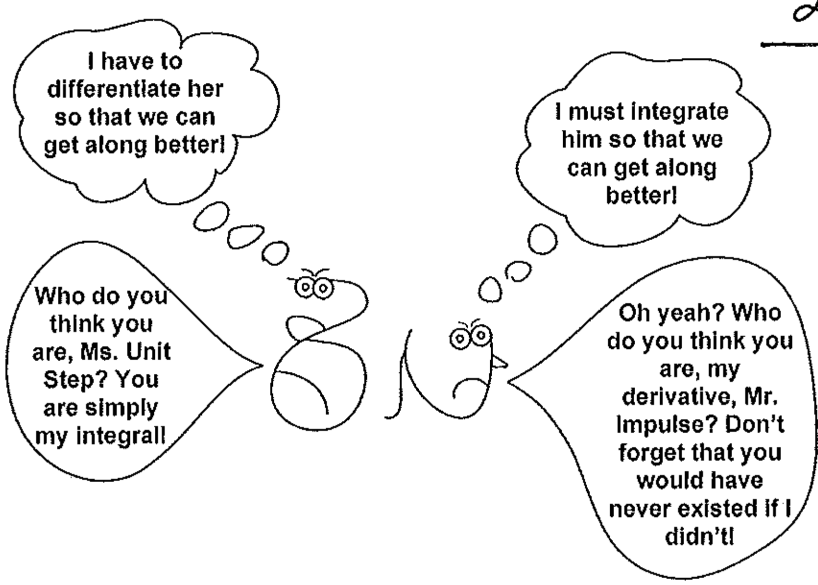
2/18/2011



University ☺f P☺rtland
Sch☺☺l ☺f Engineering

<u>EE 262-δignals & δystems-3 cr. hrs.</u>
<u>Spring 2011</u>

<u>Midterm Exam # 1</u>

(Prepared by Professor A. S. Inan)

(Friday, February 18, 2011*)
(Closed Book Exam, One formula sheet allowed.)
(Total Time: 55 mins.)

$$\begin{array}{r} 55949 \\ -\quad 39 \\ \hline 55910 \end{array}$$

Reverse of this number is 1955!

*<u>Today's math puzzle by Inan:</u> Today's date expressed as a single date number as 2182011 equals 39 × 55949 where 55949 is a prime number. Interestingly enough, the difference of numbers 55949 and 39 relate to Inan's age. Can you figure out how? ☺

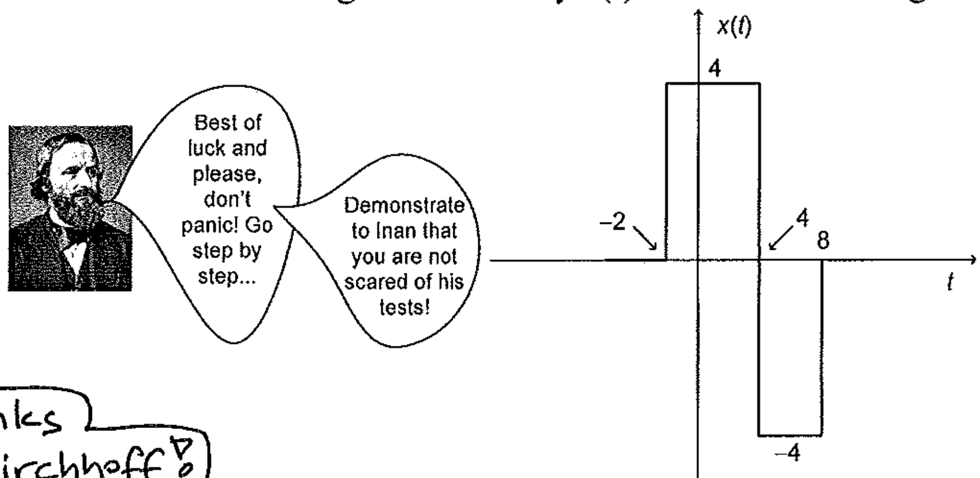
1955 must be his birth year! Why should I care?

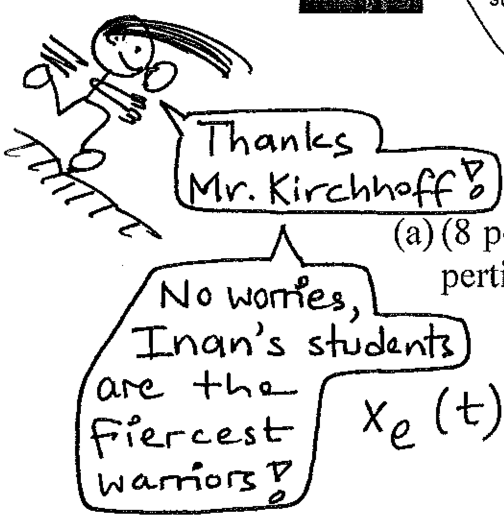
Name: SOLUTIONS !

Signature: Solution

Note: 5 Problems total, 3 in-class, 2 take-home, due next Tuesday!

