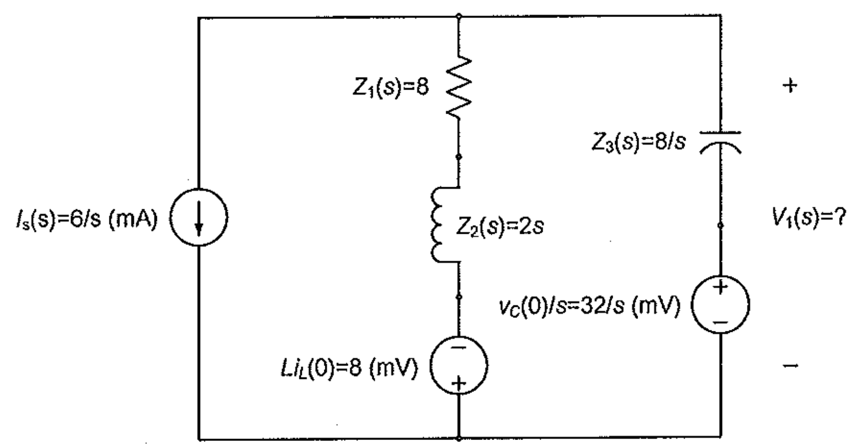
University of Portland
School of Engineering

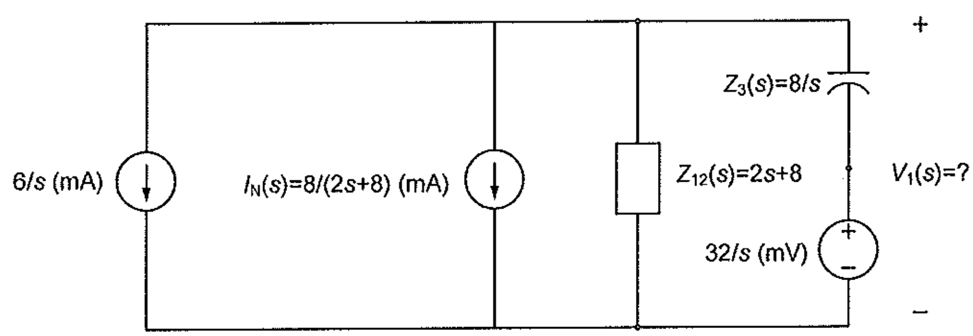
EE 262/Spring 2011
A. Inan

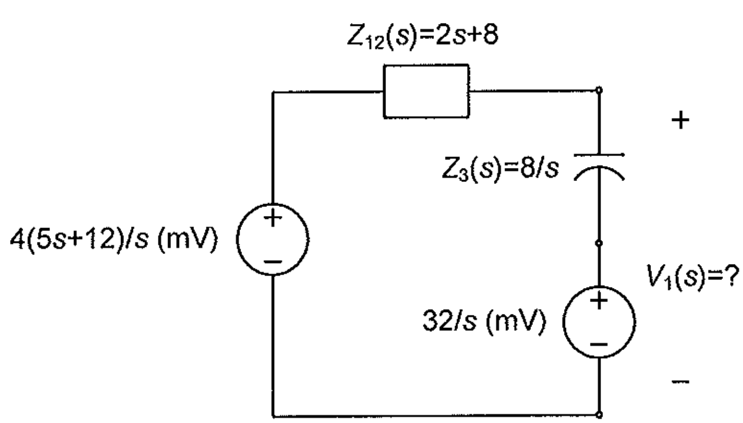
Partial Solutions for the Laplace Transform Problems of Homework # 7
(Tuesday, April 19, 2011)

1. Transforming the circuit into Laplace domain:



Laplace domain equivalent circuit





Using the voltage divider principle, one can write the output voltage $V_1(s)$ (in mV) as follows:

$$V_1(s) = \frac{\frac{8}{s}}{\frac{8}{s} + 2(s+4)}\left(\frac{4(5s+12)}{s} - \frac{32}{s}\right) + \frac{32}{s} = \frac{16(5s+4))}{s(s^2+4s+4)} + \frac{32}{s} = \frac{16(2s^2+13s+12)}{s(s+2)^2}$$
$$= \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

