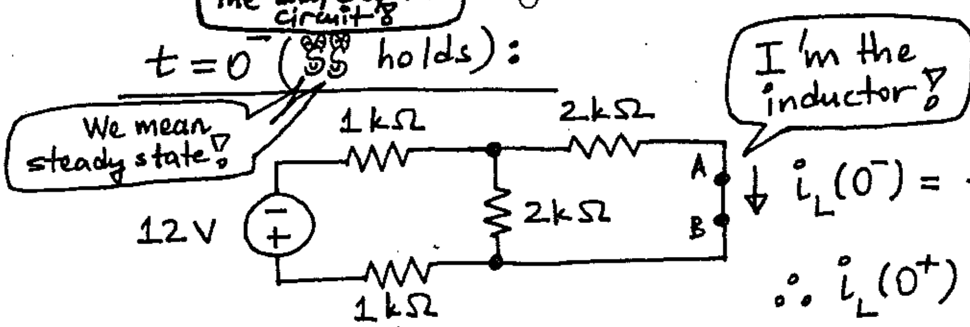
Time constant:



$R_T = 2 + 2 // (1+1) = 3\,k\Omega$

$\therefore \tau = \frac{L}{R_T} = \frac{0.12\,H}{3\,k\Omega} = 0.04\,ms$

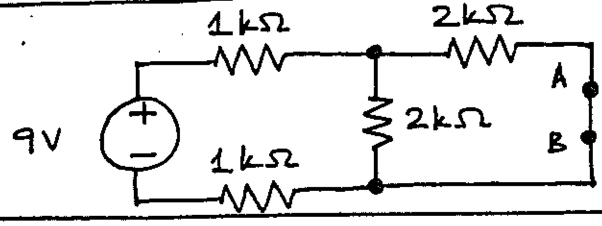
$t = 0^-$ (SS holds):



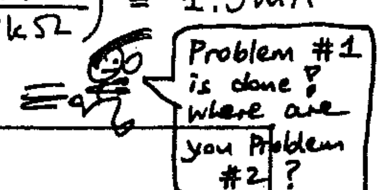
$i_L(0^-) = -\frac{1}{2}\left(\frac{12V}{3k\Omega}\right) = -2\,mA$

$\therefore i_L(0^+) = i_L(0^-) = -2\,mA$

$t = \infty$ (SS holds):



$i_L(\infty) = \frac{1}{2}\left(\frac{9V}{3k\Omega}\right) = 1.5\,mA$



$$\therefore i_L(t) = -2e^{-25{,}000t} + 1.5\left(1 - e^{-25{,}000t}\right)$$
$$= 1.5 - 3.5e^{-25{,}000t}\ mA$$
for $t \geq 0$

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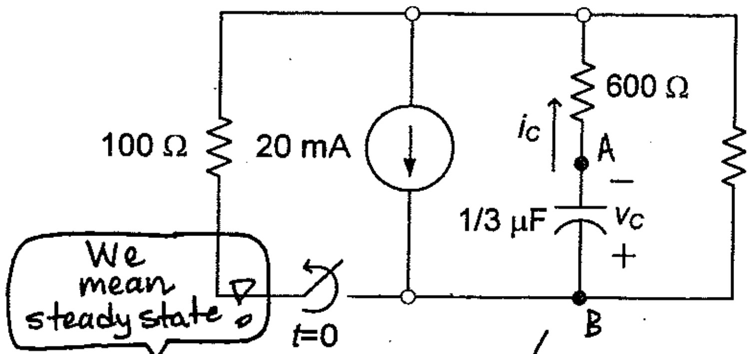
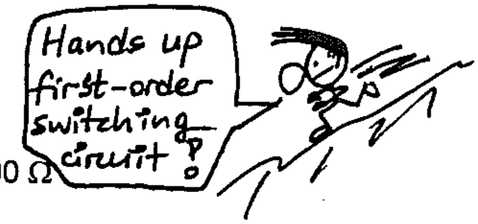
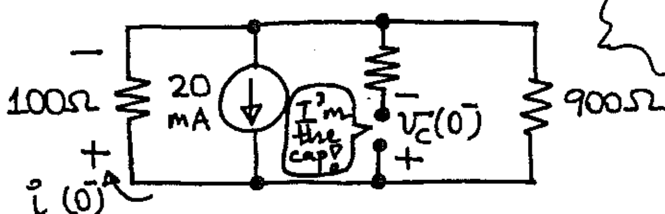
2. (15 mins., 30 Points) In the circuit shown below, the switch <u>opens</u> at t=0, after being closed for a long time. Find the complete mathematical expression for the voltage $v_C(t)$ and the current $i_C(t)$ of the 1/3 μF capacitor for $t \geq 0$. (Please show your work step by step. Also, again, provide appropriate units.)