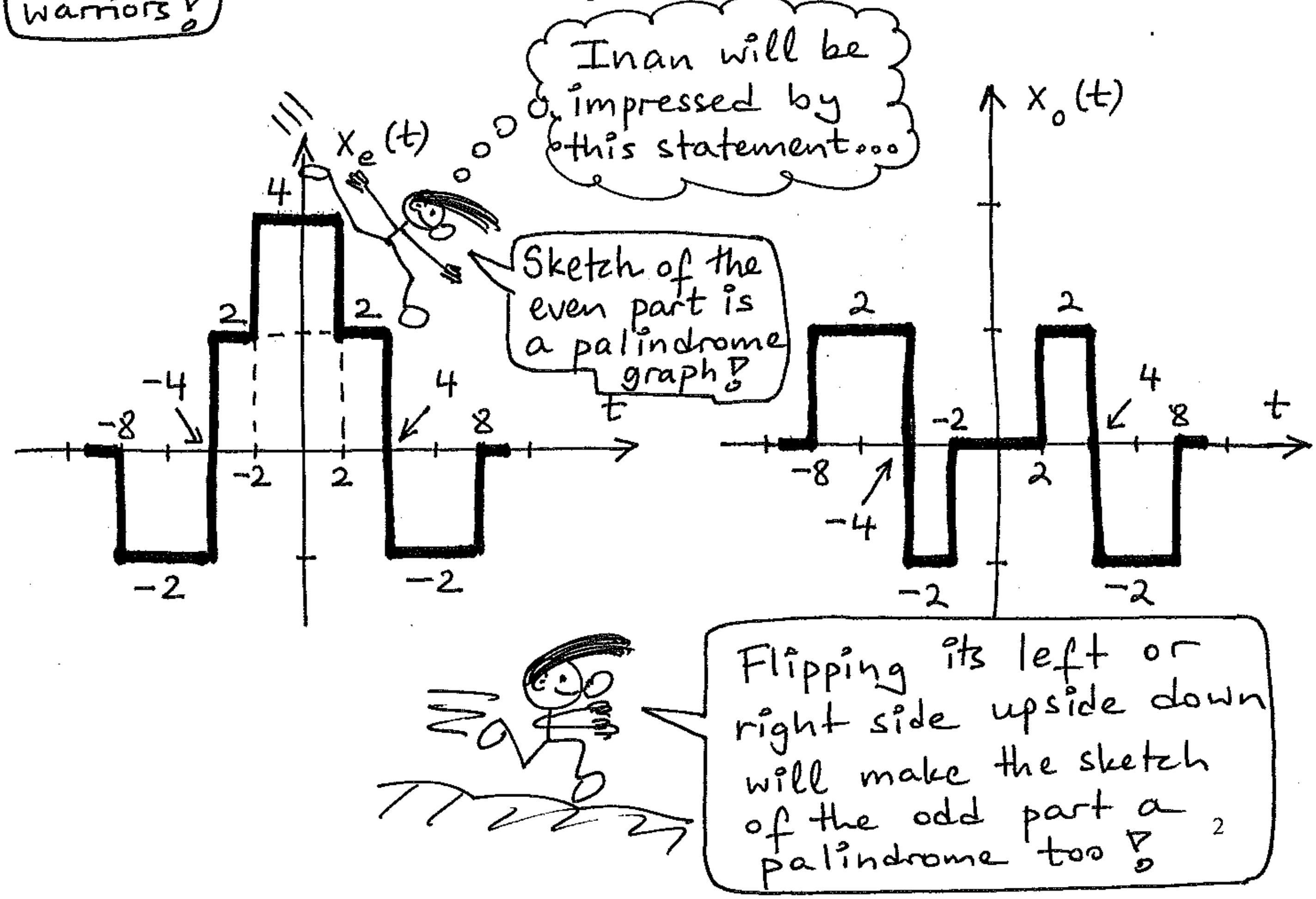
(1) (10 mins., Total: 20 points) **A continuous-time signal.** Consider the continuous-time signal denoted by $x(t)$ as shown in the figure below.

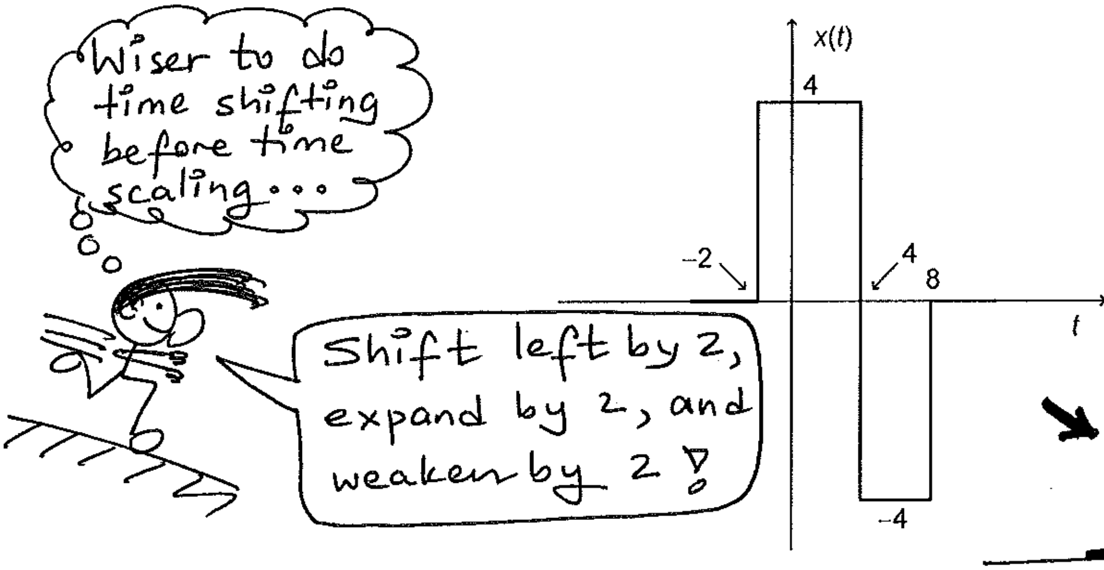


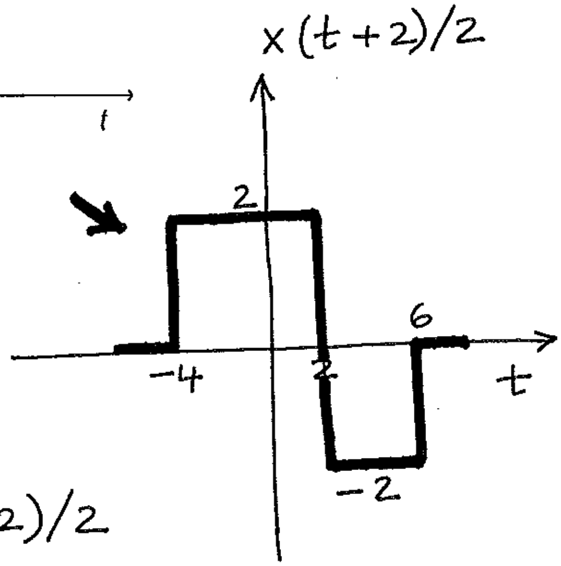


(a) (8 points) Sketch the even and odd parts of $x(t)$. Provide all the pertinent values on your sketch.

