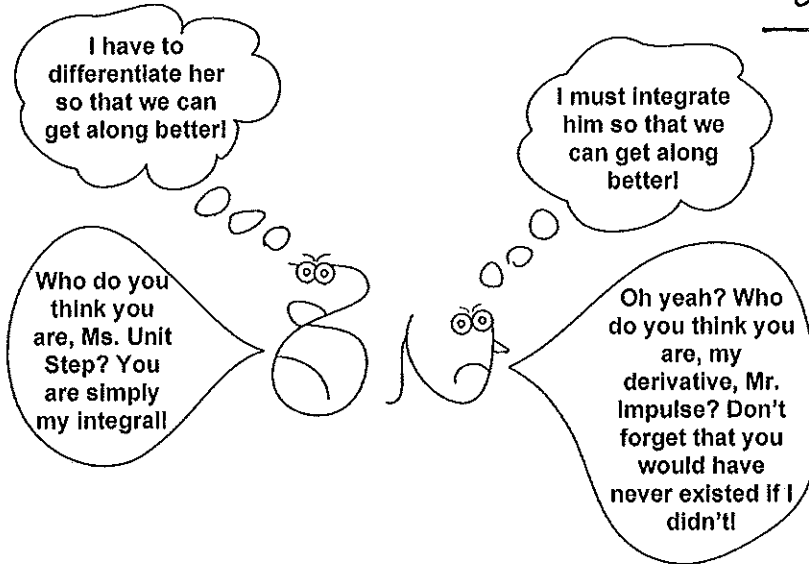


2/18/2011



**University of Portland  
School of Engineering**

**EE 262-Signals & Systems-3 cr. hrs.  
Spring 2011**

**Midterm Exam # 1**

(Prepared by Professor A. S. Inan)

(Friday, February 18, 2011\*)

(Closed Book Exam, One formula sheet allowed.)

(Total Time: 55 mins.)

$$\begin{array}{r} 55949 \\ - \quad 39 \\ \hline 55910 \end{array}$$

\*Today's math puzzle by Inan: Today's date expressed as a single date number as 2182011 equals  $39 \times 55949$  where 55949 is a prime number. Interestingly enough, the difference of numbers 55949 and 39 relate to Inan's age. Can you figure out how? ☺

Reverse of this number is 1955

Name: SOLUTIONS !

1955 must be his birth year? Why should I care? ☺

Signature: Solution ☺



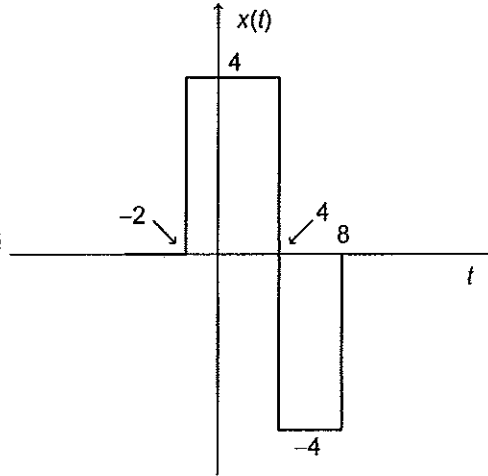
Note: 5 Problems total, 3 in-class, 2 take-home, due next Tuesday!

(1) (10 mins., Total: 20 points) A continuous-time signal. Consider the continuous-time signal denoted by  $x(t)$  as shown in the figure below.



Best of luck and please, don't panic! Go step by step...

Demonstrate to Inan that you are not scared of his tests!



Thanks Mr. Kirchhoff!