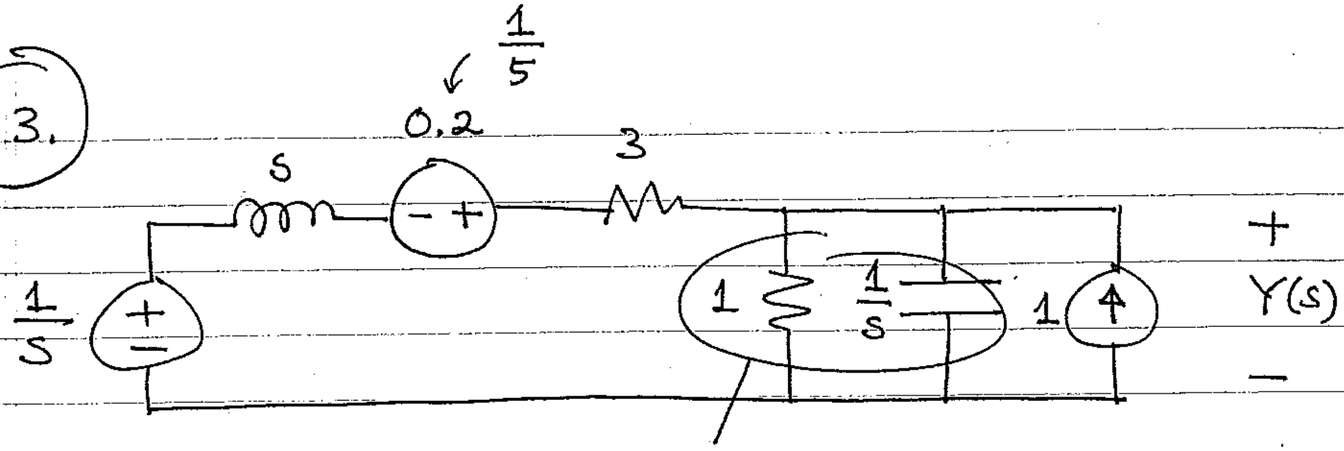
The coefficients A, B and C can be found as:

$$A = sV_1(s)\big|_{s=0} = 48 \text{ mV}$$
$$B = \frac{d}{ds}(s+2)^2 V_1(s)\Big|_{s=-2}$$
$$C = (s+2)^2 V_1(s)\Big|_{s=-2} = 48 \text{ mV}$$

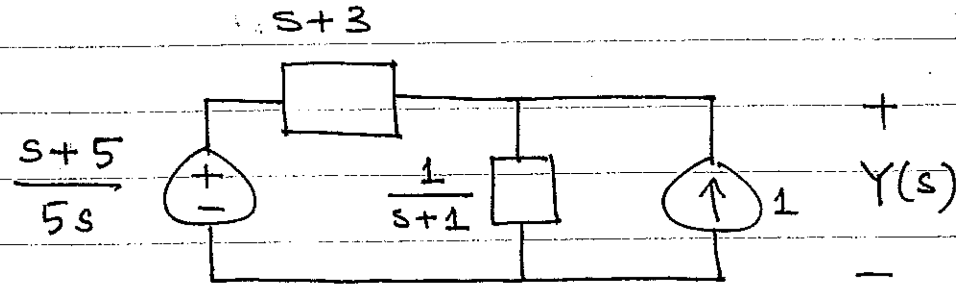
Therefore, the time-domain expression for the output voltage is

$v_1(t) = 48u(t) + Be^{-2t}u(t) + 48te^{-2t}u(t)\,(\text{mV})$ valid for $t \geq 0$

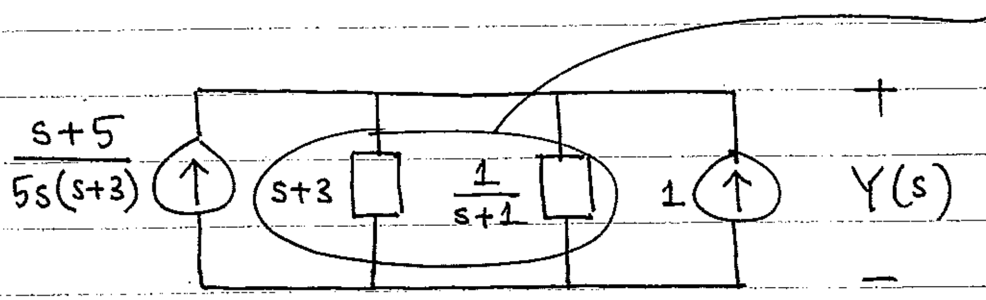
3.



$$\frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$$



$$\frac{(s+3)\frac{1}{s+1}}{s+3+\frac{1}{s+1}}$$



$$Y(s) = \frac{s+3}{s^2+4s+4}\left[\frac{s+5}{5s(s+3)} + 1\right]$$

$$= \frac{5s^2+16s+5}{5s\underbrace{(s^2+4s+4)}_{(s+2)^2}} = \frac{A}{s} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)}$$

3. continues ...

$$A = sY(s)\Big|_{s=0} = \frac{5}{5\times 4} = \frac{1}{4}$$

$$B = (s+2)^2 Y(s)\Big|_{s=-2} = \frac{5(-2)^2 + 16(-2) + 5}{5(-2)} = \frac{-7}{-10} = 7/10$$

$$C = \left[\frac{d}{ds}\left\{(s+2)^2 Y(s)\right\}\right]_{s=-2} = \left\{\frac{d}{ds}\left[\frac{5s^2+16s+5}{5s}\right]\right\}_{s=-2}$$

