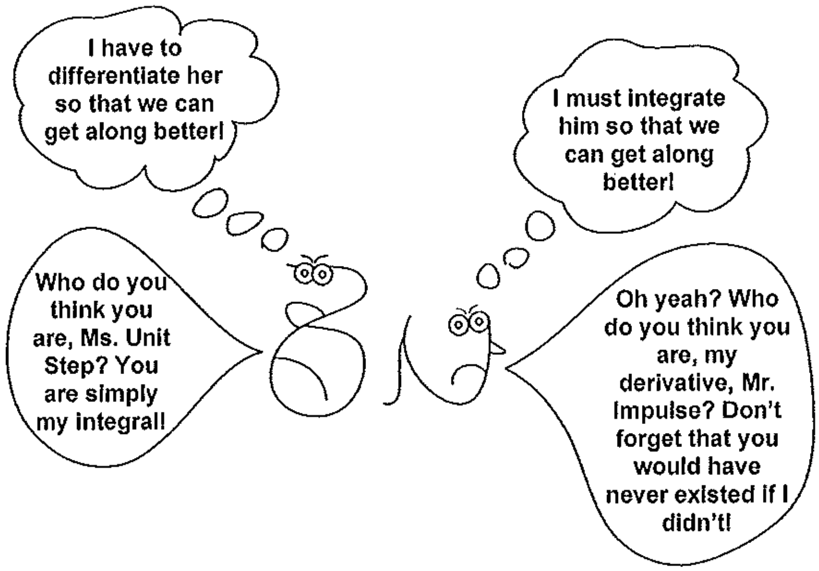


University ☺f P☺rtland

Sch☺☺l ☺f Engineering

EE 262-δignals & δystems-3 cr. hrs.
Spring 2012

Midterm Exam # 1

(Prepared by Professor A. S. Inan)

(Friday, February 17, 2012)

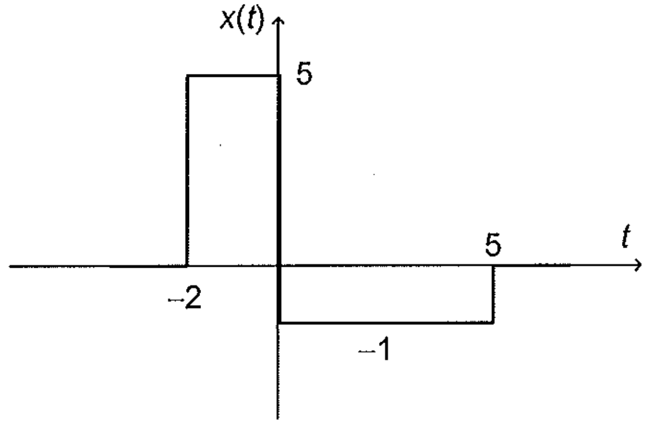
(Closed Book Exam, One formula sheet allowed.)

(Total Time: 55 mins.)

