4/20/2011

University of Portland
School of Engineering

EE 262-Signals & Systems-3 cr. hrs.
Spring 2011

Midterm Exam # 3
(Prepared by Professor A. S. Inan)

(Wednesday, April 20, 2011)

Name: SOLUTIONS ! ☺

Signature: Snoitulos ☺

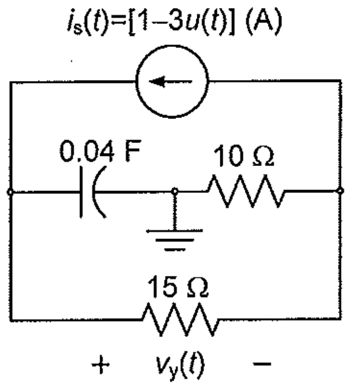
"Honesty is the best policy."
Aesop (~ 620B.C. –?)

"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

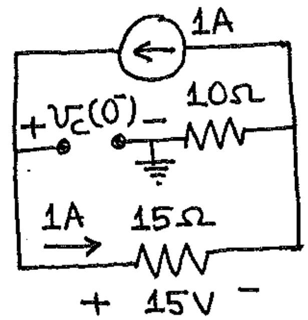
"Honest people are the true winners of the universe."
Anonymous

Two problems take-home: Do either Problem # 1 or # 2. Also, do either # 3 or # 4. The problems you choose not to do in class are take-home due to the beginning of the next class. Best of luck!

(1) (25 points). **Laplace transform in electric circuits.** Using Laplace domain equivalent circuit, find the complete simplified mathematical expression for the voltage $v_y(t)$ for $t > 0$.

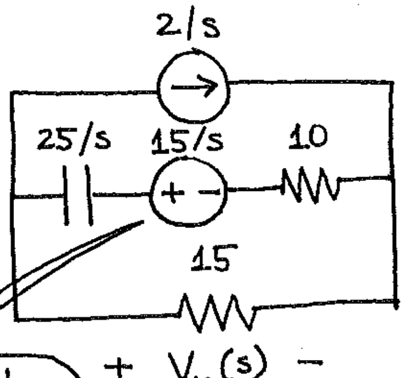


At $t=0^-$ (SS):



$V_C(0^-)=15V$

Laplace domain equivalent circuit:



I represent the initial condition of the capacitor V

Using superposition principle:

$$V_y(s) = \frac{15 \times 15/s}{15+10+\frac{25}{s}} - \frac{15\times\left(10+\frac{25}{s}\right)\left(\frac{2}{s}\right)}{15+10+\frac{25}{s}}$$

$$= \frac{9}{s+1} - \frac{6(2s+5)}{s(s+1)}$$

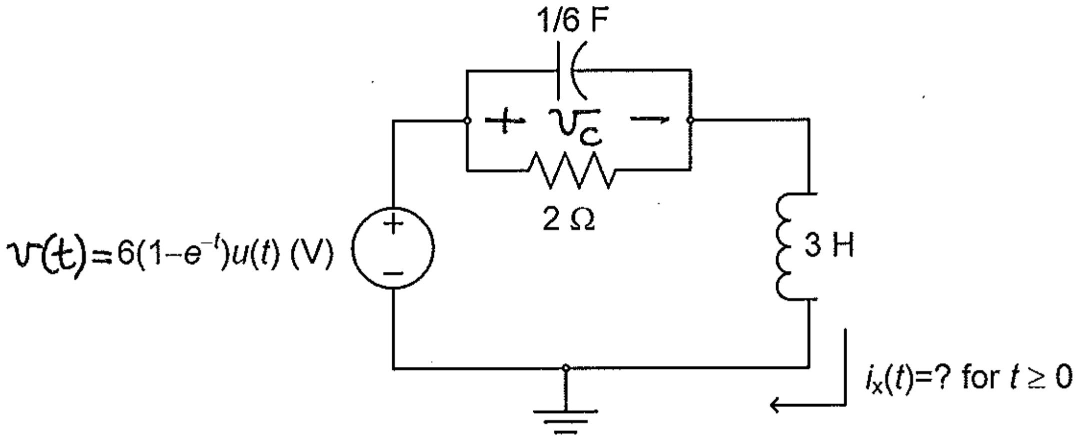
$$= \frac{-3(s+10)}{s(s+1)} = \frac{A}{s} + \frac{B}{(s+1)}$$

$$A = sV_y(s)\Big|_{s=0} = -30$$

$$B = (s+1)V_y(s)\Big|_{s=-1} = 27$$

$$\boxed{\therefore\ v_y(t) = -30u(t) + 27e^{-t}u(t)}$$

(2) (Total: 25 points) **Laplace transform in electric circuits.** For the electric circuit shown:



(a) (10 points) Draw the complete unilateral Laplace domain equivalent circuit with all the pertinent values included.