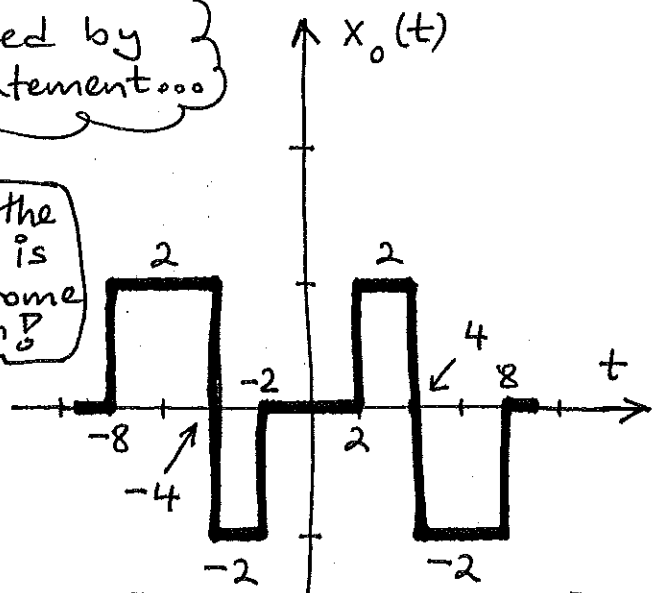
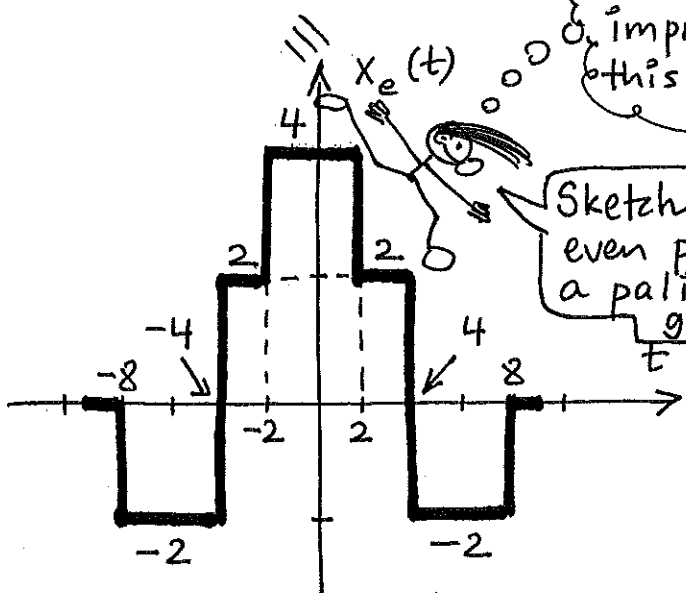
(a) (8 points) Sketch the even and odd parts of  $x(t)$ . Provide all the pertinent values on your sketch.

No worries, Inan's students are the fiercest warriors!

$$x_e(t) = \frac{x(t) + x(-t)}{2}, \quad x_o(t) = \frac{x(t) - x(-t)}{2}$$

Inan will be impressed by this statement...

Sketch of the even part is a palindrome graph!

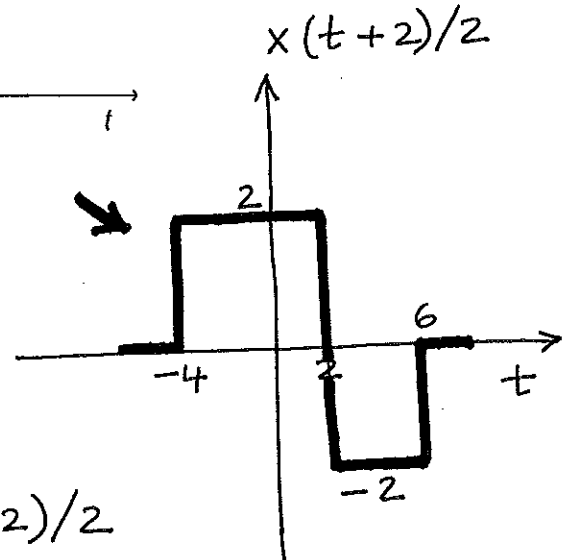
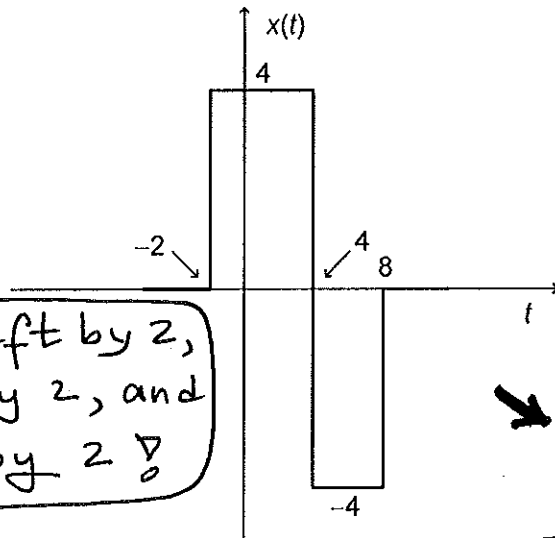


Flipping its left or right side upside down will make the sketch of the odd part a palindrome too!

Wiser to do time shifting before time scaling...

