4/21/2006

University of Portland
School of Engineering

EE 262-Signals & Systems-3 cr. hrs.
Spring 2006

Midterm Exam # 3
(Prepared by Professor A. S. Inan)

Bonjour!
Obtenez l'ensemble!
Pret? Allez!!

Hi!
Get set!
Ready?
Go!!

(Friday, April 21, 2006)
(Closed Book Exam, Formula Sheets Allowed.)
(Total Time: 55 mins.)

Name: SOLUTIONS! ☺

Signature: Solutions ☺

"Honesty is the best policy."
Aesop (~ 620B.C.–?)

"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

"Honest people are the true winners of the universe."
Anonymous

Inan: Is this an easy test or a hard test?

You will find out!

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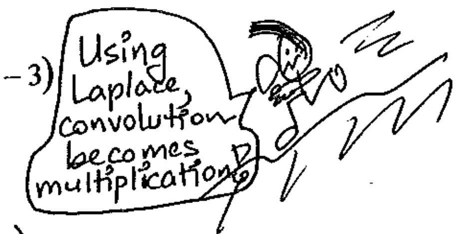
This box will be used by Inan for grading →	**Problem # 1**
	Problem # 2
	Problem # 3
Problem#4	~~**Total Score:**~~
	Total Score:

(1) (10 mins., 20 points) **Unilateral Laplace transform.** Find the unilateral Laplace transform of the signal $y(t)$ given by

$$y(t) = [\underbrace{7e^{2t-1}u(t-2)}_{f(t)} * \underbrace{e^{-t}u(t-1)}_{p(t)}] * \delta(t-3)$$



Please show your work step by step!

$$f(t) = 7e^{2t-1}u(t-2) = 7e^{3}e^{2(t-2)}u(t-2) = g(t-2)$$

$$\text{where } g(t) = 7e^{3}e^{2t}u(t) \xleftrightarrow{\mathcal{L}} G(s) = \frac{7e^{3}}{s-2}$$

$$f(t) = g(t-2) \xleftrightarrow{\mathcal{L}} F(s) = \frac{7e^{3-2s}}{s-2}$$

$$p(t) = e^{-t}u(t-1) = e^{-1}e^{-(t-1)}u(t-1) = q(t-1)$$

$$\text{where } q(t) = e^{-1}e^{-t}u(t) \xleftrightarrow{\mathcal{L}} Q(s) = \frac{e^{-1}}{s+1}$$

$$p(t) = q(t-1) \xleftrightarrow{\mathcal{L}} P(s) = \frac{e^{-(s+1)}}{s+1}$$

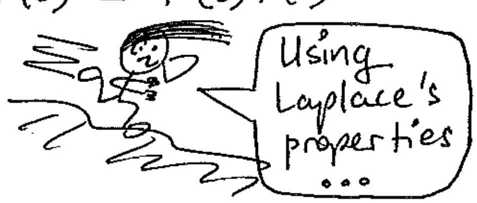
$$\therefore \mathcal{L}\{f(t) * p(t)\} = F(s)P(s) = \frac{7e^{3-2s}}{(s-2)} \cdot \frac{e^{-(s+1)}}{(s+1)}$$

$$= \frac{7e^{2-3s}}{(s-2)(s+1)}$$

$$\mathcal{L}\{\delta(t-3)\} = e^{-3s}$$

$$\therefore \mathcal{L}\{y(t)\} = Y(s) = F(s)P(s)e^{-3s} = \boxed{\frac{7e^{2-6s}}{(s-2)(s+1)}}$$

What about the other problems Inan?



This problem was a piece of cake!

Keep going!

(2) (15 mins., 25 points) **Inverse Laplace transform.** The unilateral Laplace transform of a signal $x(t)$ is given by

$$x(t) \overset{L_u}{\leftrightarrow} X(s) = 3e^{-2s}\frac{d}{ds}\underbrace{\left(\frac{(s+5)}{(s+1)^2(s^2+9)}\right)}_{F(s)}$$

Find the signal $x(t)$. Please show your work step by step!

$$F(s) = \frac{s+5}{(s+1)^2(s^2+9)} = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{Cs+D}{(s^2+9)}$$

$$A = (s+1)^2F(s)\Big|_{s=-1} = \frac{s+5}{s^2+9}\Big|_{s=-1} = \frac{4}{10} = \frac{2}{5}$$

$$B = \frac{d}{ds}\left[(s+1)^2F(s)\right]\Big|_{s=-1} = \frac{d}{ds}\left[\frac{s+5}{s^2+9}\right]\Big|_{s=-1} = \frac{(s^2+9)-2s(s+5)}{(s^2+9)^2}\Big|_{s=-1} = \frac{18}{100} = \frac{9}{50}$$