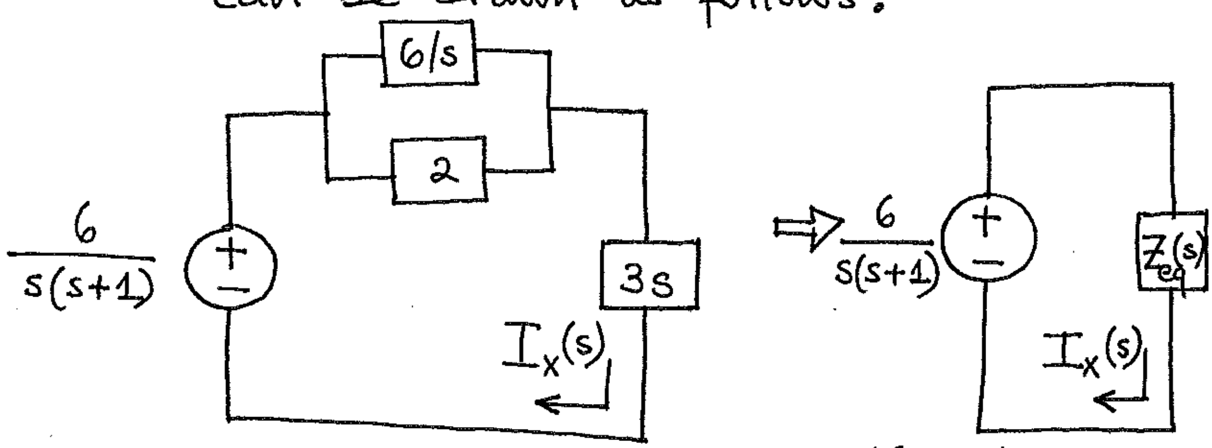
Note that the initial conditions are

$$v_c(0^+) = 0 \text{ and } i_x(0^+) = 0$$

Also, Laplace transform of $v(t)$ waveform is given by

$$V(s) = \frac{6}{s} - \frac{6}{(s+1)} = \frac{6}{s(s+1)}$$

The Laplace domain equivalent circuit can be drawn as follows:



$$Z_{eq}(s) = \left(\frac{6}{s} // 2\right) + 3s$$

(b)(15 points) Using the circuit drawn in part (a), find the complete simplified mathematical expression for the current $i_x(t)$ for $t \geq 0$.

<u>Ohm's law:</u>

$$I_x(s) = \frac{\dfrac{6}{s(s+1)}}{\left(\frac{6}{s} // 2\right) + 3s}$$

$$= \frac{\dfrac{6}{s(s+1)}}{\dfrac{6}{s+3} + 3s}$$

$$= \frac{2(s+3)}{s(s+1)^2(s+2)}$$

$$= \frac{A}{s} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)} + \frac{D}{(s+2)}$$

$$A = s I_x(s)\Big|_{s=0} = 3$$

$$B = (s+1)^2 I_x(s)\Big|_{s=-1} = \frac{2 \times 2}{-1 \times 1} = -4$$

$$C = \frac{d}{ds}\left[(s+1)^2 I_x(s)\right]\Big|_{s=-1} = \frac{2s(s+2) - (2s+2)\,2(s+3)}{[s(s+2)]^2}\Bigg|_{s=-1}$$

$$= \frac{-2 \times 1 - 0}{[-1 \times 1]^2} = -2$$

$$D = (s+2) I_x(s)\Big|_{s=-2} = \frac{2 \times 1}{-2 \times (-1)^2} = -1$$

$$\boxed{\therefore\ i_x(t) = 3u(t) - 4te^{-t}u(t) - 2e^{-t}u(t) - e^{-2t}u(t)}$$

(3) (Total: 25 points) **Fourier transforms.** Use the tables of Fourier transforms and properties to find the Fourier transform of the following functions:

(a) (12.5 points) $x(t) = 4e^{3-2t}\cos(2t-2)u(t-1)$ $= 4e\, e^{-2(t-1)}\cos(2(t-1))\,u(t-1)$

$$\left.\begin{aligned} e^{-2t}u(t) &\leftrightarrow \frac{1}{2+j\omega} \\ \cos(2t) &\leftrightarrow \frac{e^{j2t}+e^{-j2t}}{2} \end{aligned}\right\} \leftarrow \text{Fourier transform pairs:}$$