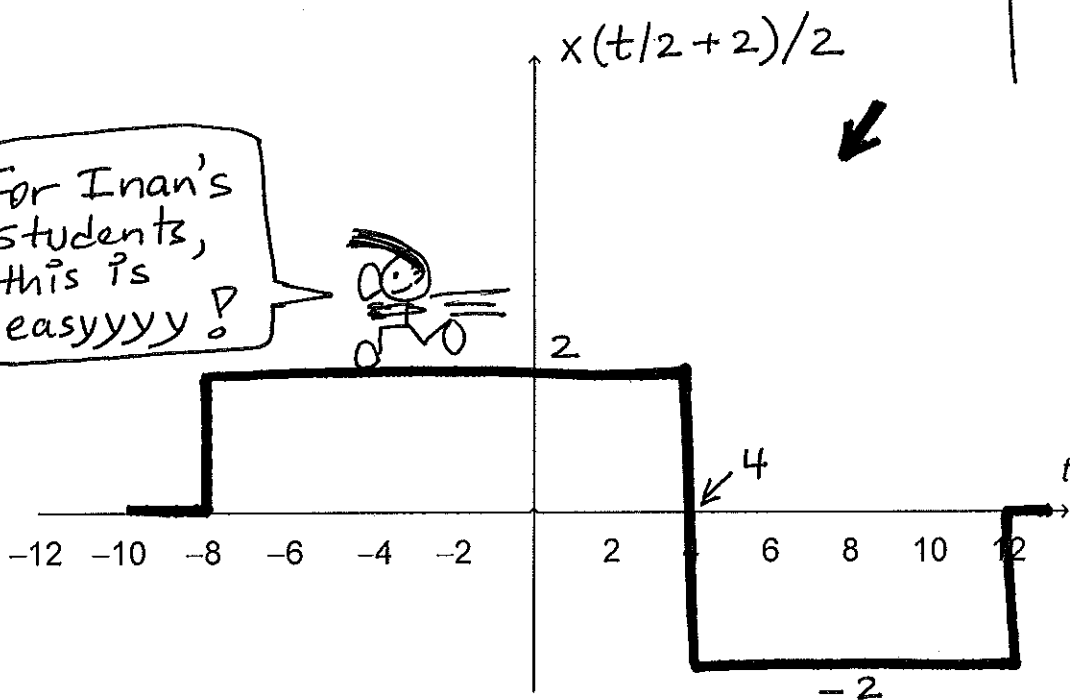


Shift left by 2, expand by 2, and weaken by 2!



(b) (6 points) Sketch  $x(t/2+2)/2$ .

For Inan's students, this is easyyyy!

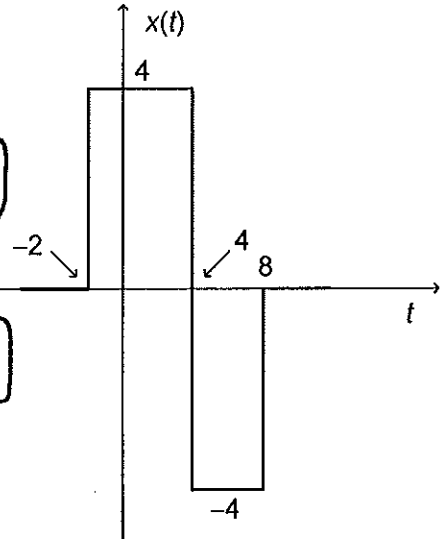


Next part! Surrender or you will regret that you did not!





Differentiating every jump point leads to an impulse function!

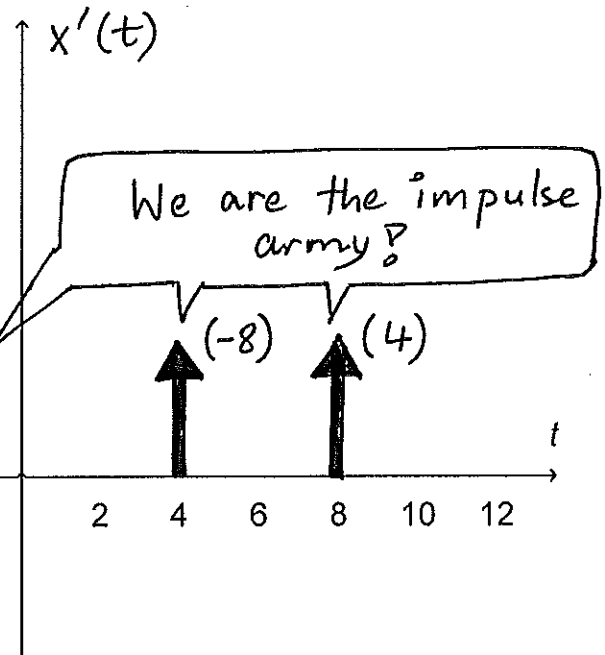


Inan's students are well trained to tackle with impulse functions!

