4/10/2006

University of Portland
School of Engineering

EE 262-Signals & Systems-3 cr. hrs.
Spring 2006

Midterm Exam # 2
(Prepared by Professor A. S. Inan)

(Friday, March 31, 2006)

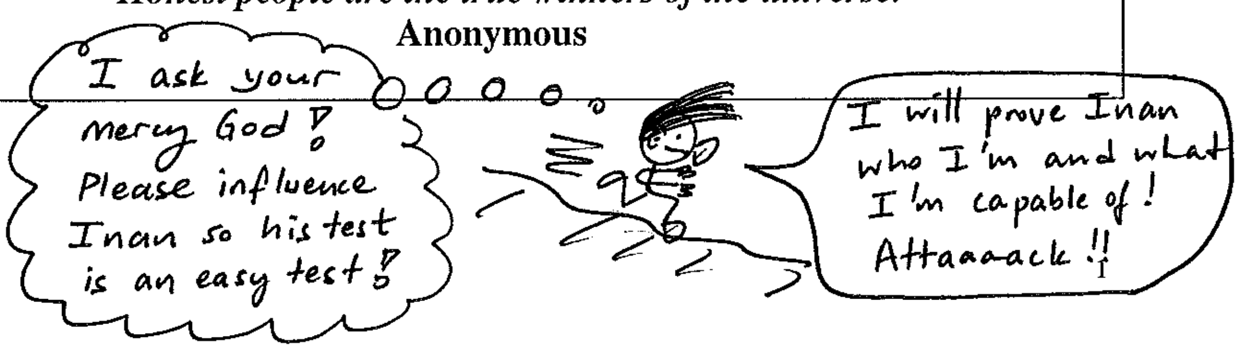
Name: SOLUTIONS ☺

Signature: ☺

"Honesty is the best policy."
Aesop (~ 620B.C. –?)

"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

"Honest people are the true winners of the universe."
Anonymous



Inan is trying to demoralize me with the word "zero" to make me think that I will get zero in this test!