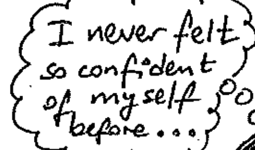
$$F(s=0) = \frac{5}{9} = A + B + \frac{D}{9} \rightarrow D = 9\left(\frac{5}{9} - \frac{2}{5} - \frac{9}{50}\right) = -\frac{11}{50}$$

$$\lim_{s\to\infty} sF(s) = 0 = B + C \rightarrow C = -B = -\frac{9}{50}$$

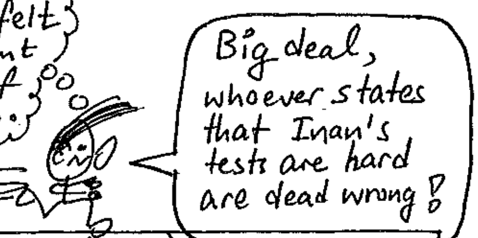
$$\therefore F(s) = \frac{2/5}{(s+1)^2} + \frac{9/50}{(s+1)} + \frac{(-9/50)s}{(s^2+9)} + \frac{(-11/50)}{(s^2+9)}$$

$$\therefore f(t) = \frac{2}{5}te^{-t}u(t) + \frac{9}{50}e^{-t}u(t) - \frac{9}{50}\cos(3t)u(t) - \frac{11}{150}\sin(3t)u(t)$$

$$\frac{d}{ds}F(s) \overset{L}{\longleftrightarrow} -tf(t)$$



$$3e^{-2s}\frac{d}{ds}F(s) \overset{L}{\longleftrightarrow} -3(t-2)f(t-2) =$$



$$\boxed{\begin{aligned}\therefore x(t) = &-\frac{6}{5}(t-2)^2e^{-(t-2)}u(t-2) - \frac{27}{50}(t-2)e^{-(t-2)}u(t-2)\\ &+ \frac{27}{50}(t-2)\cos[3(t-2)]u(t-2) + \frac{11}{50}(t-2)\sin[3(t-2)]u(t-2)\end{aligned}}$$

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(3) (15 mins., 25 points) **The transfer function and the impulse response of an LTI system.** The governing differential equation of an LTI system with input signal $x(t)$ and output signal $y(t)$ is given by

Transforming to Laplace's domain!

$$\mathcal{L}\left\{ \frac{d^2y(t)}{dt^2} + 20\frac{dy(t)}{dt} + 100y(t) = 10\frac{dx(t)}{dt} - 30x(t) \right\}$$

Laplace turns differential equations into algebraic equations...

Use Laplace transform to find the impulse response $h(t)$ of this system.

You are absolutely right on that!

$$s^2Y(s) - s\cancel{y(0^-)}^{\,0} - \cancel{\left.\frac{dy}{dt}\right|_{0^-}}^{\,0} + 20\left[sY(s) - \cancel{y(0^-)}^{\,0}\right] + 100\,Y(s)$$

$$= 10\left[sX(s) - \cancel{x(0^-)}^{\,0}\right] - 30\,X(s)$$

$$\rightarrow \quad (s^2 + 20s + 100)\,Y(s) = (10s - 30)\,X(s)$$

$$\rightarrow \quad H(s) = \frac{Y(s)}{X(s)} = \frac{10(s-3)}{(s+10)^2} = \frac{A}{(s+10)^2} + \frac{B}{(s+10)}$$

I'm the transfer function of the LTI system!

$$A = (s+10)^2 H(s)\Big|_{s=-10} = 10(s-3)\Big|_{s=-10} = -130$$

$$B = \frac{d}{ds}\left[(s+10)^2 H(s)\right]\Bigg|_{s=-10} = \frac{d}{ds}\left[10(s-3)\right]\Bigg|_{s=-10} = 10$$

$$\therefore \quad H(s) = \frac{-130}{(s+10)^2} + \frac{10}{(s+10)}$$

$$\therefore \quad h(t) = \mathcal{L}^{-1}\{H(s)\} = -130\,t\,e^{-10t}u(t) + 10e^{-10t}u(t)$$

$$= \boxed{10(1-13t)\,e^{-10t}\,u(t)}$$

I'm the unit impulse response of the LTI system!

Nice to meet you Mr. Unit Impulse Response...

This Exam test is too easy and getting a bit boring...