(c) (6 points) Find the complete mathematical expression for the function  $y(t) = dx(t)/dt$  and sketch  $y(t)$  versus  $t$ . Provide all the pertinent values on your sketch.



Salute to all the impulses!



$$x(t) = 4u(t+2) - 8u(t-4) + 4u(t-8)$$

$$x'(t) = 4\delta(t+2) - 8\delta(t-4) + 4\delta(t-8)$$



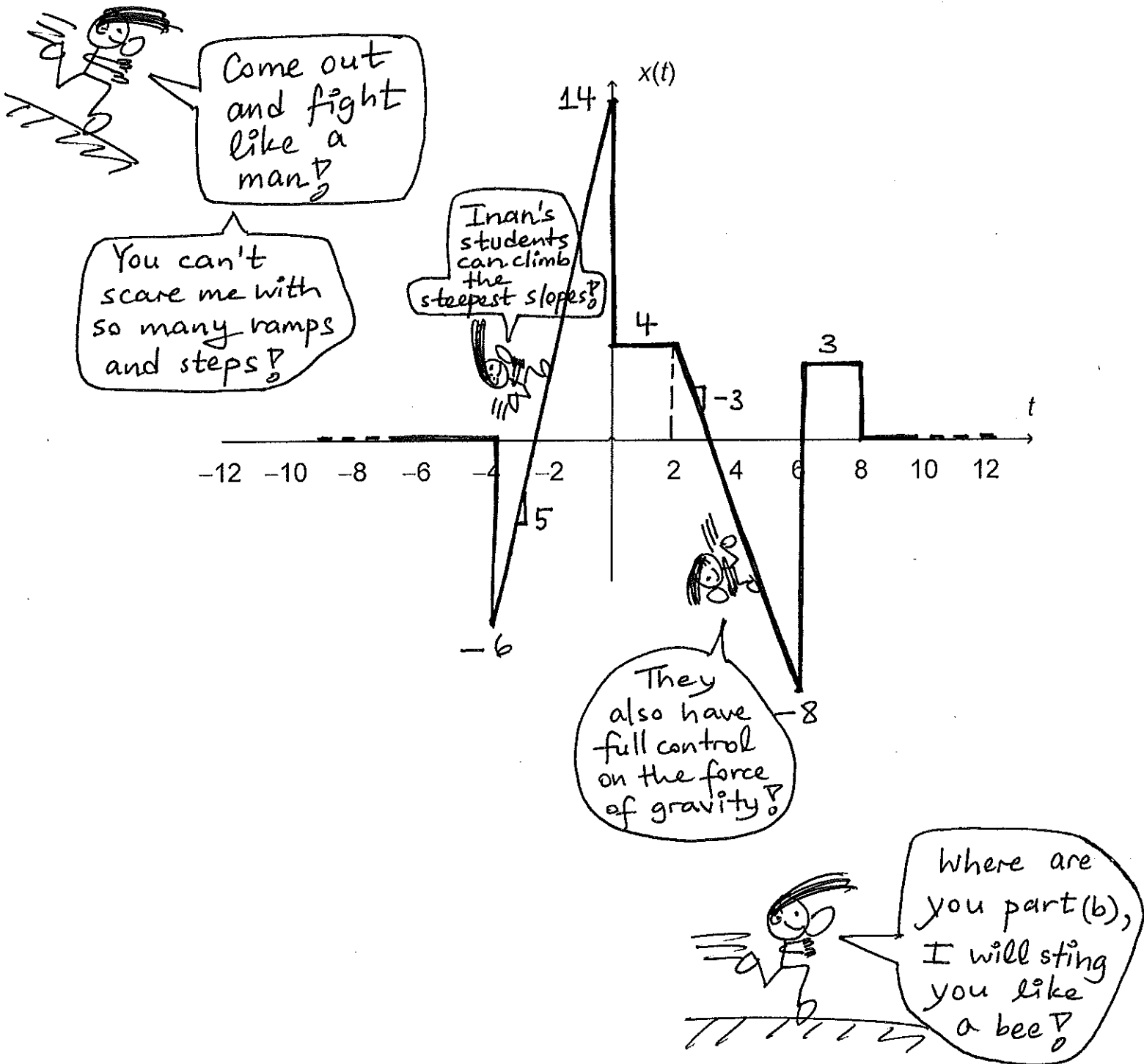
Next problem!

