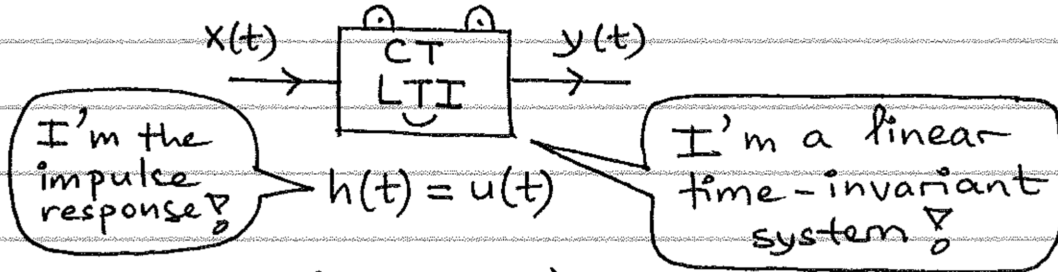
EE 262
SPRING 2012
A. INAN

SOLUTIONS TO HOMEWORK #3

3.1



a. $x(t) = 3\delta(t-2) - 3\delta(t-6)$

$$\boxed{y(t) = 3u(t-2) - 3u(t-6)}$$

b. $$x(t) = u(t) \to y(t) = y_s(t) = \int_{-\infty}^{t} h(t')\,dt' = \begin{cases} 0, & t \leq 0 \\ t, & t \geq 0 \end{cases}$$

$$= t\,u(t) = \boxed{r(t)}$$

c. $x(t) = t\,u(t) = r(t)$

$$\to y(t) = y_r(t) = \int_{-\infty}^{t} y_s(t')\,dt' = \begin{cases} 0, & t \leq 0 \\ \frac{t^2}{2}, & t \geq 0 \end{cases} = \boxed{\frac{t^2}{2}u(t)}$$

d. $x(t) = 4\left[u(t) - u(t-5)\right]$

$$y(t) = 4\left[y_s(t) - y_s(t-5)\right]$$

$$= \boxed{4\left[r(t) - r(t-5)\right]}$$

3.6

$$x(t) \longrightarrow \boxed{\text{CT LTI}} \longrightarrow y(t)$$

I'm the impulse response!

$$h(t) = u(t) - u(t-2)$$

I'm a linear time-invariant system!

$$x(t) = r(t) - 2r(t-1) + r(t-2).$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$y(t) = y_r(t) - 2y_r(t-1) + y_r(t-2).$$

Note that $y_r(t) = \int_{-\infty}^{t} y_s(t')\,dt'$. So, first we need to find the unit-step response $y_s(t)$.

I'm the unit-step response!

$$y_s(t) = \int_{-\infty}^{t} h(t')\,dt' = \begin{cases} 0, & t \le 0 \\ t, & 0 \le t \le 2 \\ 2, & t \ge 2 \end{cases}$$

$$= t\,u(t) - (t-2)\,u(t-2)$$

Next, we find $y_r(t)$.

$$y_r(t) = \int_{-\infty}^{t} y_s(t')\,dt' = \begin{cases} 0, & t \le 0 \\ t^2/2, & 0 \le t \le 2 \\ \underbrace{2 + 2(t-2)}_{2t-2}, & t \ge 2 \end{cases}$$

I'm the unit-ramp response!

$$= \frac{t^2}{2}u(t) - \frac{(t-2)^2}{2}u(t-2)$$

$$\boxed{\begin{aligned} \therefore\ y(t) = {} & \frac{t^2}{2}u(t) - \frac{(t-2)^2}{2}u(t-2) - \frac{\cancel{2}(t-1)^2}{\cancel{2}}u(t-1) \\ & + \frac{\cancel{2}(t-3)^2}{\cancel{2}}u(t-3) + \frac{(t-2)^2}{2}u(t-2) - \frac{(t-4)^2}{2}u(t-4) \end{aligned}}$$

3.8

$$x(t) \longrightarrow \boxed{\begin{matrix} \text{CT} \\ \text{LTI} \end{matrix}} \longrightarrow y(t)$$

I'm a linear time-invariant system!

I'm the impulse response! $h(t) = 5u(t-1)$

$$x(t) = u(t) - u(t-2) \longrightarrow y(t) = ?$$

$$y(t) = y_s(t) - y_s(t-2).$$

$$y_s(t) = \int_{-\infty}^{t} h(t')\,dt' = \begin{cases} 0, & t \le 1 \\ 5(t-1), & t \ge 1 \end{cases}$$

$$= 5(t-1)\,u(t-1)$$

$$\boxed{\therefore\ y(t) = 5(t-1)\,u(t-1) - 5(t-3)\,u(t-3)}$$

3.14

$$x(t) \longrightarrow \boxed{\begin{matrix} \text{CT} \\ \text{LTI} \end{matrix}} \longrightarrow y(t)$$

Me too is a linear time-invariant system!