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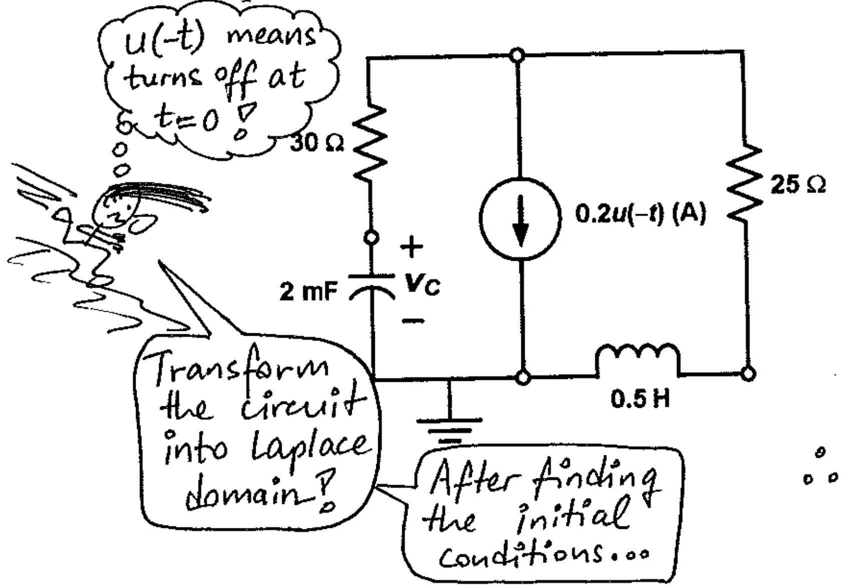
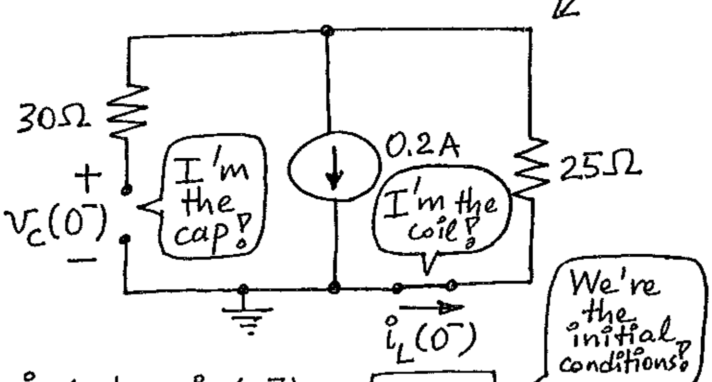
(4) (15 mins., 30 points) **Solving electric circuit problems using Laplace transform.** For the circuit shown, use Laplace transform to find the capacitor voltage $v_C(t)$ after t=0. Please show your work clearly.



At $t = 0^-$ (steady state):

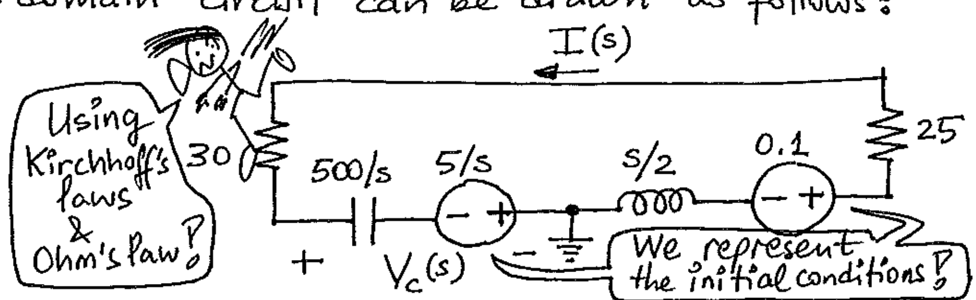


$$\therefore\ i_L(0^+) = i_L(0^-) = \boxed{0.2\,A}$$

We're the initial conditions!

$$v_C(0^+) = v_C(0^-) = -(25\Omega)(0.2A) = \boxed{-5\,V}$$

For $t>0$, the Laplace-domain circuit can be drawn as follows:



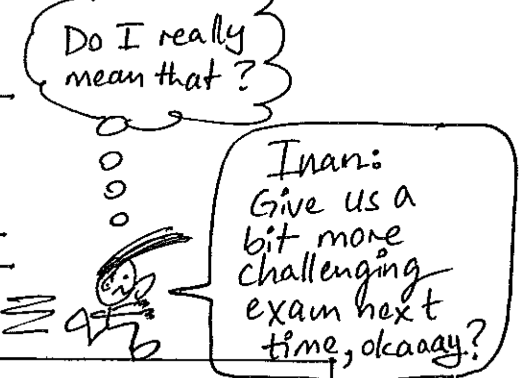
$$I(s) = \frac{5/s + 1/10}{55 + s/2 + 500/s} = \frac{s+50}{5(s^2+110s+1000)}$$

$$V_c(s) = \frac{500}{s} I(s) - \frac{5}{s} = \frac{100(s+50)}{s(s^2+110s+1000)} - \frac{5}{s} = \frac{-5s^2 - 450s}{s(s^2+110s+1000)}$$

$$= \frac{-5(s+90)}{(s+10)(s+100)} = \frac{A}{s+10} + \frac{B}{s+100}$$

$$A = (s+10)V_c(s)\Big|_{s=-10} = \frac{-5(s+90)}{s+100}\Big|_{s=-10} = -\frac{40}{9}$$

$$B = (s+100)V_c(s)\Big|_{s=-100} = \frac{-5(s+90)}{s+10}\Big|_{s=-100} = -\frac{5}{9}$$



$$\boxed{\therefore\ v_c(t) = -\frac{40}{9}e^{-10t}u(t) - \frac{5}{9}e^{-100t}u(t),\ \text{for } t \geq 0}$$