Don't hide, or you will be swallowed by a tide!

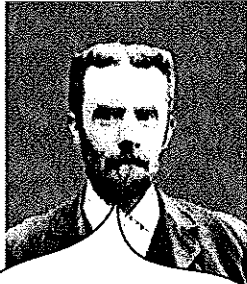
(2) (15 mins., Total: 20 points) **Impulse, step, and ramp functions.** A continuous-time signal is given by

$$x(t) = 5r(t+4) - 6u(t+4) - 10u(t) - 5r(t) - 3r(t-2) + 3r(t-6) + 11u(t-6) - 3u(t-8)$$

(a) (10 points) Sketch this signal. Provide all the necessary values on your sketch.



(b)(10 points) Using  $x(t)$  given in part (a), sketch the derivative signal,  $dx(t)/dt$ . Provide all the appropriate values on your sketch.



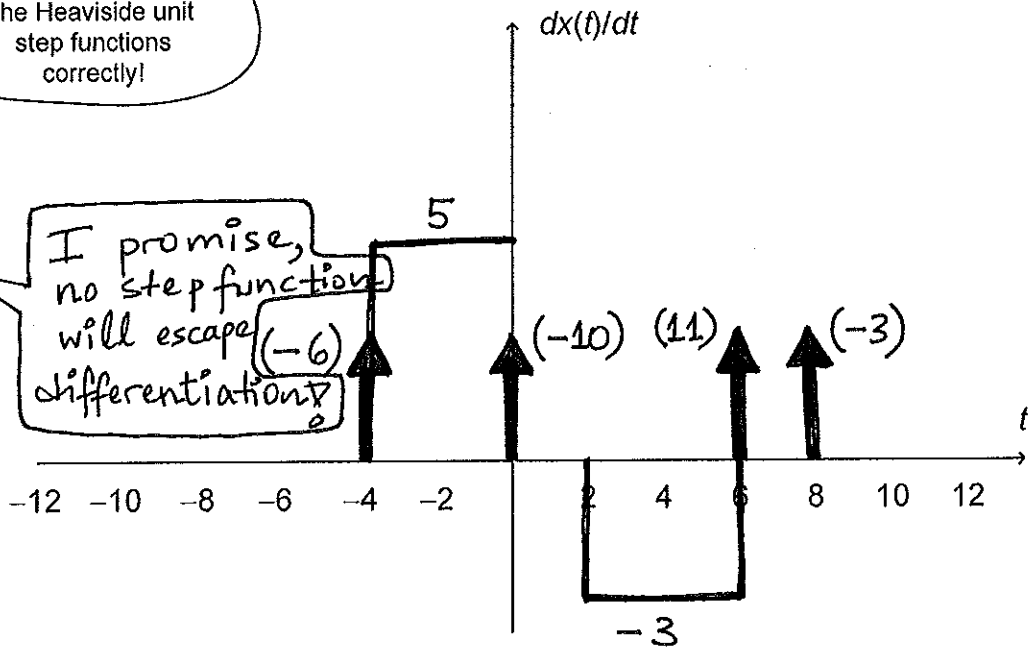
Please, differentiate the Heaviside unit step functions correctly!

No worries Mr. Heaviside, I won't let you down!

Why is there a bump in my callout?

$\frac{du}{dt} = \delta$

I promise, no step functions will escape differentiation!