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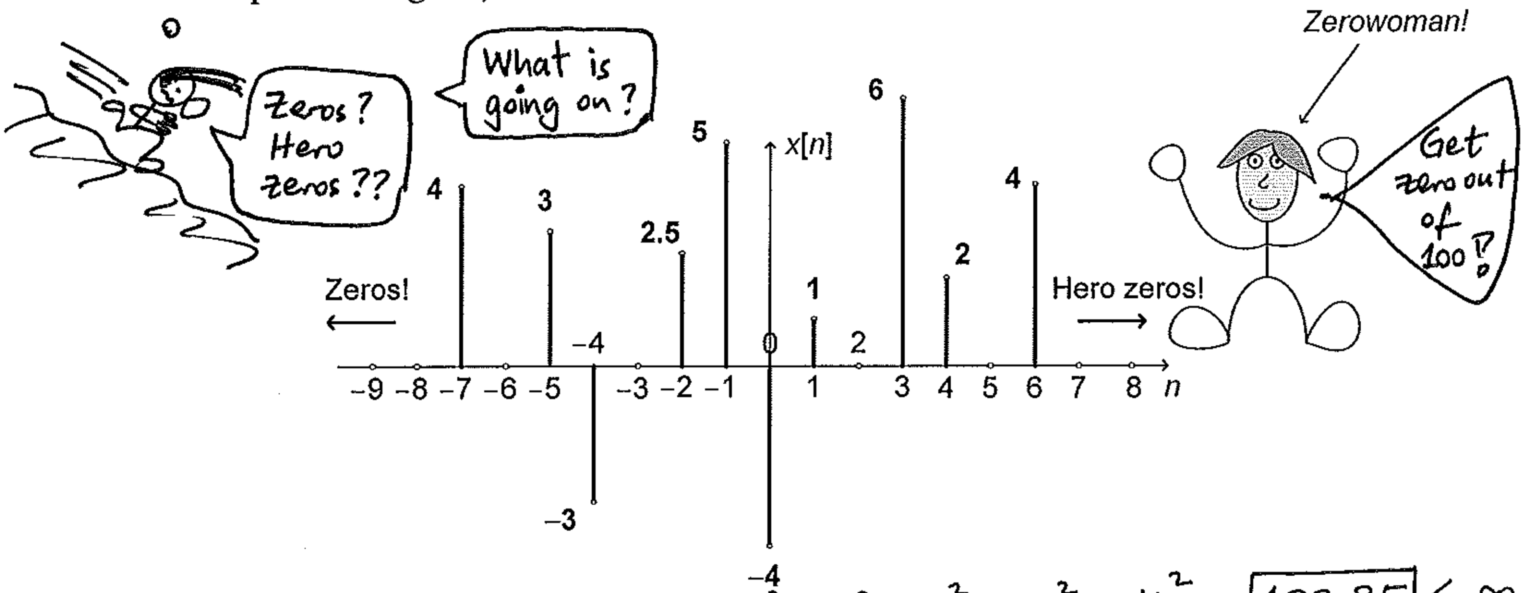
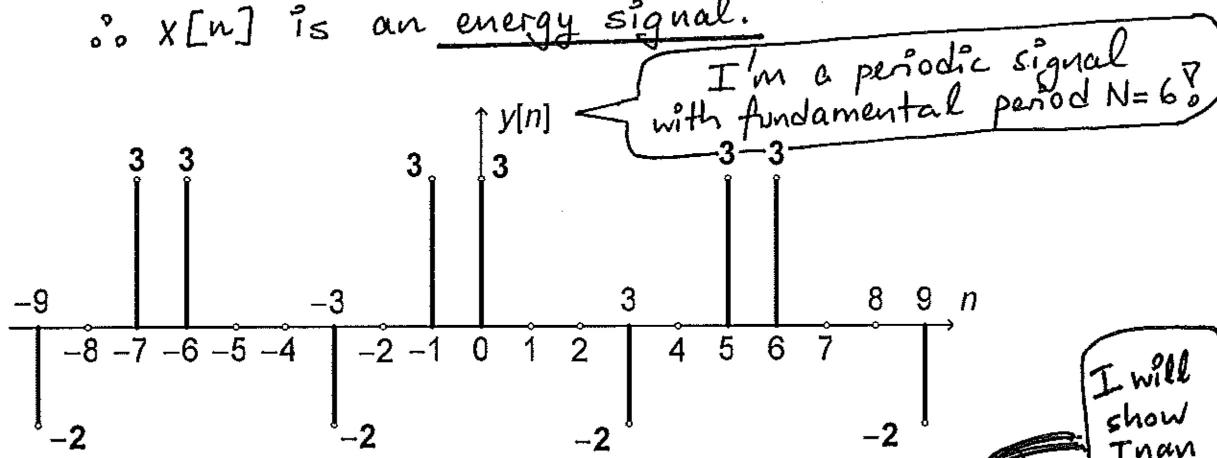
(1) (20 points) **Energy and power of a signal.** Find the energy and power of each of the two discrete-time signals $x[n]$ and $y[n]$ shown and determine which one is energy signal and which one is power signal. (Note that $y[n]$ is a periodic signal.)



$$E_x = 4^2 + 3^2 + (-3)^2 + (2.5)^2 + 5^2 + (-4)^2 + 1^2 + 6^2 + 2^2 + 4^2 = \boxed{138.25} < \infty$$

$P_x = \boxed{0}$ $\therefore$ $x[n]$ is an energy signal.



$E_y = \boxed{\infty}$

$$P_y = \frac{(-2)^2 + 3^2 + 3^2}{6} = \frac{22}{6} = \frac{11}{3} \cong \boxed{3.67}$$

$\therefore$ Since $0 < P_y < \infty$, $y[n]$ is a power signal.