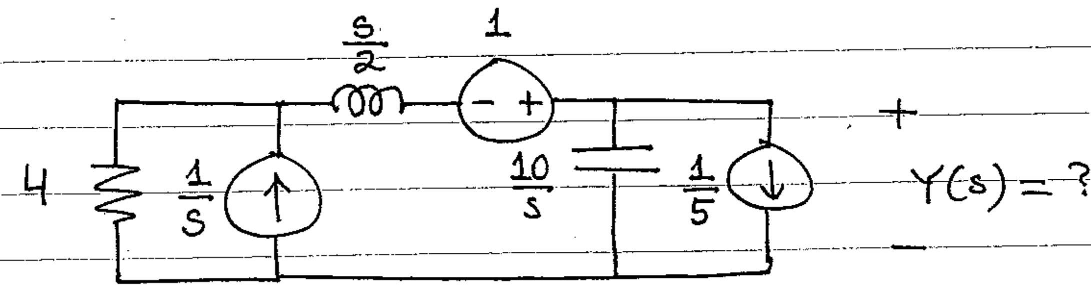
$$= \frac{(10s+16)(5s) - 5(5s^2+16s+5)}{25s^2}\Bigg|_{s=-2}$$

$$= \frac{40+35}{25(-2)^2} = \frac{75}{100} = \frac{3}{4}$$

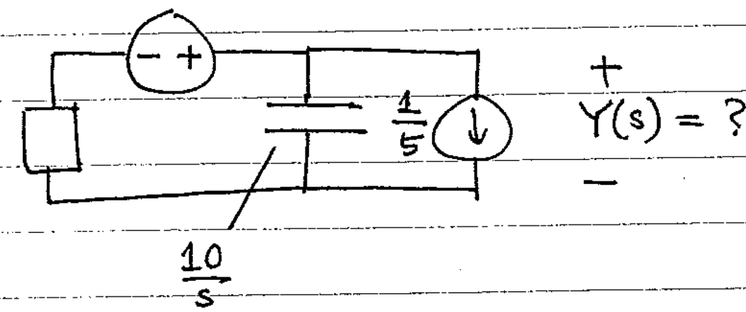
$$\therefore Y(s) = \frac{1/4}{s} + \frac{7/10}{(s+2)^2} + \frac{3/4}{(s+2)}$$

$$\boxed{\therefore y(t) = \frac{1}{4}u(t) + \frac{7}{10}te^{-2t}u(t) + \frac{3}{4}e^{-2t}u(t)}$$

5.

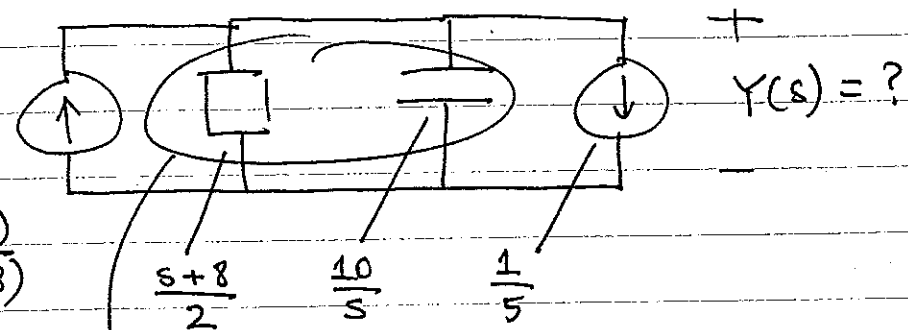


$$\left[\frac{4}{s}+1\right] = \frac{s+4}{s}$$



$$\underbrace{\left[\frac{s}{2}+4\right]}_{\frac{s+8}{2}}$$

$$\frac{\frac{s+4}{s}}{\frac{s+8}{2}} = \frac{2(s+4)}{s(s+8)}$$



$$\frac{\left(\frac{s+8}{2}\right)\left(\frac{10}{s}\right)}{\frac{s+8}{2}+\frac{10}{s}} = \frac{10(s+8)}{s^2+8s+20}$$

$$\therefore Y(s) = \left[\frac{\overset{2}{\cancel{10}}\cancel{(s+8)}}{s^2+8s+20}\right]\underbrace{\left[\frac{2(s+4)}{s(s+8)} - \frac{1}{5}\right]}_{\frac{10(s+4)-s(s+8)}{\cancel{5}s\cancel{(s+8)}}}$$

$$-s^2+2s+40$$

$$= \frac{2(-s^2+2s+40)}{s(s^2+8s+20)}$$

5. continues...