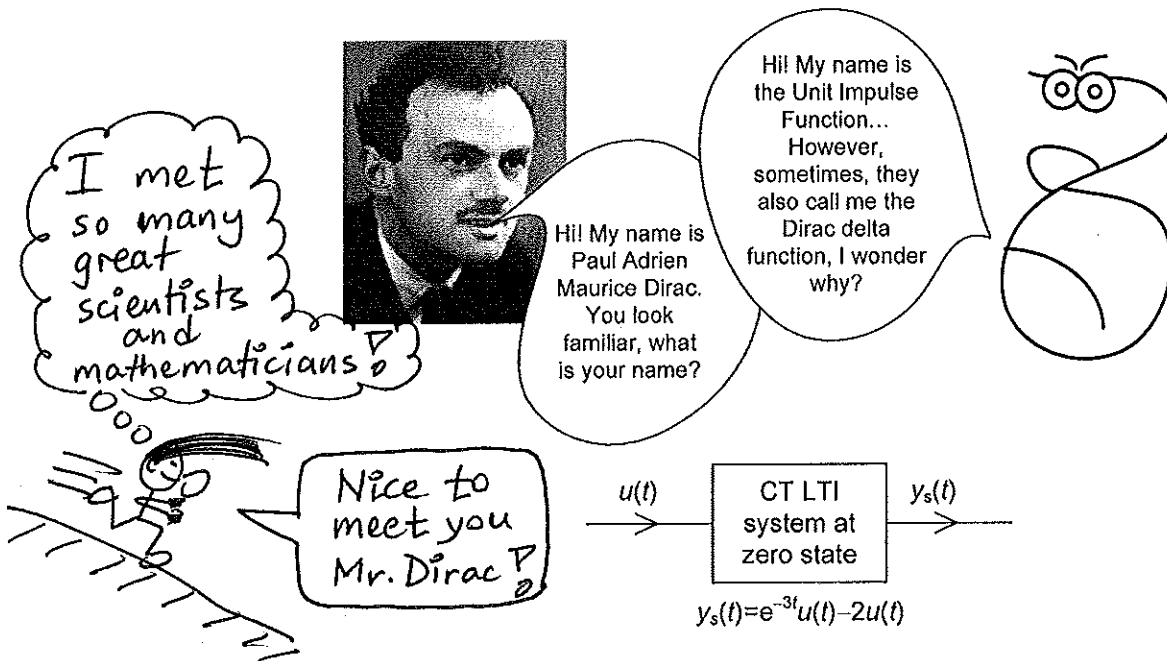
The mathematical expression for  $x'(t)$  is given by:

$$x'(t) = 5u(t+4) - 6\delta(t+4) - 10\delta(t) - 5u(t) - 3u(t-2) + 3u(t-6) + 11\delta(t-6) - 3\delta(t-8)$$



All step functions of  $x(t)$  are successfully differentiated!

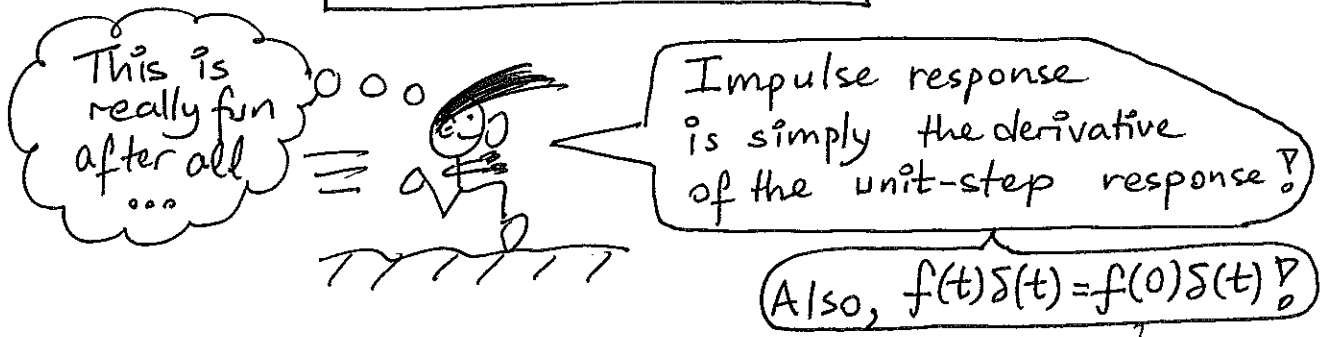
Mission complete! Next problem!



(3) (15 mins., Total: 20 points) **CT LTI system.** The unit-step response  $y_s(t)$  of a continuous-time (CT) linear time-invariant (LTI) system is given as shown.