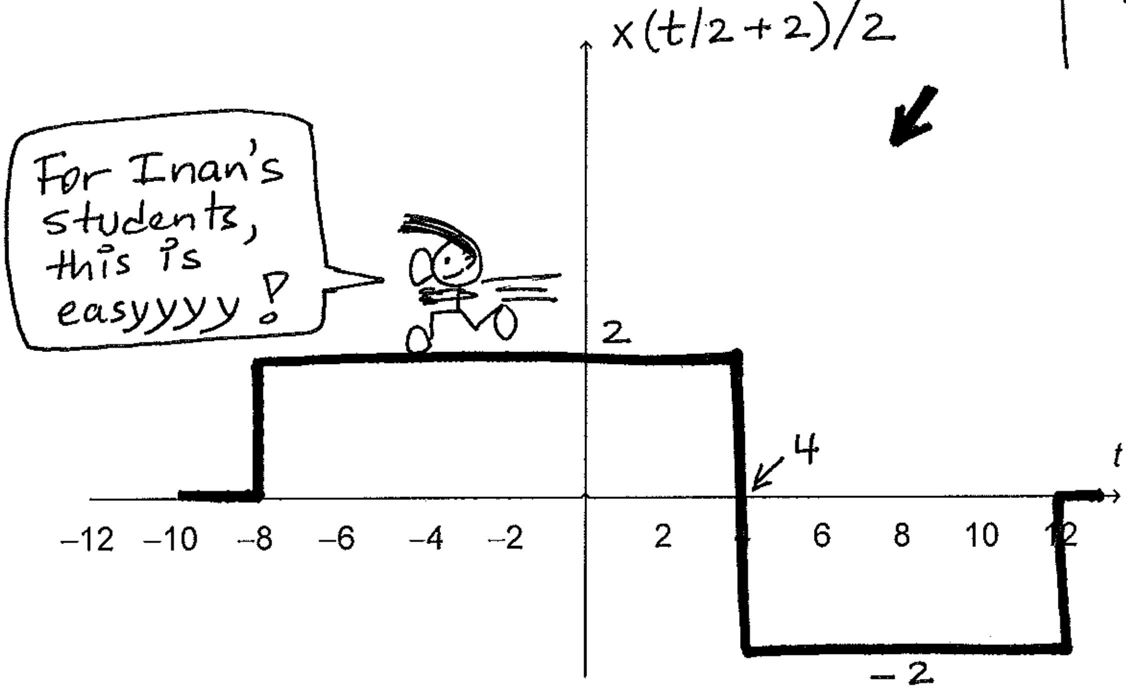
$$x_e(t) = \frac{x(t) + x(-t)}{2}, \quad x_o(t) = \frac{x(t) - x(-t)}{2}$$