$$= \frac{-2(s^2-2s-40)}{s\left[(s+4)^2+4\right]} = \frac{A}{s} + \frac{Bs+C}{s^2+8s+20}$$

$A+B = -2$

$8A+C = 4$

$20A = 80 \rightarrow A = 4 \rightarrow B = -6$

$\searrow C = -28$

$$\therefore Y(s) = \frac{4}{s} - \frac{6s+28}{s^2+8s+20}$$

$$= \frac{4}{s} \cancel{- \frac{6s}{[(s+4)^2+4]} - \frac{28}{[(s+4)^2+4]}}$$

$\downarrow \mathcal{L}^{-1}$

$4u(t) \qquad - \frac{6(s+4)}{(s+4)^2+4} - \frac{4}{(s+4)^2+4}$

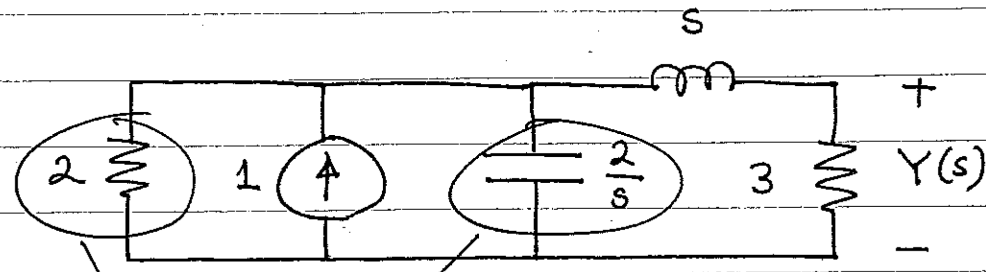
$\downarrow \qquad \downarrow$

$-2e^{-4t}\sin(2t)u(t)$

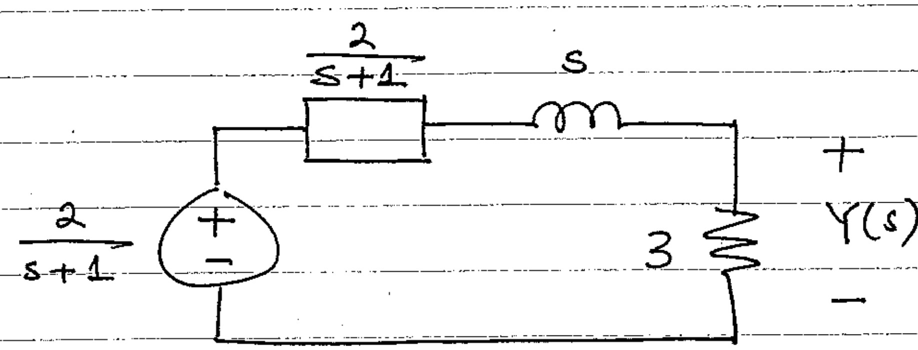
$-6e^{-4t}\cos(2t)u(t)$

$$\boxed{\therefore y(t) = 4u(t) - 6e^{-4t}\cos(2t)u(t) - 2e^{-4t}\sin(2t)u(t)}$$

7.



$$\frac{(2)\left(\frac{2}{s}\right)}{2+\frac{2}{s}} = \frac{2}{s+1}$$



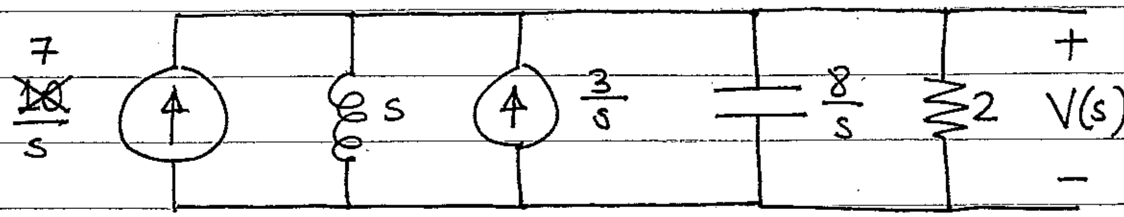
$$\therefore\ Y(s) = \frac{3}{3+s+\frac{2}{s+1}}\left(\frac{2}{s+1}\right)$$

$$= \frac{6}{s^2+4s+3+2} = \frac{6}{s^2+4s+5}$$

$$= \frac{6}{(s+2)^2+1}$$

$$\boxed{\therefore\ y(t) = 6e^{-2t}\sin t\, u(t)}$$

2.



$$V(s) = \left(\frac{10}{s}\right)\frac{1}{\frac{1}{s}+\frac{s}{8}+\frac{1}{2}} = \frac{80}{8+s^2+4s}$$

$$= \frac{80}{(s+2)^2+4}$$