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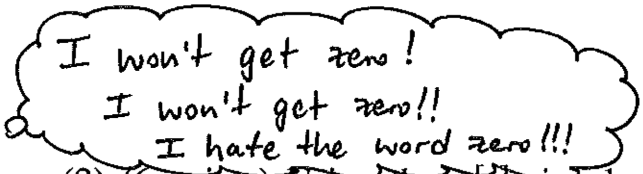
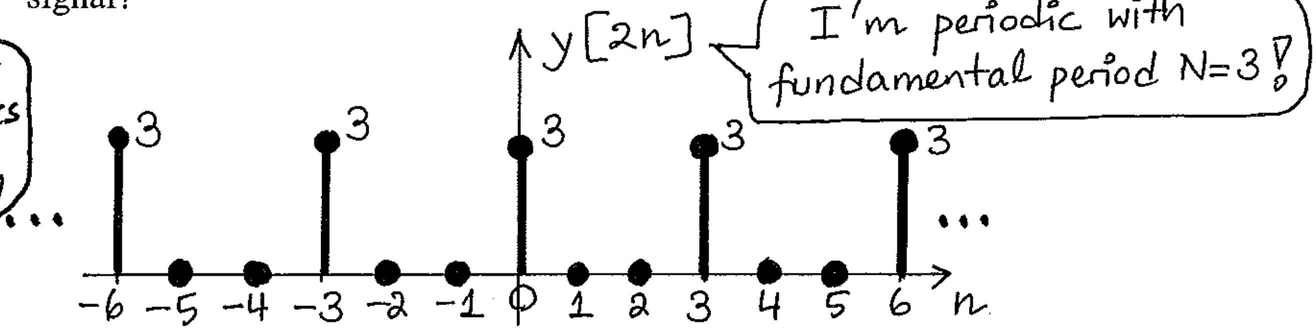
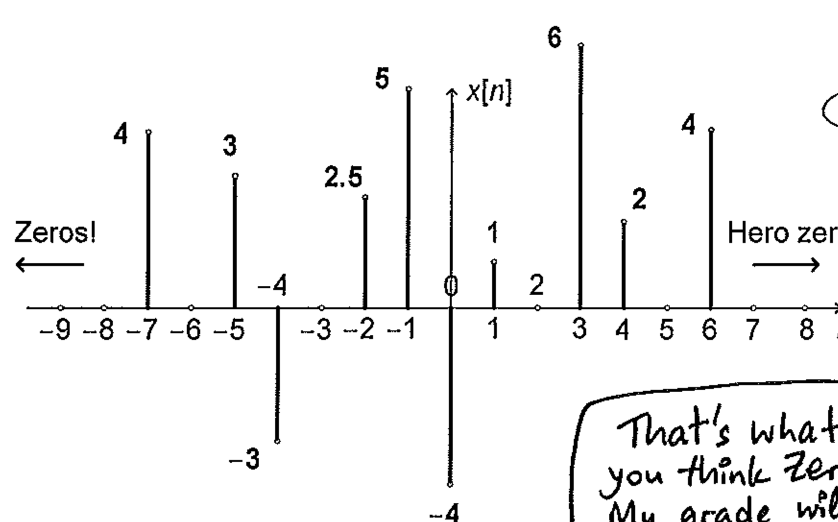
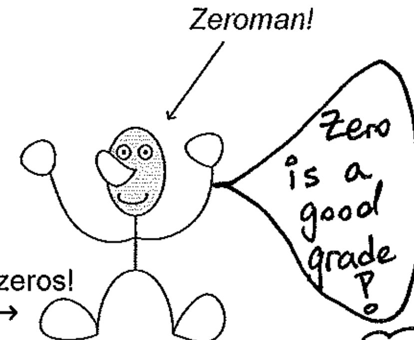


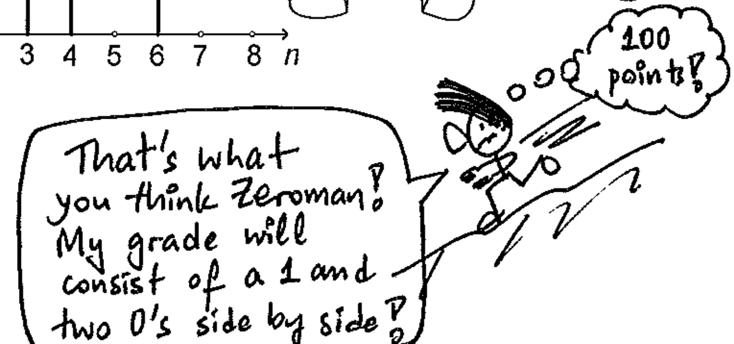
(2) (6 points) Using the $y[n]$ signal given on the previous page, is the signal $y[2n]$ periodic? If yes, what is the fundamental period of this signal?