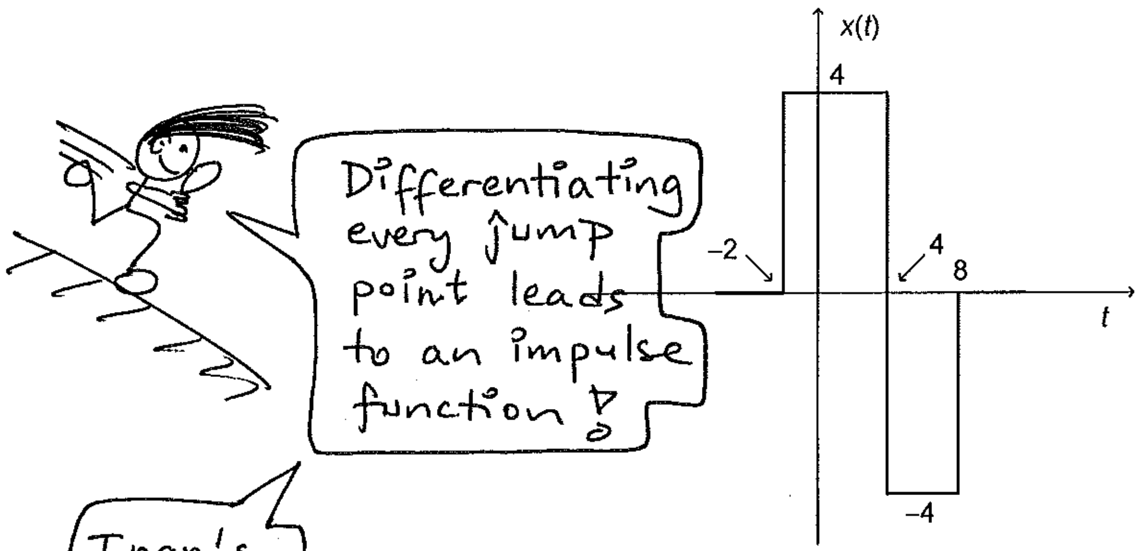


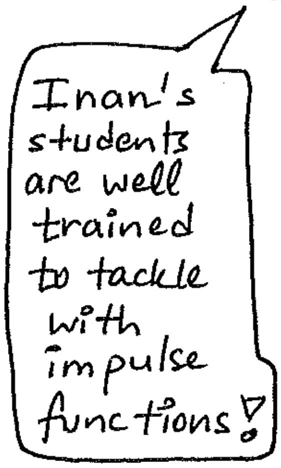




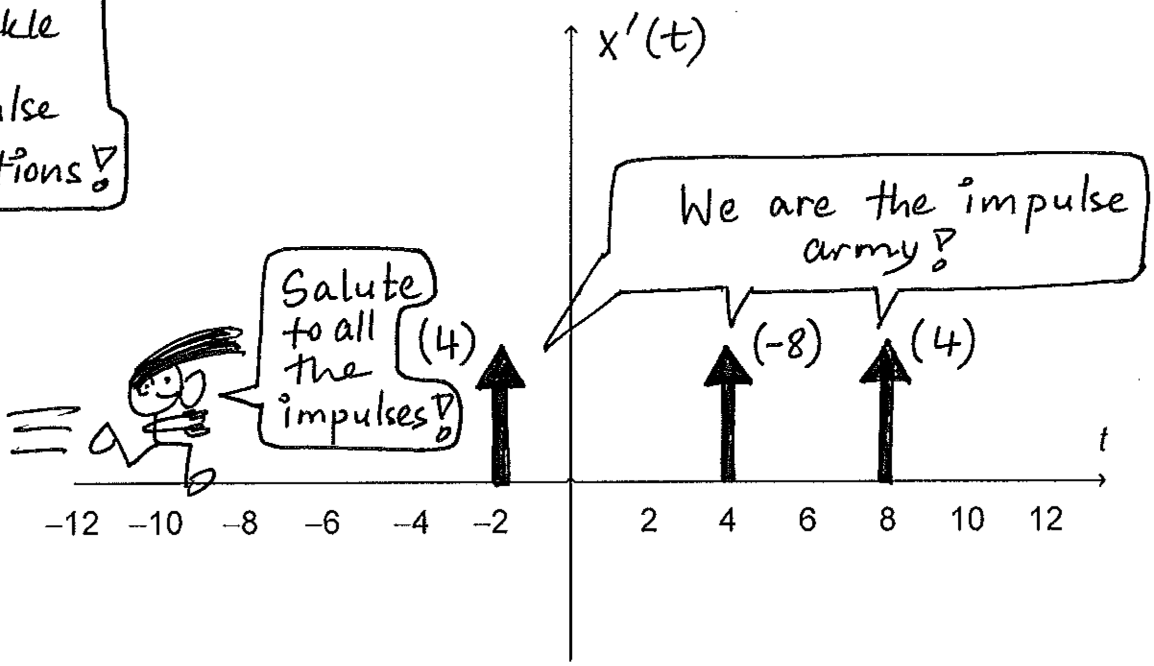
(b) (6 points) Sketch $x(t/2+2)/2$.





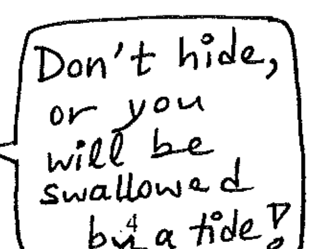


(c) (6 points) Find the complete mathematical expression for the function $y(t)=dx(t)/dt$ and sketch $y(t)$ versus t. Provide all the pertinent values on your sketch.



$$x(t) = 4u(t+2) - 8u(t-4) + 4u(t-8)$$

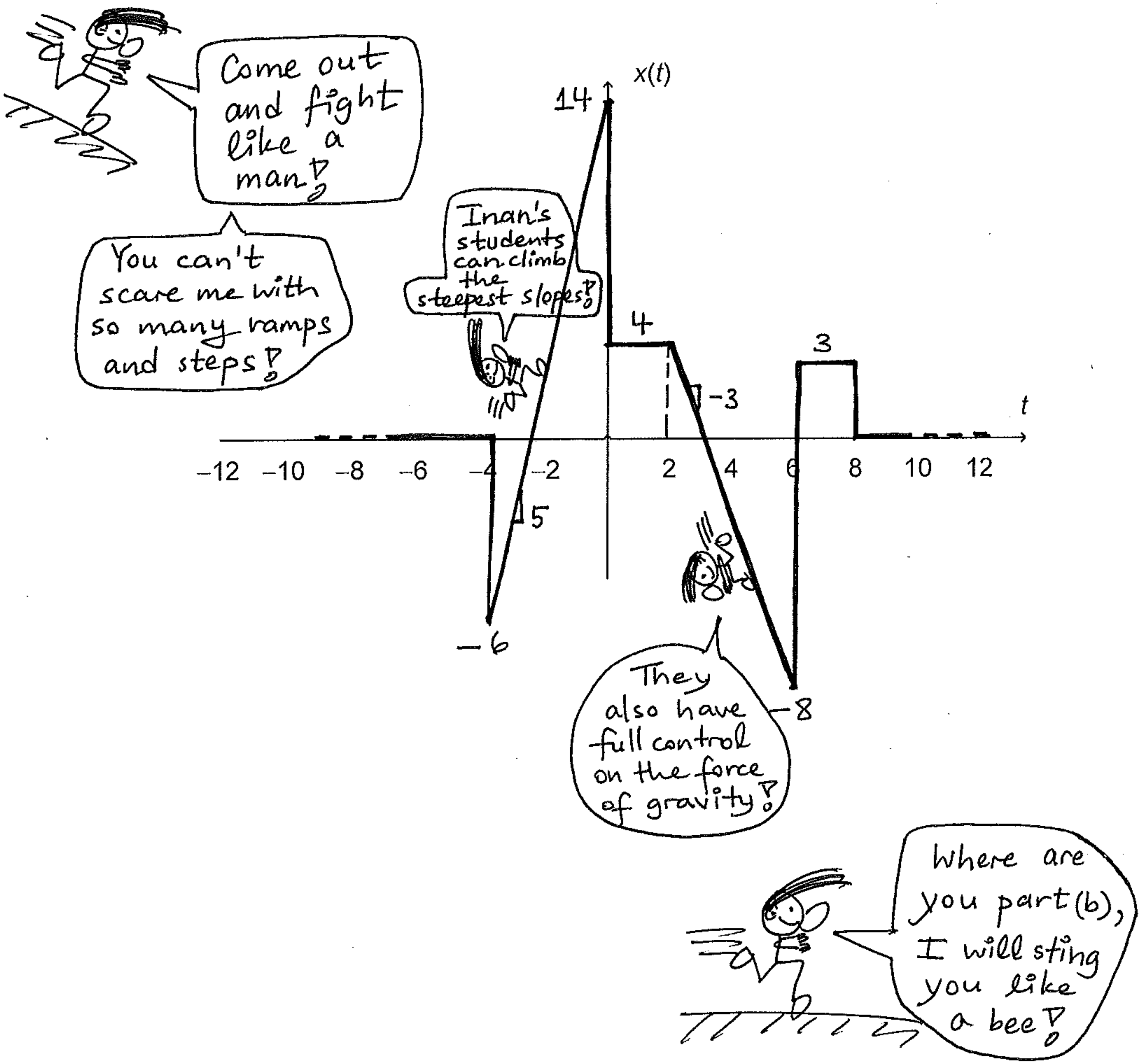
$$x'(t) = 4\delta(t+2) - 8\delta(t-4) + 4\delta(t-8)$$