Modulation property $\rightarrow$ $\cos(2t)e^{-2t}u(t) \leftrightarrow \underbrace{\frac{1}{2}\left[\frac{1}{2+j(\omega-2)} + \frac{1}{2+j(\omega+2)}\right]}_{\frac{2+j\omega}{[2+j(\omega-2)][2+j(\omega+2)]}}$

Time-shift property $\rightarrow$ $x(t) \leftrightarrow \boxed{\frac{4e^{1-j\omega}(2+j\omega)}{[2+j(\omega-2)][2+j(\omega+2)]}}$

(b) (12.5 points) $x(t) = 3t\dfrac{d^2}{dt^2}\left(e^{-\pi|2t-1|}\right)$

$e^{-\pi|t|} \leftrightarrow \dfrac{2\pi}{\pi^2+\omega^2}$ $\leftarrow$ Fourier transform pair

Time-shift property $\rightarrow$ $e^{-\pi|t-1|} \leftrightarrow \dfrac{2\pi e^{-j\omega}}{\pi^2+\omega^2}$

Time-scaling property $\rightarrow$ $e^{-\pi|2t-1|} \leftrightarrow \dfrac{1}{\cancel{2}}\,\dfrac{\cancel{2}\pi e^{-j\omega/2}}{\pi^2+(\omega/2)^2}$

Time-derivative property $\rightarrow$ $\dfrac{d^2}{dt^2}\left(e^{-\pi|2t-1|}\right) \leftrightarrow \dfrac{-\pi\omega^2 e^{-j\omega/2}}{\pi^2+(\omega/2)^2}$

Multiplication by t $\rightarrow$ $3t\dfrac{d^2}{dt^2}\left(e^{-\pi|2t-1|}\right) \leftrightarrow \boxed{j3\dfrac{d}{d\omega}\left[\dfrac{-\pi\omega^2 e^{-j\omega/2}}{\pi^2+(\omega/2)^2}\right]}$

Please show your work step by step.

(4) (Total: 25 points) **Inverse Fourier transforms.** Using tables and properties, find the inverse Fourier transform of

(a) (12.5 points) $X(\omega)=\dfrac{2j\omega\cos(3\omega)}{(2+j\omega)(3+j\omega)}$

Using PFE: $\dfrac{2}{(2+j\omega)(3+j\omega)} = \dfrac{2}{2+j\omega} - \dfrac{2}{3+j\omega} \leftrightarrow 2e^{-2t}u(t) - 2e^{-3t}u(t)$

Since $\cos(3\omega) = \dfrac{e^{j3\omega}+e^{-j3\omega}}{2}$ (Euler's formula)

Time-delay property $\rightarrow$ $\dfrac{2\cos(3\omega)}{(2+j\omega)(3+j\omega)} \leftrightarrow e^{-2(t+3)}u(t+3) - e^{-3(t+3)}u(t+3) + e^{-2(t-3)}u(t-3) - e^{-3(t-3)}u(t-3)$

Time-derivative property $\rightarrow$ $\dfrac{2j\omega\cos(3\omega)}{(2+j\omega)(3+j\omega)} \leftrightarrow \boxed{\dfrac{d}{dt}\left[e^{-2(t+3)}u(t+3) - e^{-3(t+3)}u(t+3) + e^{-2(t-3)}u(t-3) - e^{-3(t-3)}u(t-3)\right]}$

(b) (12.5 points) $X(\omega)=\dfrac{d}{d\omega}\left[\dfrac{3\sin(2\omega)\sin(4\omega)}{\omega}\right]$

$\dfrac{e^{j2\omega}-e^{-j2\omega}}{2j}$

$\dfrac{3\sin(4\omega)}{\omega} \leftrightarrow \dfrac{3}{2}u(t+4) - \dfrac{3}{2}u(t-4)$ $\leftarrow$ Fourier transform pair

Time-delay property $\rightarrow$ $\dfrac{3\sin(2\omega)\sin(4\omega)}{\omega} \leftrightarrow \dfrac{3}{4j}u(t+6) - \dfrac{3}{4j}u(t-2) - \dfrac{3}{4j}u(t+2) + \dfrac{3}{4j}u(t-6)$

Multiplication by t $\rightarrow$ $\dfrac{d}{d\omega}\left[\dfrac{3\sin(2\omega)\sin(4\omega)}{\omega}\right] \leftrightarrow \boxed{-\dfrac{3t}{4}u(t+6) + \dfrac{3t}{4}u(t-2) + \dfrac{3t}{4}u(t+2) - \dfrac{3t}{4}u(t-6)}$

Please provide your work step by step.