I'm the unit-step responce! $y_s(t) = 2u(t) - e^{-5t}u(t)$

I'm the impulse response $h(t) = \dfrac{dy_s(t)}{dt} = 2\delta(t) + 5e^{-5t}u(t) - \underbrace{e^{-5t}}_{1}\delta(t)$

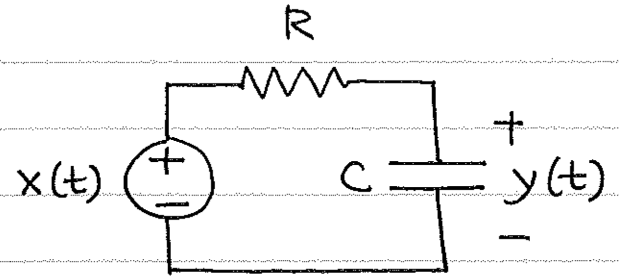
$$= \boxed{\delta(t) + 5e^{-5t}u(t)}$$

If $x(t) = 3\left[u(t-2) - u(t-6)\right]$, then:

$$y(t) = 3\left[y_s(t-2) - y_s(t-6)\right]$$

$$= \boxed{3\left[2u(t-2) - e^{-5(t-2)}u(t-2) - 2u(t-6) + e^{-5(t-6)}u(t-6)\right]}$$

5. a.



I'm the unit-step response!

If $x(t) = u(t) \rightarrow y(t) = y_s(t) = \begin{cases} 0, & t \leq 0 \\ 1 - e^{-t/(RC)}, & t \geq 0 \end{cases}$

$$= \boxed{\left(1 - e^{-t/(RC)}\right) u(t)}$$

If $x(t) = \delta(t) \rightarrow y(t) = h(t) = \dfrac{dy_s(t)}{dt}$

I'm the impulse response!

$$= \frac{1}{RC} e^{-t/(RC)} u(t) + \underbrace{\left(1 - e^{-t/(RC)}\right)\delta(t)}_{0}$$

$$= \boxed{\frac{1}{RC} e^{-t/(RC)} u(t)}$$

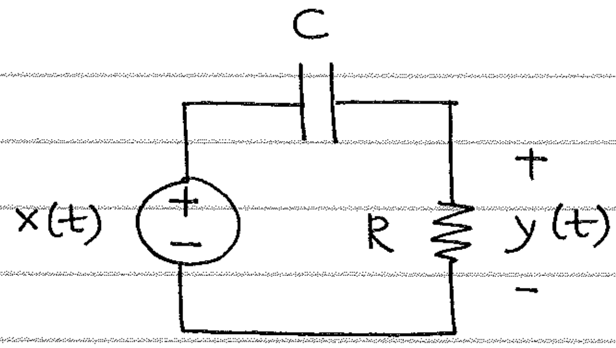
I'm the unit-ramp response!

If $x(t) = r(t) \rightarrow y(t) = y_r(t) = \displaystyle\int_{-\infty}^{t} y_s(t')\,dt'$

$$\rightarrow y_r(t) = \begin{cases} 0, & t \leq 0 \\ \underbrace{\int_0^t \left[1 - e^{-t'/(RC)}\right] dt'}_{\left[t' + RCe^{-t'/(RC)}\right]_0^t}, & t \geq 0 \end{cases}$$

$$= \boxed{\left[t - RC\left(1 - e^{-t/(RC)}\right)\right] u(t)}$$

5. b.



I'm the unit-step response ∇

$$\text{If } x(t) = u(t) \rightarrow y(t) = y_s(t) = \begin{cases} 0, & t < 0 \\ e^{-t/(RC)}, & t > 0 \end{cases}$$

$$= \boxed{e^{-t/(RC)}\,u(t)}$$

$$\text{If } x(t) = \delta(t) \rightarrow y(t) = h(t) = \frac{dy_s(t)}{dt}$$

I'm the impulse response ∇

$$= -\frac{1}{RC}e^{-t/(RC)}u(t) + \underbrace{e^{-t/(RC)}\delta(t)}_{1}$$

$$= \boxed{\delta(t) - \frac{1}{RC}e^{-t/(RC)}u(t)}$$

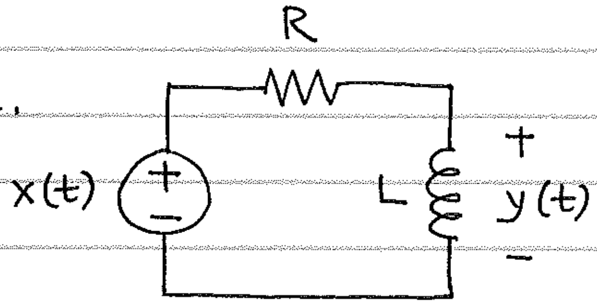
$$\text{If } x(t) = r(t) \rightarrow y(t) = y_r(t) = \int_{-\infty}^{t} y_s(t')\,dt'$$