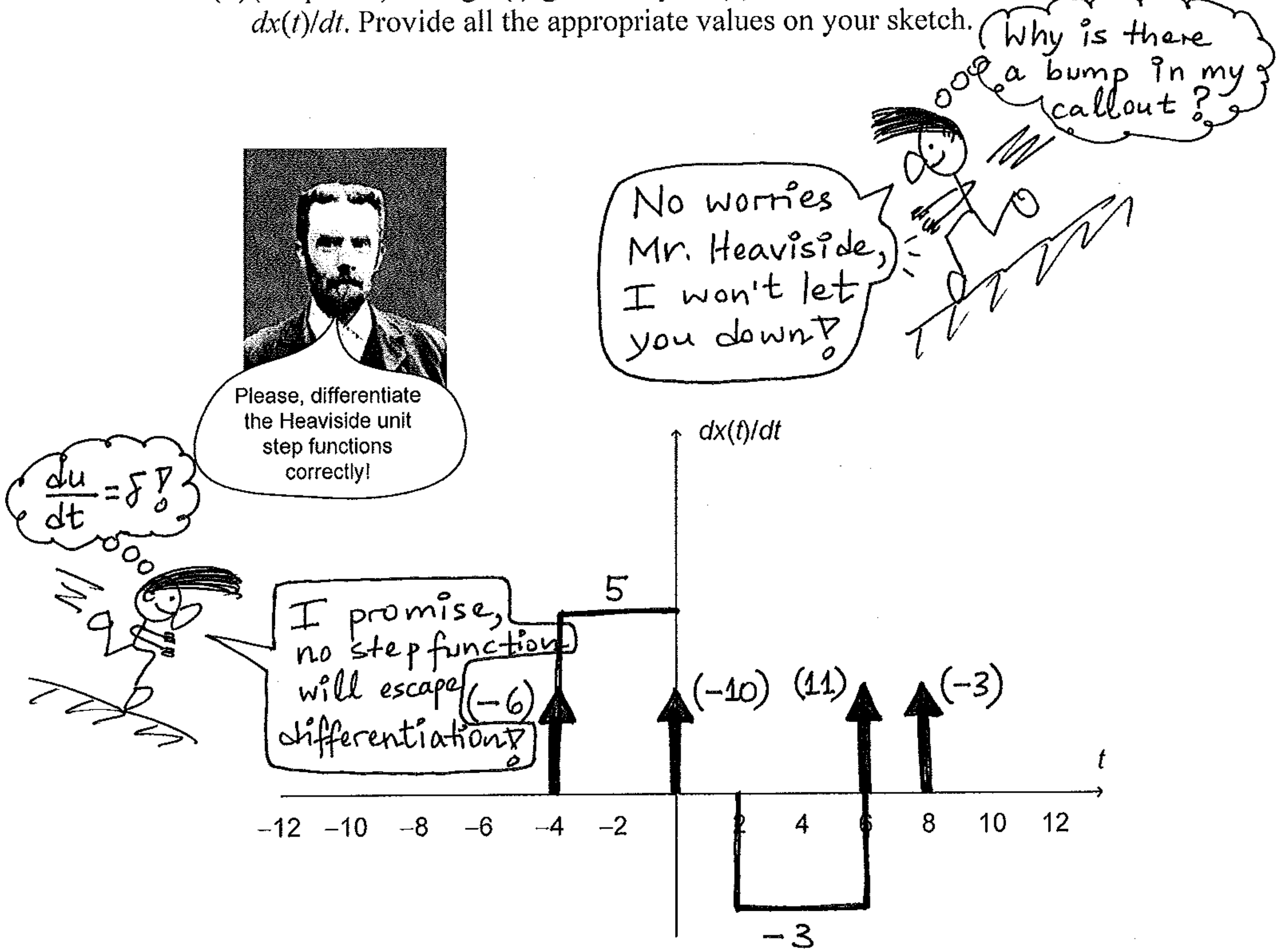
(2) (15 mins., <u>Total:</u> 20 points) **Impulse, step, and ramp functions.** A continuous-time signal is given by

$$x(t) = 5r(t+4) - 6u(t+4) - 10u(t) - 5r(t) - 3r(t-2) + 3r(t-6) + 11u(t-6) - 3u(t-8)$$

(a) (10 points) Sketch this signal. Provide all the necessary values on your sketch.