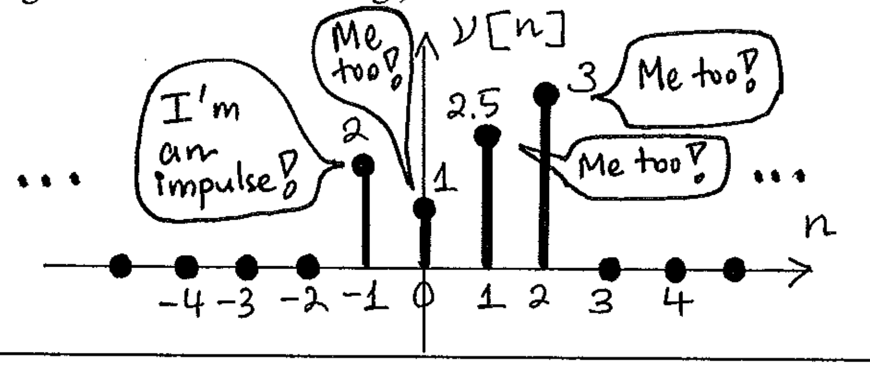
(3) (Total: 30 points) **Time scaling and shifting.** Given the discrete-time signal $x[n]$ as shown, find the following:







(a) (10 points) Find and sketch the signal $v[n] = x[1-3n]$. (Do the time shifting before the time scaling.)



$$\therefore v[n] = 2\delta[n+1] + \delta[n] + 2.5\delta[n-1] + 3\delta[n-2]$$

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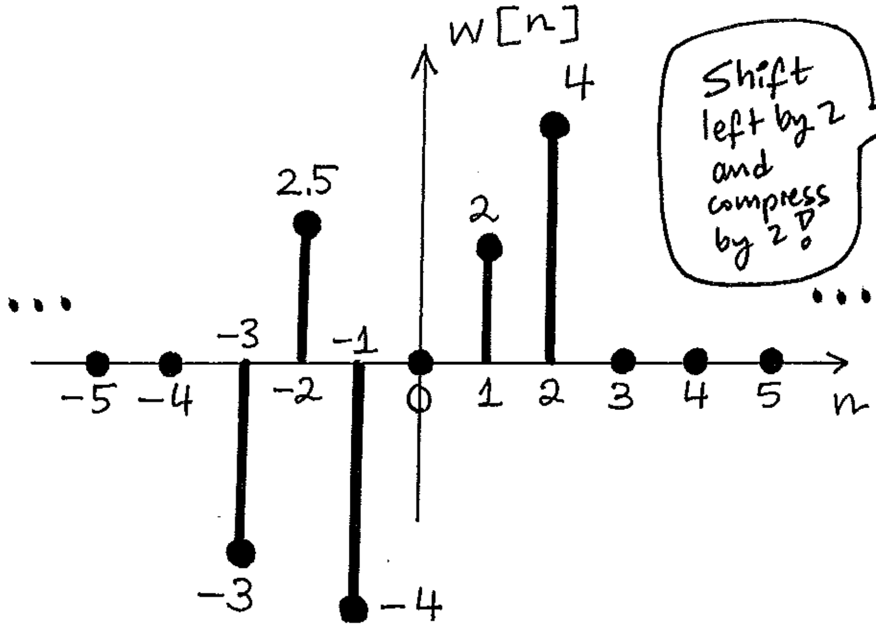
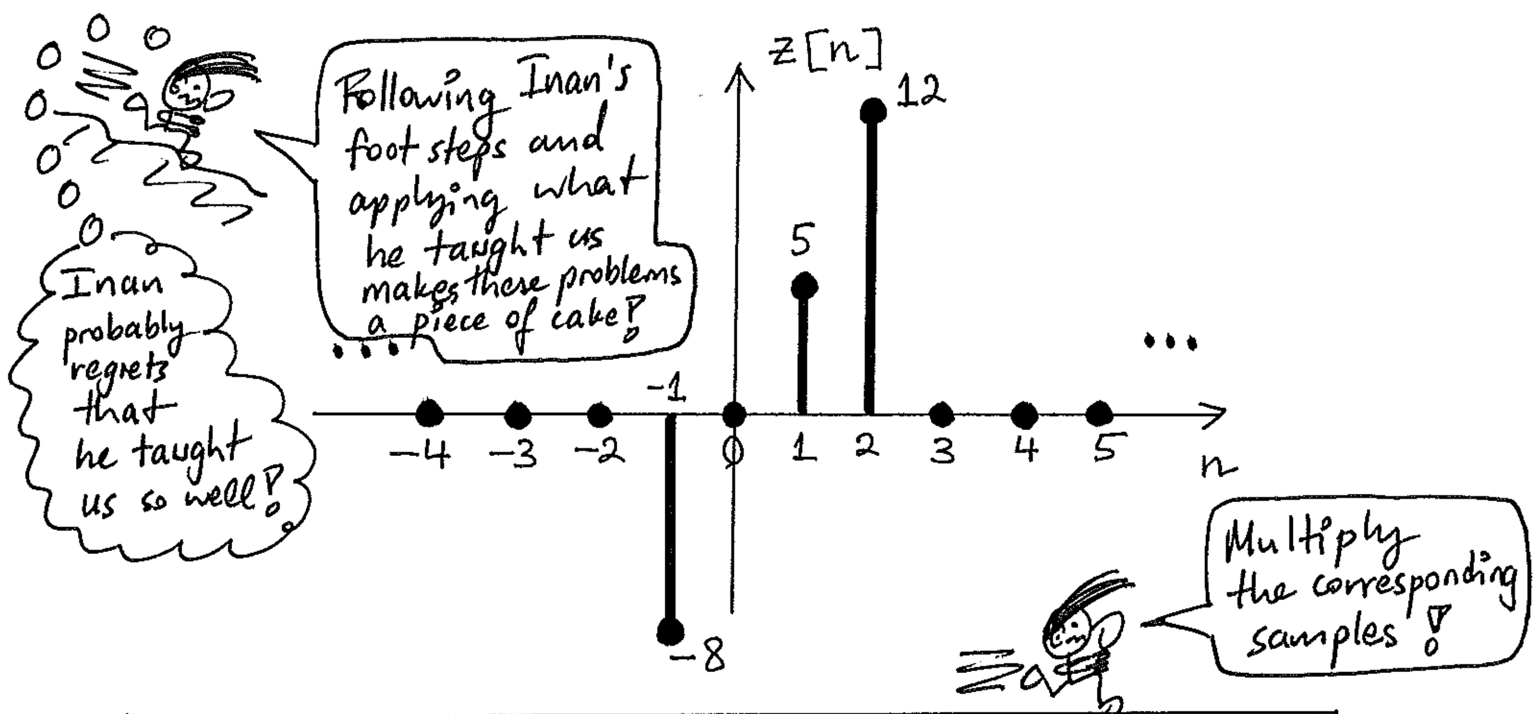
(b) (10 points) Find and sketch the signal $w[n] = x[2+2n]$. (Do the time shifting before the time scaling.)