(a) (10 points) Find the impulse response  $h(t)$  of this system.

$$\begin{aligned}
 h(t) &= \frac{dy_s(t)}{dt} = \frac{d}{dt} \left[ (e^{-3t} - 2) u(t) \right] \\
 &= -3e^{-3t} u(t) + (e^{-3t} - 2) \frac{du(t)}{dt} \\
 &= -3e^{-3t} u(t) + (e^{-3t} - 2) \delta(t) \\
 &= \boxed{-3e^{-3t} u(t) - \delta(t)}
 \end{aligned}$$




(b) (10 points) Find the response of this system due to an input signal given by  $x(t) = 2\delta(t-1) + 3u(t-2)$ . (Assume zero initial conditions.)

$$x(t) = x_1(t) + x_2(t) = 2\delta(t-1) + 3u(t-2)$$

$$y(t) = y_1(t) + y_2(t) = 2h(t-1) + 3y_s(t-2)$$


$$= -6e^{-3(t-1)}u(t-1) - 2\delta(t-1) \\ + 3e^{-3(t-2)}u(t-2) - 6u(t-2)$$

$$\therefore y(t) = -6e^{-3(t-1)}u(t-1) - 2\delta(t-1) \\ + 3(e^{-3(t-2)} - 2)u(t-2)$$



LTI systems are fun!

Combination of impulse and step responses!



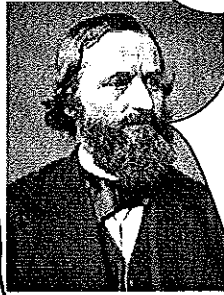
Inan's students have no barriers! They are trained to tackle even the unsolvable!

Their determination is like a hard rock and their mission is always a success!



Inan is trying to scare me with a ghost-shaped callout but I will pretend like I didn't even notice it...

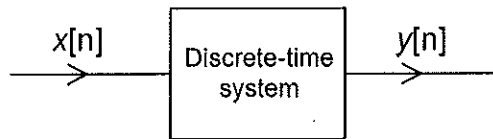
(4) (15 mins., Total: 20 points) **Properties of a discrete-time system.** The input  $x[n]$  and the output  $y[n]$  relationship of a discrete-time system is given by  $y[n] = (n-1)e^{x[n/2]}$ .



Hurry up!

Never ever give up! Until your score is top!

You can be rest assured Mr. Kirchhoff!



(a) (3 points) Is this system memory-less? (Provide a clearly stated justification for your answer.)

Since  $y[2]$  depends on  $x[1]$ , this system is not memoryless!

Inan's tests transform some UP students with memory into "memoryless" students!

(b) (3 points) Is this system causal? (Clear justification required!)

Since  $y[n] \neq 0$  for  $n < 0$  when  $x[n] = 0$  for  $n < 0$  (because  $y[n] = (n-1)e^0 = n-1$ ), this system is noncausal!

At least for a while...

(c) (3 points) Is this system BIBO stable? (Justification required!)

Even if  $x[n]$  is bounded,  $y[n]$  is not bounded because of the  $(n-1)$  term.  
 $\therefore$  Not BIBO stable!

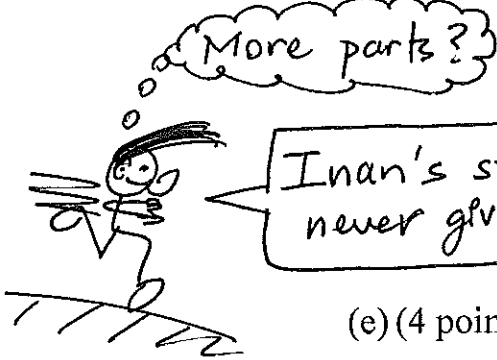
You will pay for this Inan!

I was BIBO stable until I took Inan's classes!

(d) (3 points) Is this system invertible? (Justification required!)

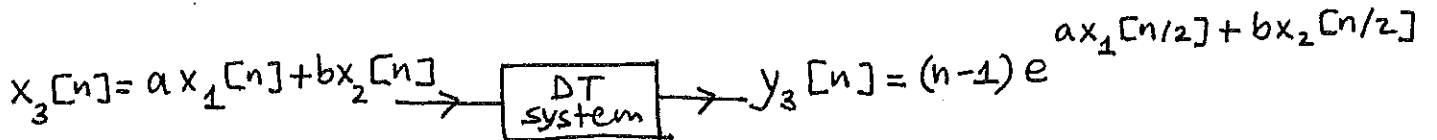
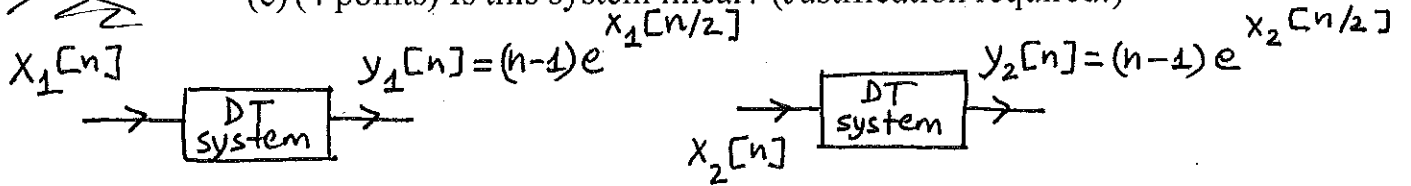
$$y[2n] = (2n-1)e^{x[n]} \rightarrow x[n] = \ln\left(\frac{y[2n]}{2n-1}\right)$$