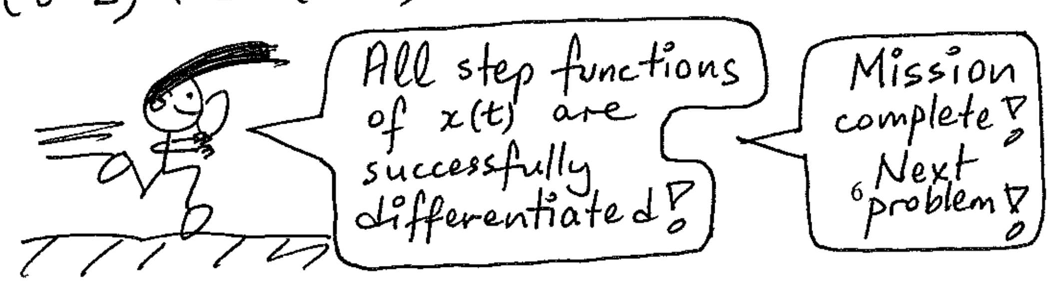


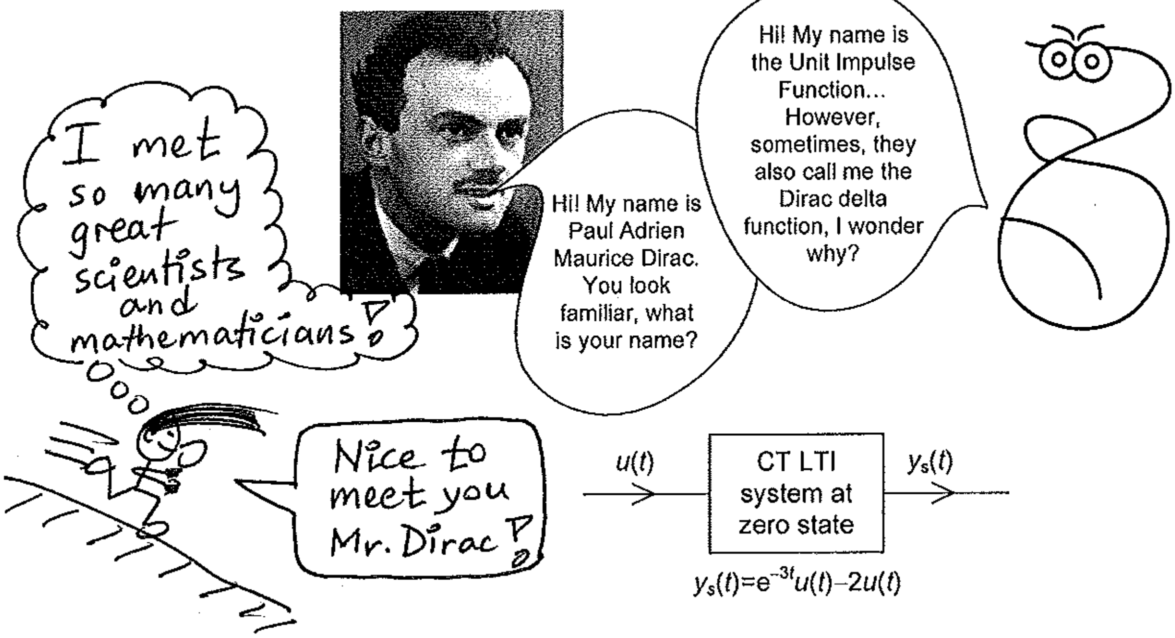
(b)(10 points) Using $x(t)$ given in part (a), sketch the derivative signal, $dx(t)/dt$. Provide all the appropriate values on your sketch.



The mathematical expression for $x'(t)$ is given by:

$$x'(t) = 5u(t+4) - 6\delta(t+4) - 10\delta(t) - 5u(t) - 3u(t-2) + 3u(t-6) + 11\delta(t-6) - 3\delta(t-8)$$





(3) (15 mins., Total: 20 points) **CT LTI system.** The unit-step response $y_s(t)$ of a continuous-time (CT) linear time-invariant (LTI) system is given as shown.

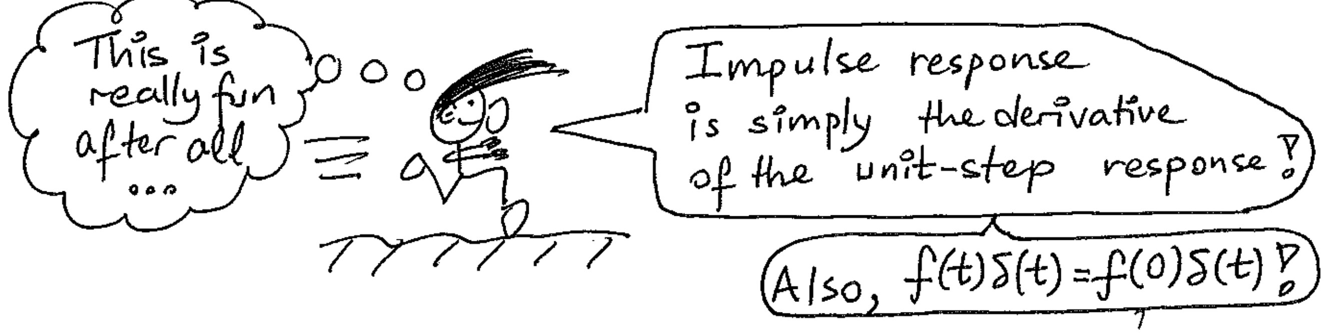
(a) (10 points) Find the impulse response $h(t)$ of this system.

$$h(t) = \frac{dy_s(t)}{dt} = \frac{d}{dt}\left[(e^{-3t}-2)\,u(t)\right]$$

$$= -3e^{-3t}u(t) + (e^{-3t}-2)\frac{du(t)}{dt}$$

$$= -3e^{-3t}u(t) + (e^{-3t}-2)\,\delta(t)$$

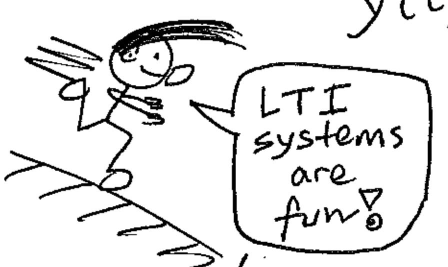
$$= \boxed{-3e^{-3t}u(t) - \delta(t)}$$



(b) (10 points) Find the response of this system due to an input signal given by $x(t)=2\delta(t-1)+3u(t-2)$. (Assume zero initial conditions.)

