EE 262
SPRING 2012
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SOLUTIONS TO HOMEWORK #4

(#1) (a)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\alpha)\, h(\alpha)\, d\alpha$$

$$= \int_{-\infty}^{\infty} e^{-a(t-\alpha)} u(t-\alpha)\, e^{-b\alpha} u(\alpha)\, d\alpha$$

I'm zero for $\alpha<0$ & 1 for $\alpha>0$!

$$= \int_{0}^{\infty} e^{-a(t-\alpha)} u(t-\alpha)\, e^{-b\alpha}\, d\alpha$$

I'm zero for $\alpha > t$ & 1 for $\alpha < t$!

$$= \int_{0}^{t} e^{-a(t-\alpha)} e^{-b\alpha}\, d\alpha$$

$$= e^{-at} \int_{0}^{t} e^{(a-b)\alpha}\, d\alpha$$

$$= e^{-at} \left[\frac{e^{(a-b)\alpha}}{(a-b)} \right]_0^t$$

$$= \frac{e^{-at}}{(a-b)} \left[e^{(a-b)t} - 1 \right] = \frac{e^{-bt} - e^{-at}}{(a-b)}$$

$$\boxed{\therefore\ y(t) = \frac{e^{-bt} - e^{-at}}{(a-b)}\, u(t)}$$

(Note that one can also evaluate this convolution integral using the graphical approach.)

(b) Using the result of part (a) with $b=0$:

$$y(t) = e^{-at}u(t) * u(t) = \left[\frac{e^{-bt} - e^{-at}}{(a-b)} u(t)\right]_{b=0}$$

$$= \boxed{\frac{1-e^{-at}}{a} u(t)}$$

(c) $y(t) = e^{-at}u(t) * \delta(t-b) = \boxed{e^{-a(t-b)} u(t-b)}$

(d) $y(t) = e^{-2t}u(t) * [3u(t-1) - 2\delta(t-3)]$

$$= e^{-2t}u(t) * 3u(t-1) - e^{-2t}u(t) * 2\delta(t-3)$$

$$= \boxed{\underbrace{\frac{3}{2}\left[1-e^{-2(t-1)}\right]u(t-1)}_{\text{Using the result of part (b) along with time-delay property}} - 2e^{-2(t-3)}u(t-3)}$$

(e) $y(t) = e^{-2t}u(t-1) * 3u(t-4)$

$$= \frac{1}{e^2} e^{-2(t-1)} u(t-1) * 3u(t-4)$$

$$= \boxed{\underbrace{\frac{3}{e^2}\left[\frac{1-e^{-2(t-5)}}{2}\right]u(t-5)}_{\text{Using the result of part (b) \& time-delay property of convolution}}}$$

(#2) (a) $h(t) = \dfrac{dy_s(t)}{dt} = 2e^{-t}u(t) + \underbrace{(3-2e^{-t})}_{1}\delta(t)$

$$= \boxed{2e^{-t}u(t) + \delta(t)}$$

(b) $y_r(t) = \int_{-\infty}^{t} y_s(\tau)\,d\tau$

$$= \begin{cases} 0, & t<0 \\ \int_0^t (3-2e^{-\tau})\,d\tau, & t>0 \end{cases}$$

$$= \begin{cases} 0, & t<0 \\ \left[3\tau+2e^{-\tau}\right]_0^t, & t>0 \end{cases}$$

$$= \boxed{\left[3t+2(e^{-t}-1)\right]u(t)}$$

(c) $y(t) = 2\left[y_s(t-1) - y_s(t-3)\right]$

$$= \boxed{2\left[(3-2e^{-(t-1)})u(t-1) - (3-2e^{-(t-3)})u(t-3)\right]}$$

(#3) (a) $y(t) = u(t) * r(t)$

I'm zero for $\alpha<0$ & α for $\alpha>0$!

$$= \int_{-\infty}^{\infty} u(t-\alpha)\,r(\alpha)\,d\alpha$$

I'm zero for $\alpha > t$ & 1 for $\alpha < t$!

$$= \int_0^{\infty} u(t-\alpha)\,\alpha\,d\alpha$$

Since $\alpha>0$, then, $t>0$ for the case when $t>\alpha$!

