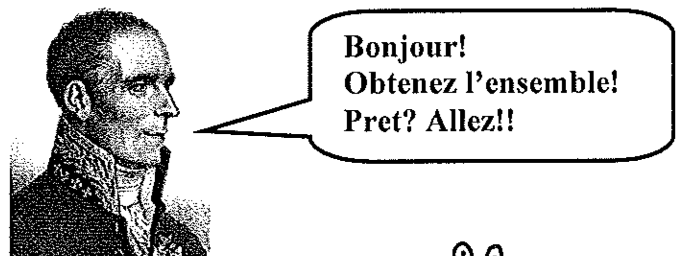
3/30/2011

University of Portland
School of Engineering

EE 262-Signals & Systems-3 cr. hrs.
Spring 2011

Midterm Exam # 2
(Prepared by Professor A. S. Inan)



(Wednesday, March 30, 2011)

Name: SOLUTIONS ☺

Signature: Solutions ☺

"Honesty is the best policy."
Aesop (~ 620B.C. –?)

"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

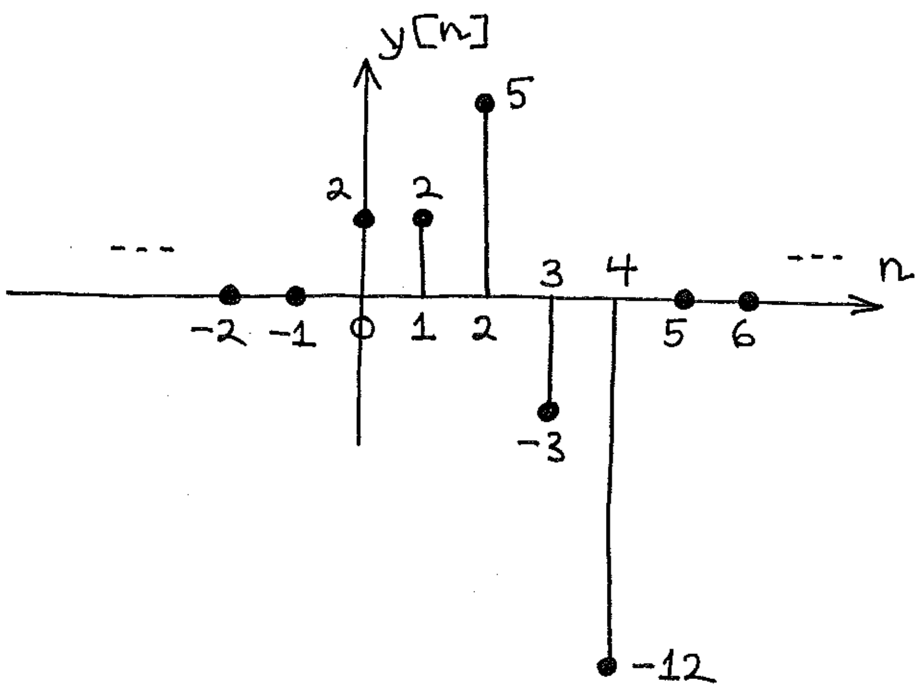
"Honest people are the true winners of the universe."
Anonymous

Select and do any 4 of the 5 problems assigned during class time. The fifth problem is a take-home problem due class time on Friday, April 1, 2011.

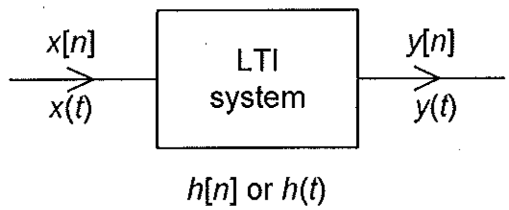
(1) (20 points). **DT LTI system.** The unit impulse of a DT LTI system is given below. Find and sketch the zero-state response $y_{zs}[n]$ of this system due to input signal $x[n] = 2\delta[n] - 3\delta[n-2]$.