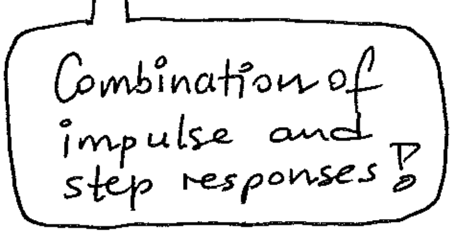
$$x(t) = x_1(t) + x_2(t) = 2\delta(t-1) + 3u(t-2)$$

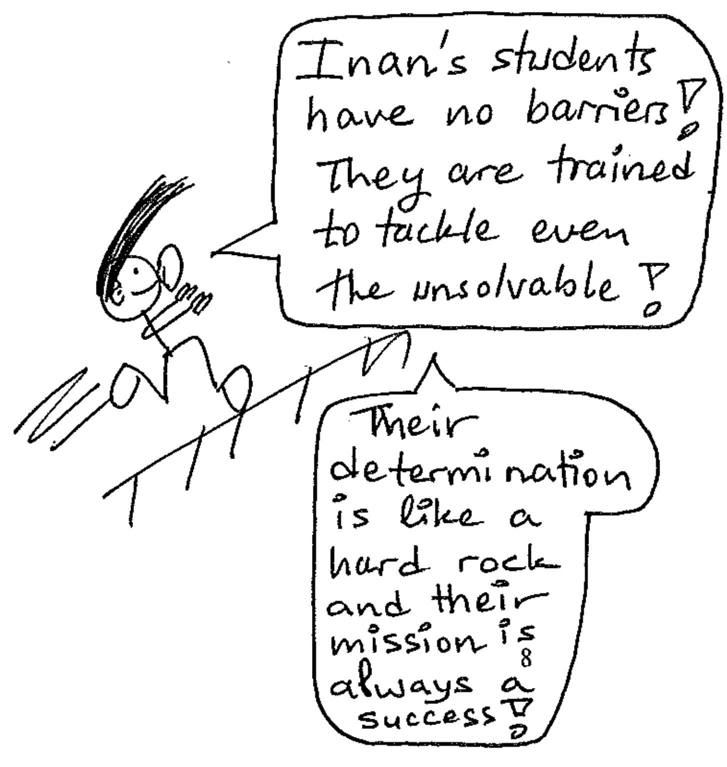
$$y(t) = y_1(t) + y_2(t) = 2h(t-1) + 3y_s(t-2)$$



$$= -6e^{-3(t-1)}u(t-1) - 2\delta(t-1) + 3e^{-3(t-2)}u(t-2) - 6u(t-2)$$

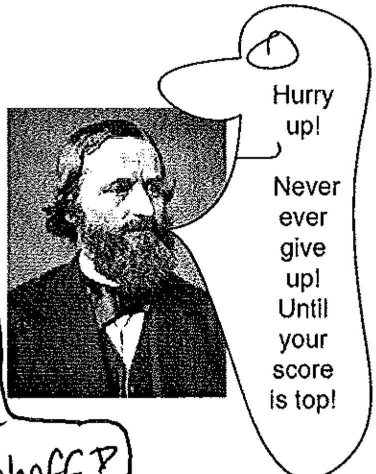
$$\boxed{\therefore\ y(t) = -6e^{-3(t-1)}u(t-1) - 2\delta(t-1) + 3\left(e^{-3(t-2)} - 2\right)u(t-2)}$$

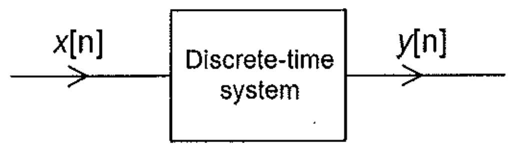




Inan is trying to scare me with a ghost-shaped callout but I will pretend like I didn't even notice it...

(4) (15 mins., <u>Total:</u> 20 points) **Properties of a discrete-time system.** The input $x[n]$ and the output $y[n]$ relationship of a discrete-time system is given by $y[n]=(n-1)e^{x[n/2]}$.





You can be rest assured Mr. Kirchhoff!

Inan's students always target nothing but the highest test scores!

(a) (3 points) Is this system memory-less? (Provide a clearly stated justification for your answer.)

Since $y[2]$ depends on $x[1]$, this system is <u>not</u> memoryless!

Inan's tests transform some UP students with memory into "memoryless" students!

(b) (3 points) Is this system causal? (Clear justification required!)

At least for a while...

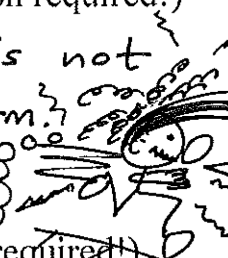
Since $y[n] \neq 0$ for $n<0$ when $x[n]=0$ for $n<0$ (because $y[n]=(n-1)e^{0}=n-1$), this system is <u>noncausal</u>!

(c) (3 points) Is this system BIBO stable? (Justification required!)

Even if $x[n]$ is bounded, $y[n]$ is not bounded because of the $(n-1)$ term.

$\therefore$ <u>Not</u> BIBO stable!