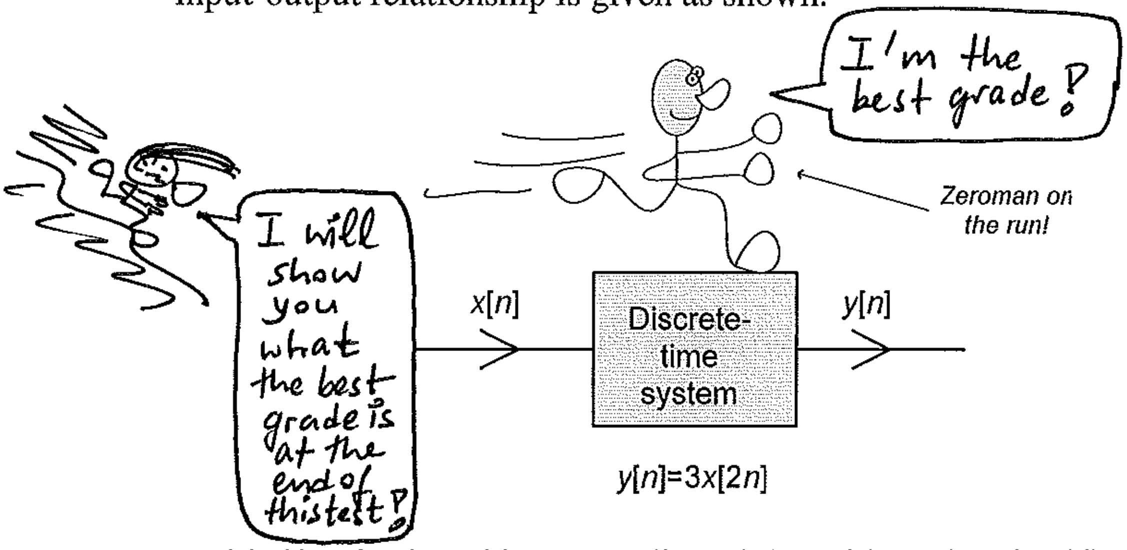
$$\therefore \; w[n] = -3\delta[n+3] + 2.5\delta[n+2] - 4\delta[n+1] + 2\delta[n-1] + 4\delta[n-2]$$