$h[n] = u[n] + 3\delta[n-2] - u[n-3]$

$$y[n] = h[n] * x[n] = 2u[n] + 6\delta[n-2] - 2u[n-3]$$
$$- 3u[n-2] - 9\delta[n-4] + 3u[n-5]$$
$$= \boxed{2\delta[n] + 2\delta[n-1] + 5\delta[n-2] - 3\delta[n-3] - 12\delta[n-4]}$$



(2) (Total: 15 points) **LTI system.** For each of the following unit impulse responses corresponding to an LTI system, determine whether the system is (a) Memoryless, (b) Causal, and (c) BIBO stable. Provide a clear justification for each answer.



(a) (7.5 points) $h[n] = (1-n)u[n-1]$

(a) Not memoryless since $h[n] \neq K\delta[n]$.

(b) Causal since $h[n] = 0$ for $n < 0$.

(c) Unstable since $\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=1}^{\infty} (k-1) \to \infty$

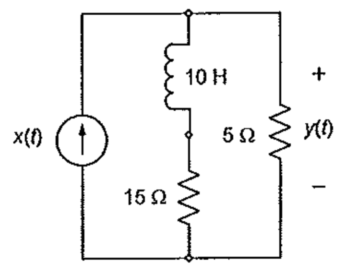
(b) (7.5 points) $h(t) = 2\sin(\pi t)u(t-1)$

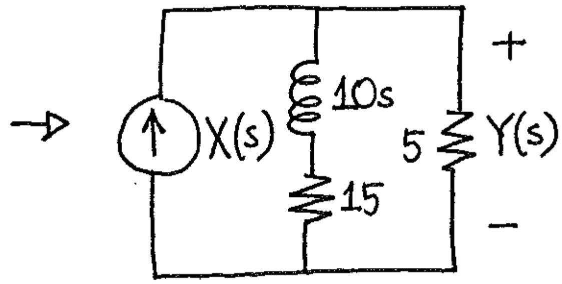
(a) Not memoryless since $h(t) \neq K\delta(t)$.

(b) Causal since $h(t) = 0$ for $t < 0$.

(c) Unstable since $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{1}^{\infty} 2|\sin(\pi t)|\,dt \to \infty$

(3) (Total: 25 points) **Electric circuits.** For the electric circuit shown:





(a) (10 points) Find its transfer function $H(s)$. Provide your work step by step.

$$H(s) = \frac{Y(s)}{X(s)} = Z_{eq}(s) = 5 // (10s+15)$$

$$= \frac{5(10s+15)}{5+10s+15}$$

$$= \boxed{\frac{5(2s+3)}{2(s+2)}}$$

(b) (10 points) Find the impulse response $h(t)$. Show your work.

$$H(s) = \frac{5}{2}\,\frac{2s+3}{s+2} = \frac{5}{2}\left[2 - \frac{1}{s+2}\right]$$

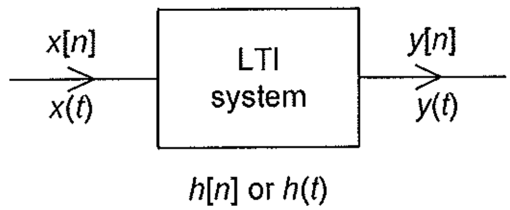
$$\therefore\ h(t) = \mathcal{L}^{-1}\{H(s)\} = \boxed{5\delta(t) - \frac{5}{2}e^{-2t}u(t)}$$

(c) (5 points) Find the unit step response $y_s(t)$. Provide your work.

$$y_s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau = \begin{cases} 0, & t<0 \\ 5+\left[\frac{5}{4}e^{-2\tau}\right]_0^t, & t>0 \end{cases}$$

$$= \begin{cases} 0, & t<0 \\ \frac{15}{4}+\frac{5}{4}e^{-2t}, & t>0 \end{cases} = \boxed{\left(\frac{15}{4}+\frac{5}{4}e^{-2t}\right)u(t)}$$

(4) (<u>Total:</u> 20 points) **LTI system.** Consider an LTI system.