You will pay for this Inan! *@*#*

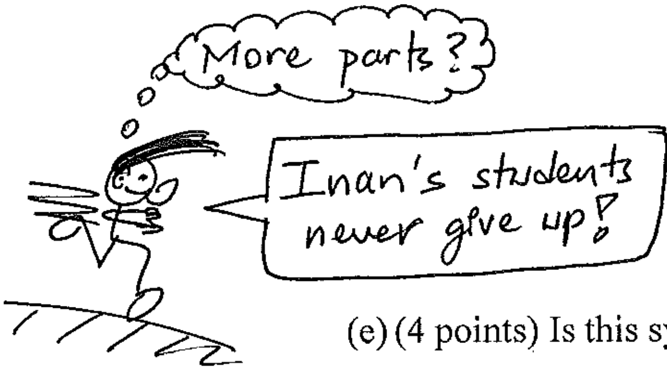


I was BIBO stable until I took Inan's classes!

(d) (3 points) Is this system invertible? (Justification required!)

$$y[2n]=(2n-1)e^{x[n]} \rightarrow x[n]=\ln\left(\frac{y[2n]}{2n-1}\right)$$

<u>Yes</u>, invertible system!



(e) (4 points) Is this system linear? (Justification required!)

$x_1[n] \rightarrow \boxed{\text{DT system}} \rightarrow y_1[n] = (n-1)e^{x_1[n/2]}$

$x_2[n] \rightarrow \boxed{\text{DT system}} \rightarrow y_2[n] = (n-1)e^{x_2[n/2]}$

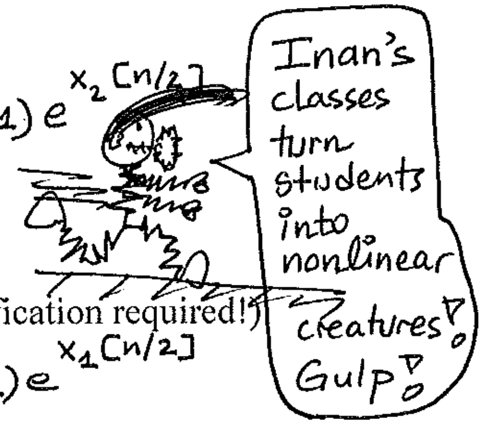
$x_3[n] = a x_1[n] + b x_2[n] \rightarrow \boxed{\text{DT system}} \rightarrow y_3[n] = (n-1)e^{a x_1[n/2] + b x_2[n/2]}$

$$\text{Since } y_3[n] = (n-1)e^{a x_1[n/2] + b x_2[n/2]}$$

$$\neq a y_1[n] + b y_2[n]$$

$$= a(n-1)e^{x_1[n/2]} + b(n-1)e^{x_2[n/2]}$$

$\therefore$ This system is <u>nonlinear</u>!



(f) (4 points) Is this system time invariant? (Justification required!)

$x_1[n] \rightarrow \boxed{} \rightarrow y_1[n] = (n-1)e^{x_1[n/2]}$

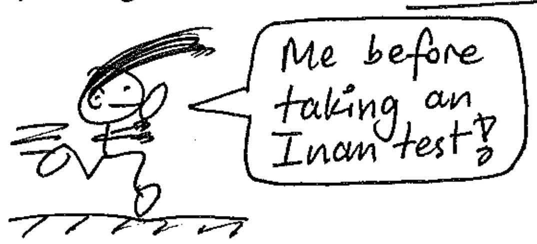
$x_2[n] = x_1[n-n_0] \rightarrow \boxed{} \rightarrow y_2[n] = (n-1)e^{x_1[n/2-n_0]}$

$$\text{Since } y_2[n] = (n-1)e^{x_1[n/2-n_0]}$$

$$\neq y_1[n-n_0]$$

$$= (n-n_0-1)e^{x_1[(n-n_0)/2]}$$

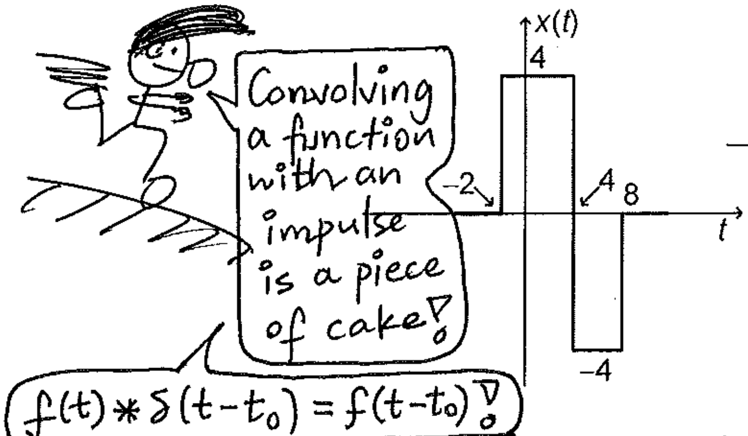
$\therefore$ This system is <u>time variant</u>!

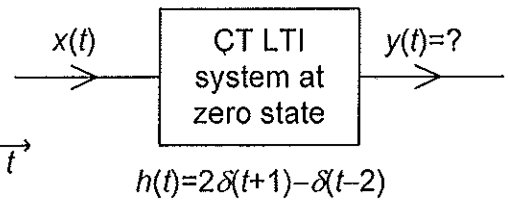


Inan must be jealous about my confidence

I need to keep my focus on the test and demonstrate my perseverance

(5) (15 mins., 20 points) **LTI system.** Find the complete mathematical expression and sketch the output response $y(t)$ of the linear time-invariant (LTI) system due to the input signal $x(t)$ as shown.





Note that $x(t) = 4u(t+2) - 8u(t-4) + 4u(t-8)$

$$y(t) = x(t) * h(t) = x(t) * 2\delta(t-1) - x(t) * \delta(t-2)$$

$$= \boxed{8u(t+3) - 16u(t-3) + 8u(t-7) - 4u(t) + 8u(t-6) - 4u(t-10)}$$