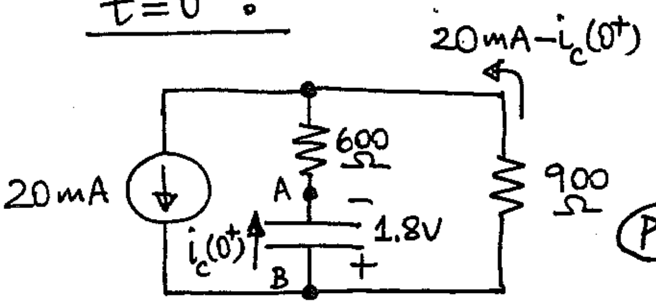
$t=0^-$ (SS holds):



CDP → $i_x(0^-) = \frac{900}{100+900}(20mA) = 18mA$

∴ $v_C(0^-) = 100 i_x(0^-) = 1.8V$

$t=0^+$:

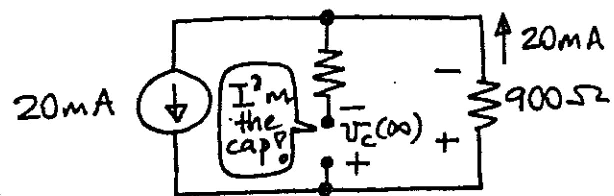


Note that $v_c(0^+) = v_c(0^-) = 1.8V$

KVL → $1.8 + 600 i_c(0^+) - 900(20mA - i_c(0^+)) = 0$

→ $i_c(0^+) = \frac{18-1.8}{1500} = 10.8\ mA$

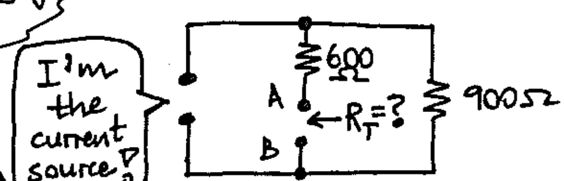
$t=\infty$ (SS holds):

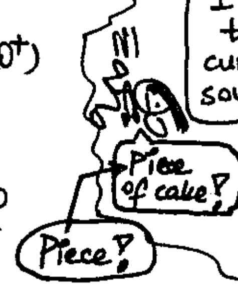


∴ $v_c(\infty) = (20mA)(900\Omega)$

$= 18V$

Time constant:

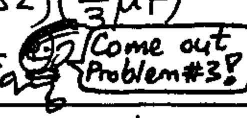




$R_T = 600 + 900 = 1.5k\Omega$

∴ $\tau = R_T C = (1.5k\Omega)(\frac{1}{3}\mu F)$

$= 5 \times 10^{-4} s$



∴ $i_c(t) = 10.8 e^{-2000t}$ (mA) &

$v_c(t) = 18 - 16.2 e^{-2000t}$ (V)

for $t > 0$

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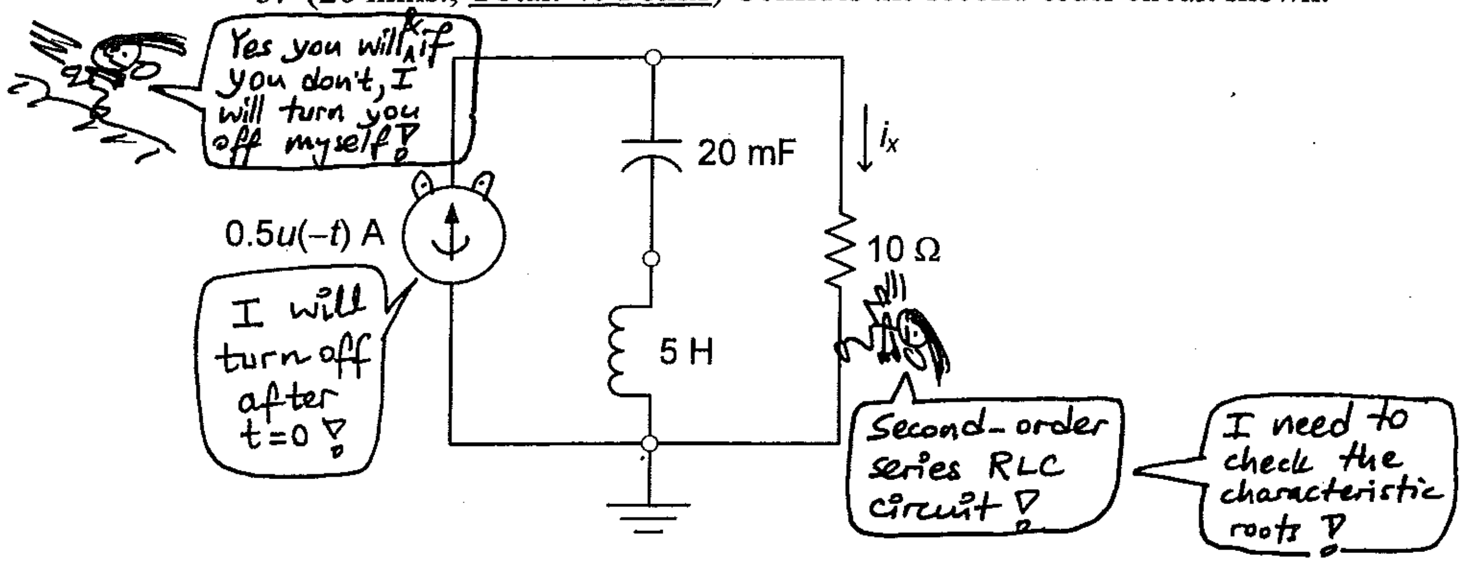
3. (20 mins., Total: 40 Points) Consider the second-order circuit shown.