(a) (10 points) If the system is continuous time, given the unit-step response of the system to be

$$y_s(t) = \left(5te^{-t} + 11e^{-t} - 3\right)u(t)$$

Find the unit-impulse response $h(t)$ of the system. What is $h(2)$?

$$h(t) = \frac{dy_s(t)}{dt} = \left(5e^{-t} - 5te^{-t} - 11e^{-t}\right)u(t)$$

$$+\underbrace{\left(5te^{-t} + 11e^{-t} - 3\right)}_{8}\delta(t)$$

$$= \boxed{-(5t+6)e^{-t}u(t) + 8\delta(t)}$$

$$h(2) = -10e^{-2} - 6e^{-2} = \boxed{-\frac{16}{e^2}}$$

(b) (10 points) If the system is discrete time, given the impulse response of the system to be

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

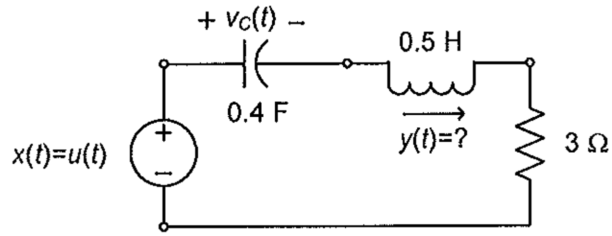
Find the unit-step response $y_s[n]$ of the system. What is $y_s[2]$?

$$y_s[n] = \sum_{k=0}^{\infty} h[n-k] = \boxed{\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{n-k} u[n-k]}$$

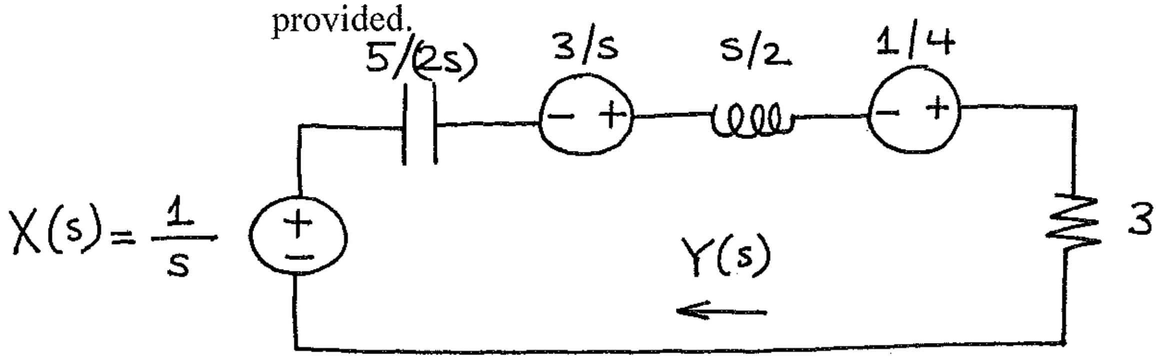
$$= \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1] + \left(\frac{1}{2}\right)^{n-2} u[n-2] + \left(\frac{1}{2}\right)^{n-3} u[n-3] + \cdots$$

$$y_s[2] = \left(\frac{1}{2}\right)^{2} + \left(\frac{1}{2}\right)^{1} + \left(\frac{1}{2}\right)^{0} = \frac{1}{4} + \frac{1}{2} + 1 = \boxed{\frac{7}{4}}$$

(5) (Total: 20 points) **Electric circuits.** In the circuit shown, assuming $v_C(0^-) = -3$ V and $y(0^-) = 0.5$ A, do the following:



(a) (10) Draw the Laplace domain circuit with all the numerical values provided.



(b) (10) Using part (a) circuit, obtain $Y(s)$ and perform inverse Laplace on it to obtain $y(t)$ for $t > 0$.

$$Y(s) = \frac{\frac{1}{s} + \frac{3}{s} + \frac{1}{4}}{\frac{5}{2s} + \frac{s}{2} + 3} = \frac{s+16}{\cancel{4s}_{\,2}} \cdot \frac{\cancel{2s}}{s^2+6s+5}$$

$$= \frac{s+16}{2(s+1)(s+5)} = \frac{K_1}{s+1} + \frac{K_2}{s+5}$$

$(s+1)Y(s)\big|_{s=-1}$

$$K_1 = \left.\frac{s+16}{2(s+5)}\right|_{s=-1} = \frac{15}{8}$$

$(s+5)Y(s)\big|_{s=-5}$

$$K_2 = \left.\frac{s+16}{2(s+1)}\right|_{s=-5} = -\frac{11}{8}$$

$$\boxed{\therefore\ y(t) = \frac{15}{8} e^{-t} u(t) - \frac{11}{8} e^{-5t} u(t)}$$