(c) (10 points) Find and sketch the signal $z[n] = x[1-3n]x[2+2n]$.



$$\therefore \; z[n] = -8\delta[n+1] + 5\delta[n-1] + 12\delta[n-2]$$

(4) (<u>Total:</u> 24 points) **Discrete-time system.** A discrete-time system with its input-output relationship is given as shown.



(a) (4 points) Is this system linear? (Provide a clear justification for your answer.)

Testing additivity property:

$x_1[n] \rightarrow y_1[n] = 3x_1[2n]$

$x_2[n] \rightarrow y_2[n] = 3x_2[2n]$

$x_3[n] = x_1[n] + x_2[n] \rightarrow y_3[n] = 3x_3[2n]$

$= 3(x_1[2n] + x_2[2n])$

$\stackrel{?}{=} y_1[n] + y_2[n]$ Yes!

Testing homogeneity property:

$x_4[n] = kx_1[n] \rightarrow y_4[n] = 3x_4[2n] = 3kx_1[2n] \stackrel{?}{=} ky_1[n]$ Yes!

$\therefore$ Both properties satisfied **$\therefore$ Linear!**

(b) (4 points) Is this system time-invariant? (Provide a clear justification.)

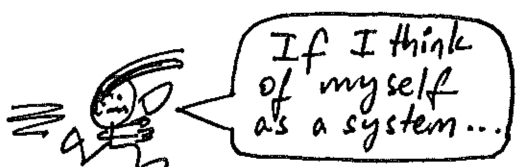
$x_1[n] \rightarrow y_1[n] = 3x_1[2n]$

$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = 3x_2[2n]$

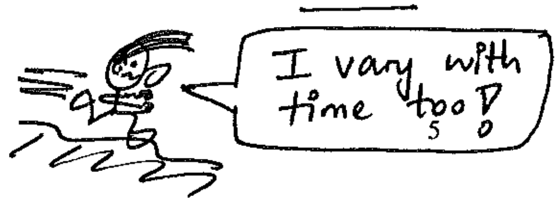
$= 3x_1[2n-n_0]$

$\stackrel{?}{=} y_1[n-n_0] = 3x_1[2(n-n_0)]$

No!



$\therefore$ Time variant!



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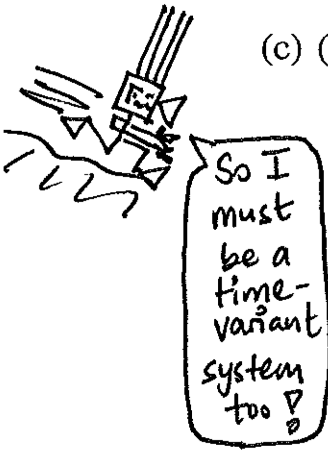
(c) (4 points) Is this system invertible? (Provide a clear justification.)



Since $y[n] = 3x[2n]$, in the process of compression, the samples of $x[n]$ at odd values of n are lost, so, these samples can not be recovered from the $y[n]$ signal.

∴ Not invertible!

(d) (4 points) Is this system memory-less? (Provide a clear justification.)

Since $y[n]$ in general depends on past or future samples of $x[n]$ (for example, $y[1] = 3x[2]$, or $y[-1] = 3x[-2]$), this system **has a memory!**

(I'm a future value of $x[n]$!)

(I'm a past value of $x[n]$!)

(e) (4 points) Is this system causal? (Provide a clear justification.)