(a) (15 points) Solve for the roots (s_1 and s_2) of the characteristic equation of the above circuit for $t \geq 0$.

Characteristic equation of a series RLC circuit is given by $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

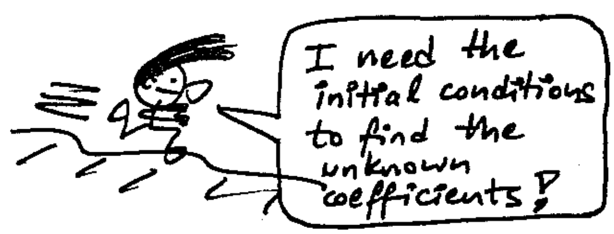
$$\rightarrow s^2 + \frac{10}{5}s + \frac{1}{(5)(20m)} = 0 \rightarrow s^2 + 2s + 10 = 0$$

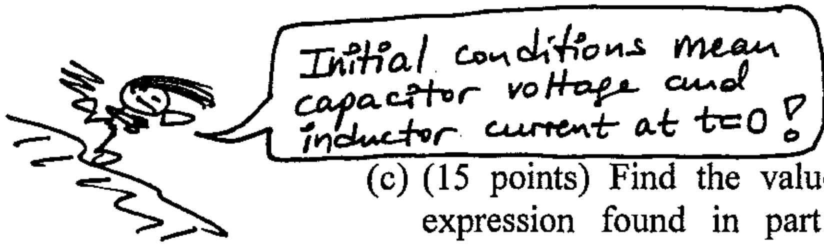
$$\therefore s_1, s_2 = \frac{-1 \pm \sqrt{1 - 10}}{1} = -1 \pm \hat{j}3$$

(b) (10 points) Based on the results of part (a), write the general mathematical expression for the current $i_x(t)$ for $t \geq 0$. (At this stage, leave the coefficients in your answer as unknown variables.)

Since the characteristic roots are complex numbers, it's underdamped response.

$$\therefore i_x(t) = A_1 e^{-t}\cos(3t) + A_2 e^{-t}\sin(3t) + A_3$$





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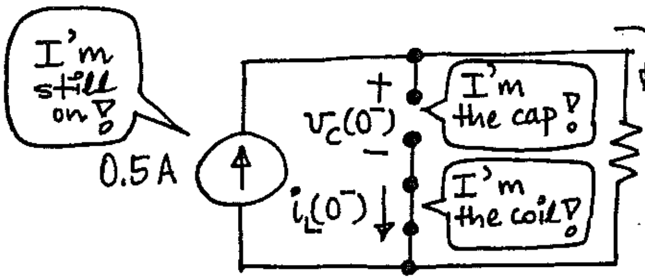
(c) (15 points) Find the values of the coefficients of the $i_x(t)$ expression found in part (b) using the initial and final conditions.

$t = 0^-$ (SS holds):



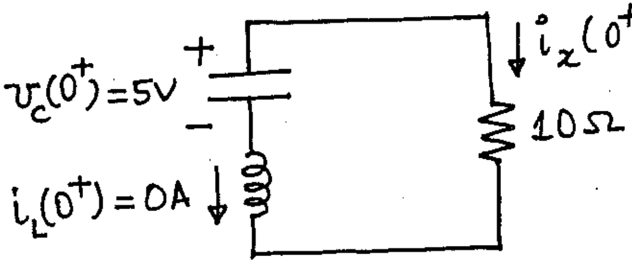
$\therefore v_C(0^-) = (10\Omega)(0.5A) = 5V$

$i_L(0^-) = 0$

$\therefore v_C(0^+) = v_C(0^-) = 5V$

$i_L(0^+) = i_L(0^-) = 0$

$t = 0^+$:



$i_x(0^+) = -i_L(0^+) = 0$

$t = \infty$ (SS holds):

$i_x(\infty) = 0$ since the capacitor will become open.

$\therefore A_3 = 0.$

$\therefore i_x(0^+) = A_1 + A_3 = A_1 = 0$

$\therefore i_x(t) = A_2 e^{-t}\sin(3t)$

Using KVL $\rightarrow v_C(t) + L\dfrac{di_L(t)}{dt} - Ri_x(t) = 0$ ($di_L(t)$: $-i_x(t)$)

$\rightarrow v_C(t) - 5\left[-A_2 e^{-t}\sin(3t) + 3A_2 e^{-t}\cos(3t)\right] - 10A_2 e^{-t}\sin(3t) = 0$

At $t = 0^+ \rightarrow 5 - 15A_2 = 0 \rightarrow \therefore A_2 = \frac{1}{3}$

$\therefore i_x(t) = \frac{1}{3}e^{-t}\sin(3t)$, for $t > 0$