I'm the unit-ramp response ∇

$$\rightarrow y_r(t) = \begin{cases} 0, & t \leq 0 \\ \underbrace{\int_0^t e^{-t'/(RC)}dt'}_{RC(1-e^{-t/(RC)})}, & t \geq 0 \end{cases}$$

$$= \boxed{RC(1-e^{-t/(RC)})\,u(t)}$$

5.c.



I'm the unit-step response!

$$\text{If } x(t) = u(t) \rightarrow y(t) = y_s(t) = \begin{cases} 0, & t < 0 \\ e^{-Rt/L}, & t > 0 \end{cases}$$

$$= \boxed{e^{-Rt/L}\, u(t)}$$

$$\text{If } x(t) = \delta(t) \rightarrow y(t) = h(t) = \frac{dy_s(t)}{dt}$$

I'm the impulse response!

$$= -\frac{R}{L} e^{-Rt/L} u(t) + \underbrace{e^{-Rt/L}}_{1} \delta(t)$$

$$= \boxed{\delta(t) - \frac{R}{L} e^{-Rt/L} u(t)}$$

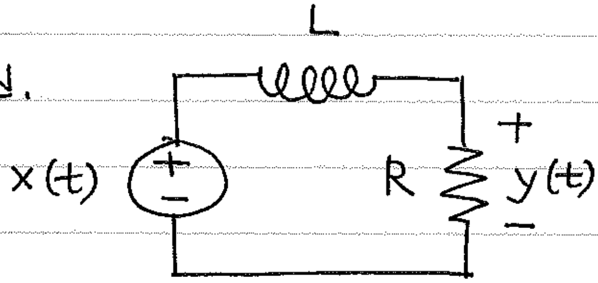
$$\text{If } x(t) = r(t) \longrightarrow y(t) = y_r(t) = \int_{-\infty}^{t} y_s(t')\,dt'$$

I'm the unit-ramp response!

$$\rightarrow y_r(t) = \begin{cases} 0, & t \leq 0 \\ \underbrace{\int_0^t e^{-Rt'/L}\,dt'}_{\frac{L}{R}\left(1 - e^{-Rt/L}\right)}, & t \geq 0 \end{cases}$$

$$= \boxed{\frac{L}{R}\left(1 - e^{-Rt/L}\right) u(t)}$$

5. d.



I'm the unit-step response!

If $x(t) = u(t) \rightarrow y(t) = y_s(t) = \begin{cases} 0, & t \leq 0 \\ 1 - e^{-Rt/L}, & t \geq 0 \end{cases}$

$$= \boxed{\left(1 - e^{-Rt/L}\right) u(t)}$$

If $x(t) = \delta(t) \rightarrow y(t) = h(t) = \dfrac{dy_s(t)}{dt}$

Me the impulse response!

$$= \frac{R}{L} e^{-Rt/L} u(t) + \underbrace{\left(1 - e^{-Rt/L}\right)}_{0} \delta(t)$$

$$= \boxed{\frac{R}{L} e^{-Rt/L} u(t)}$$

If $x(t) = r(t) \rightarrow y(t) = y_r(t) = \displaystyle\int_{-\infty}^{t} y_s(t')\, dt'$

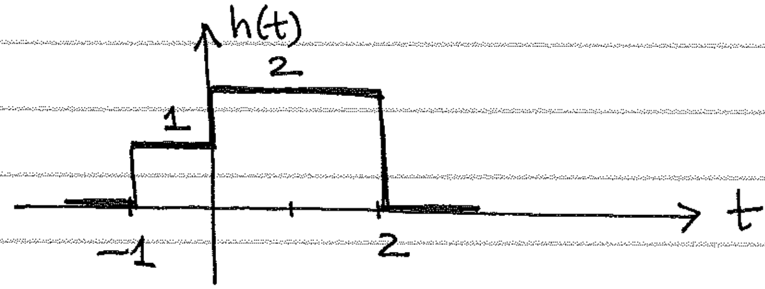
And I'm the unit-ramp responce!

$$\rightarrow y_r(t) = \begin{cases} 0, & t \leq 0 \\ \underbrace{\int_0^t \left[1 - e^{-Rt'/L}\right] dt'}_{\left[t' + \frac{L}{R} e^{-Rt'/L}\right]_0^t}, & t \geq 0 \end{cases}$$

$$= \boxed{\left[t - \frac{L}{R}\left(1 - e^{-Rt/L}\right)\right] u(t)}$$

I'm the impulse response of an LTI system!