$$= \int_0^{t>0} \alpha \, d\alpha = \frac{\alpha^2}{2}\Big|_0^t = \frac{t^2}{2} \quad \text{for } t>0$$

$$\therefore\ y(t) = u(t) * r(t) = \frac{t^2}{2} u(t)$$

(b) $y(t) = u(t-a) * r(t-b)$ (Using time-delay property of convolution)

$$= \boxed{\frac{(t-a-b)^2}{2} u(t-a-b)}$$

(c) $y(t) = [2r(t-1) + u(t-3)] * [u(t) - 3\delta(t-2)]$

$$= \frac{\cancel{2}(t-1)^2}{\cancel{2}} u(t-1) - 6r(t-3)$$

$$+ r(t-3) - 3u(t-5)$$

$$= \boxed{(t-1)^2 u(t-1) - 6r(t-3) - 3u(t-5)}$$

(#4) $y(t) = x(t) * h(t)$

$$= t\,[u(t-1) - u(t-3)] * 2\,[u(t) - u(t-2)]$$

$$= [(t-1)u(t-1) - (t-3)u(t-3) + u(t-1) - 3u(t-3)]$$
$$* 2\,[u(t) - u(t-2)]$$

$$= [r(t-1) - r(t-3) + u(t-1) - 3u(t-3)]$$
$$* 2\,[u(t) - u(t-2)]$$

$$= \cancel{2}\frac{(t-1)^2}{\cancel{2}}u(t-1) - \cancel{2}\frac{(t-3)^2}{\cancel{2}}u(t-3)$$

$$- \cancel{2}\frac{(t-3)^2}{\cancel{2}}u(t-3) + \cancel{2}\frac{(t-5)^2}{\cancel{2}}u(t-5)$$

$$+ 2r(t-1) - 2r(t-3)$$

$$- 6r(t-3) + 6r(t-5)$$

$$= \boxed{(t-1)^2u(t-1) - 2(t-3)^2u(t-3) + (t-5)^2u(t-5) + 2r(t-1) - 8r(t-3) + 6r(t-5)}$$

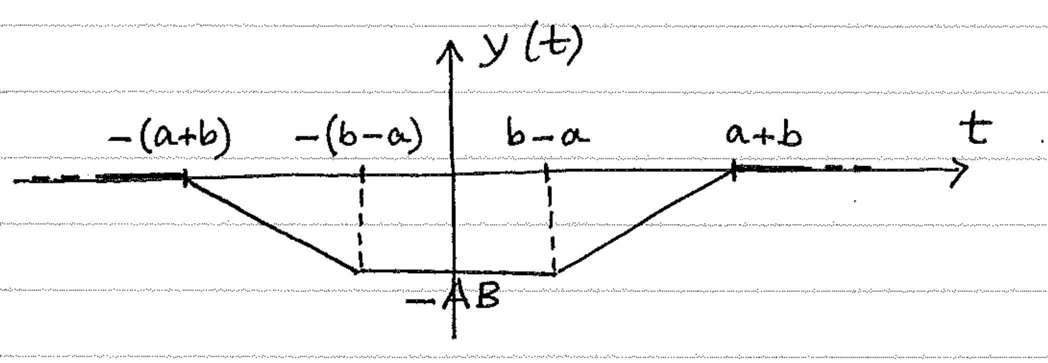
(#5) $y(t) = x(t) * h(t)$

$$= A\left[u(t-a) - u(t+a)\right] * B\left[u(t+b) - u(t-b)\right]$$

$$= ABr(t-a+b) - ABr(t-a-b)$$

$$- ABr(t+a+b) + ABr(t+a-b)$$

$$= \boxed{-ABr(t+a+b) + ABr(t+b-a) + ABr(t-b+a) - ABr(t-a-b)}$$



(#6) (a) $y[n] = x[n] * h[n]$

$$= \left(\delta[n-3] - 2\delta[n-1]\right) * \left(3u[n+1] - 2u[n-1] - u[n-3]\right)$$

$$= \underbrace{3u[n-2]} \cancel{-2u[n-4]} - u[n-6]$$

$$- 6u[n] + \underbrace{4u[n-2]} \cancel{+2u[n-4]}$$

$$= \boxed{-6u[n] + 7u[n-2] - u[n-6]}$$

(b) $y[n] = x[n] * h[n]$

$$= \left(2\delta[n+1] - 3\delta[n-2]\right) * \left(2u[n] - 5u[n-2] + 3u[n-4]\right)$$

$$= \boxed{\begin{array}{l} 4u[n+1] - 10u[n-1] + 6u[n-3] \\ - 6u[n-2] + 15u[n-4] - 9u[n-6] \end{array}}$$

(#7) $y[n] = h[n] * x[n]$

$$= \left(\delta[n+1] - 3\delta[n-2]\right) * n\left(u[n] - u[n-4]\right)$$

$$= \boxed{\begin{array}{l} (n+1)u[n+1] - (n+1)u[n-3] \\ -3(n-2)u[n-2] + 3(n-2)u[n-6] \end{array}}$$

FINITO ☺