Yes, invertible system!



Inan's students never give up!

(e) (4 points) Is this system linear? (Justification required!)



Since  $y_3[n] = (n-1)e^{ax_1[n/2] + bx_2[n/2]}$

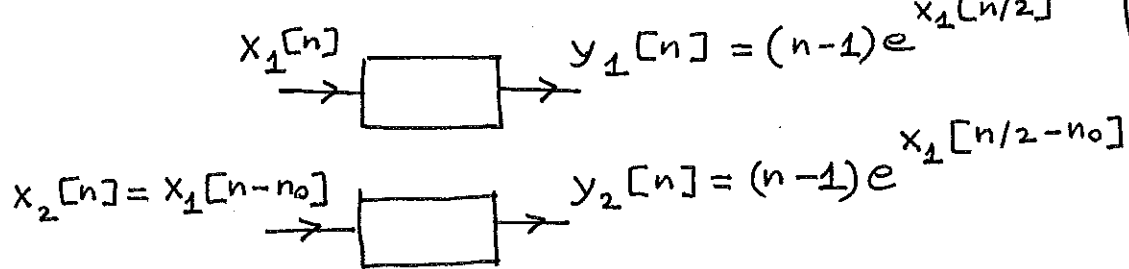
$\neq ay_1[n] + by_2[n]$

$= a(n-1)e^{x_1[n/2]} + b(n-1)e^{x_2[n/2]}$

∴ This system is nonlinear!

Inan's classes turn students into nonlinear creatures! Gulp!

(f) (4 points) Is this system time invariant? (Justification required!)

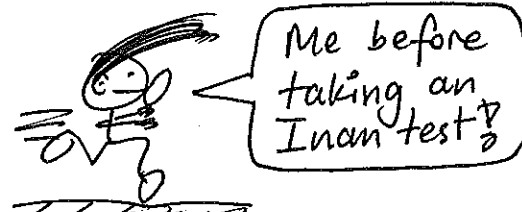


Since  $y_2[n] = (n-1)e^{x_1[n/2 - n_0]}$

$\neq y_1[n - n_0]$   
 $= (n - n_0 - 1)e^{x_1[(n - n_0)/2]}$

∴ This system is time variant!

Arghhh!



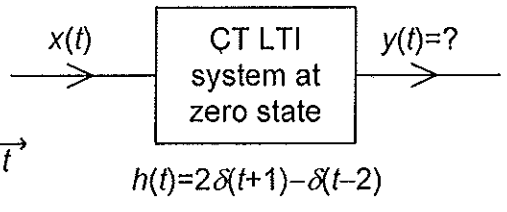
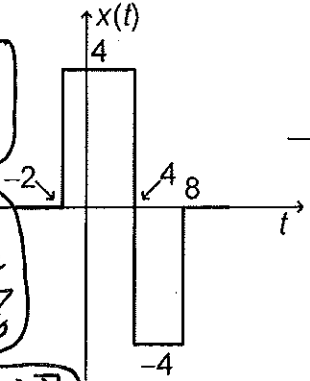
Inan must be jealous about my confidence

I need to keep my focus on the test and demonstrate my perseverance

(5) (15 mins., 20 points) LTI system. Find the complete mathematical expression and sketch the output response  $y(t)$  of the linear time-invariant (LTI) system due to the input signal  $x(t)$  as shown.



Convolving a function with an impulse is a piece of cake



$f(t) * \delta(t-t_0) = f(t-t_0)$

Note that  $x(t) = 4u(t+2) - 8u(t-4) + 4u(t-8)$

$y(t) = x(t) * h(t) = x(t) * 2\delta(t-1) - x(t) * \delta(t-2)$

$= 8u(t+3) - 16u(t-3) + 8u(t-7) - 4u(t) + 8u(t-6) - 4u(t-10)$