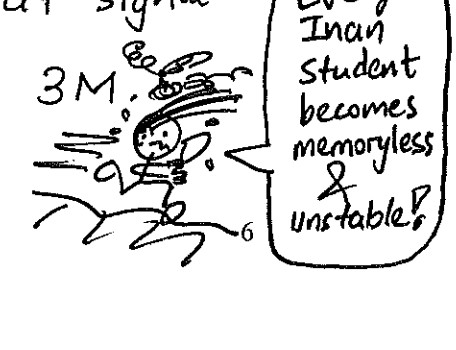
Since $y[n]$ is some cases depends on future values of $x[n]$ (for example, $y[1] = 3x[2]$), **noncausal system!**

(I'm a future value of $x[n]$!)

(f) (4 points) Is this system BIBO stable? (Provide a clear justification.)

If the input signal $x[n]$ is bounded, that is $|x[n]| \leq M$, then, since $y[n] = 3x[2n]$, $y[n]$ output signal will be bounded as $|y[n]| \leq 3M$.

∴ Stable system!



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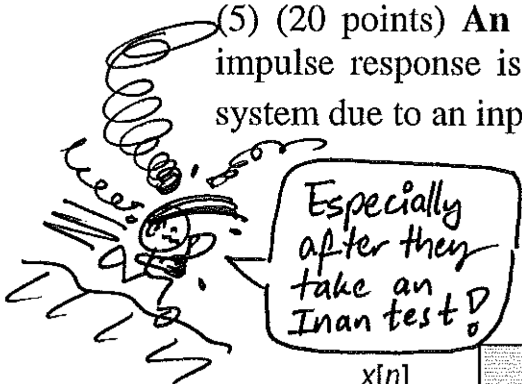
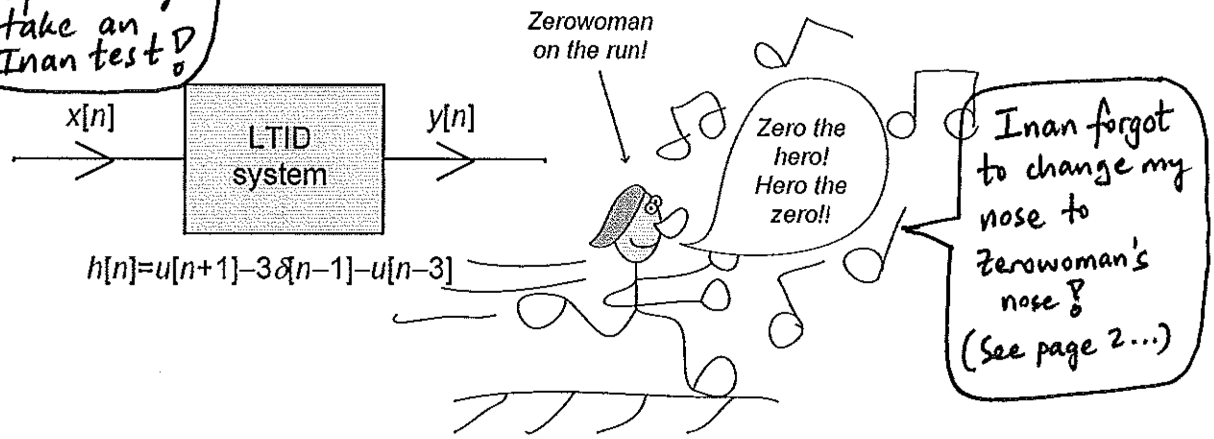
(5) (20 points) **An LTID system.** An LTID system is as shown. Its unit impulse response is also given. Find [and sketch] the zero-state response $y_{zs}[n]$ of this system due to an input signal given by



$$x[n] = 3\delta[n+2] - 2\delta[n]$$



Note that $h[n] = \delta[n+1] + \delta[n] - 2\delta[n-1] + \delta[n-2]$

So, $y[n] = x[n] * h[n]$

$$= \left(3\delta[n+2] - 2\delta[n]\right) * \left(\delta[n+1] + \delta[n] - 2\delta[n-1] + \delta[n-2]\right)$$

$$= 3\delta[n+3] + 3\delta[n+2] - 6\delta[n+1] + 3\delta[n]$$
$$- 2\delta[n+1] - 2\delta[n] + 4\delta[n-1] - 2\delta[n-2]$$

$$\therefore y[n] = 3\delta[n+3] + 3\delta[n+2] - 8\delta[n+1] + \delta[n] + 4\delta[n-1] - 2\delta[n-2]$$