6\. a. $h(t) = u(t+1) - 2u(t-2) + u(t)$



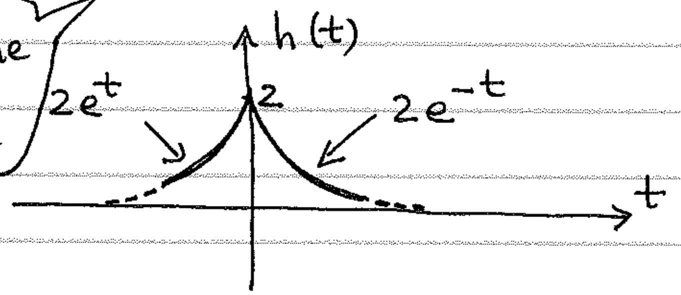
(i) Since $h(t) \neq K\delta(t) \rightarrow$ $\boxed{\text{Not memoryless!}}$

(ii) Since $h(t) \neq 0$ for $t<0 \rightarrow$ $\boxed{\text{Not causal!}}$

(iii) Since $\int_{-\infty}^{\infty} |h(t)|\,dt = 1+4 = 5 < \infty \rightarrow$ $\boxed{\text{BIBO stable!}}$

6\. b. $h(t) = 2e^{-|t|}$

I'm also the impulse response of an LTI system!



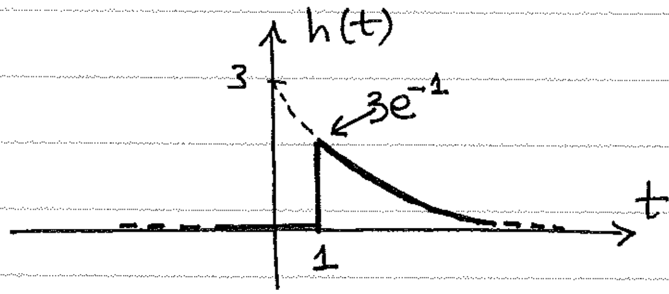
(i) Since $h(t) \neq K\delta(t) \rightarrow$ $\boxed{\text{Not memoryless!}}$

(ii) Since $h(t) \neq 0$ for $t<0 \rightarrow$ $\boxed{\text{Not causal!}}$

(iii) Since $\int_{-\infty}^{\infty} |h(t)|\,dt = 2\int_{0}^{\infty} 2e^{-t}\,dt = 4 < \infty \rightarrow$ $\boxed{\text{BIBO stable!}}$

I'm the impulse response of an LTI system!

6.c. $h(t) = 3e^{-t}u(t-1)$



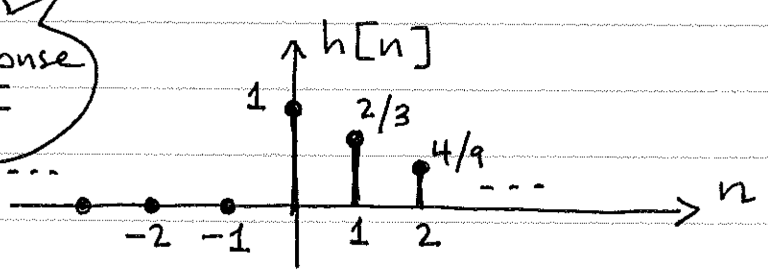
(i) Since $h(t) \neq K\delta(t) \rightarrow$ $\boxed{\text{Not memoryless!}}$

(ii) Since $h(t) = 0$ for $t < 0 \rightarrow$ $\boxed{\text{Causal!}}$

(iii) Since $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{1}^{\infty} 3e^{-t}\,dt = 3/e < \infty \rightarrow$ $\boxed{\text{BIBO stable!}}$

6.d. $h[n] = \left(\frac{2}{3}\right)^n u[n]$

I'm the impulse response of an LTI system!



(i) Since $h[n] \neq K\delta[n] \rightarrow$ $\boxed{\text{Not memoryless!}}$

(ii) Since $h[n] = 0$ for $n < 0 \rightarrow$ $\boxed{\text{Causal!}}$

(iii) Since $\sum_{k=-\infty}^{\infty} |h[k]| = 1 + \frac{2}{3} + \frac{4}{9} + \dots = \frac{1}{1-\frac{2}{3}} = 3 < \infty$

$\downarrow$

$\boxed{\text{BIBO stable!}}$