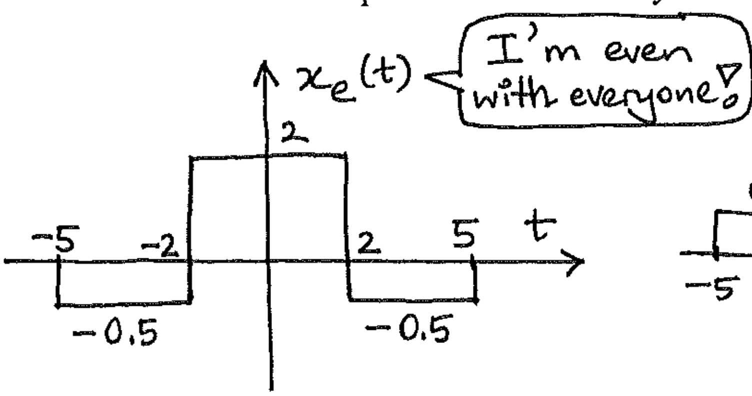
Name: SOLUTIONS ☺

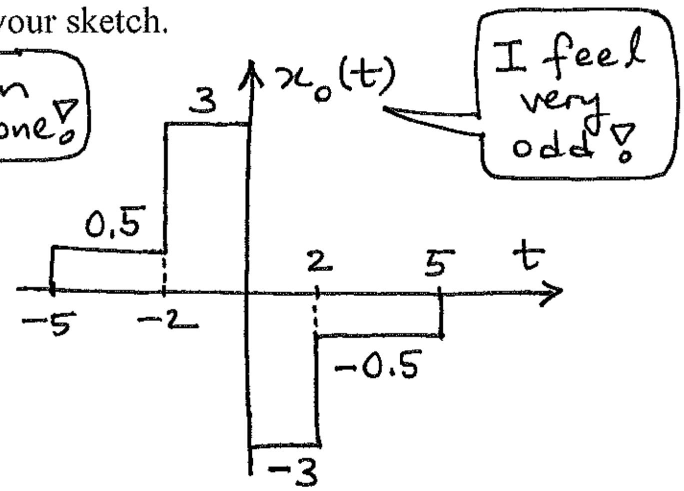
Signature: Solutions ☺

(1) (15 mins., <u>Total:</u> 25 points) **A continuous-time signal.** Consider a continuous-time signal denoted by $x(t)$ as shown in the figure below.



(a) (12.5 points) <u>Sketch</u> the even and odd parts of $x(t)$. Provide all the pertinent values on your sketch.

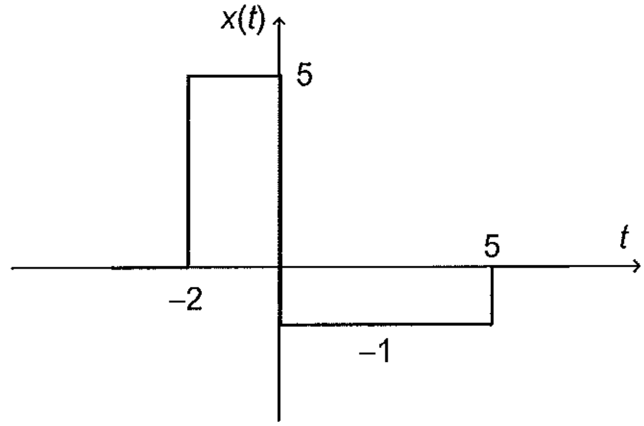




$$x_e(t) = -\frac{1}{2}u(t+5) + \frac{5}{2}u(t+2) - \frac{5}{2}u(t-2) + \frac{1}{2}u(t-5)$$

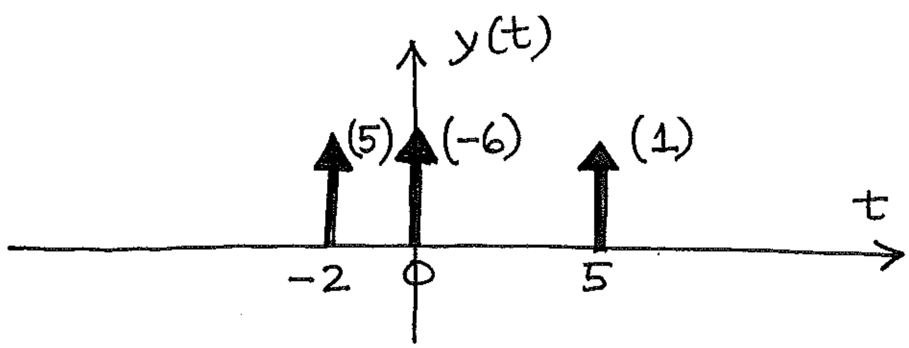
$$x_o(t) = \frac{1}{2}u(t+5) + \frac{5}{2}u(t+2) - 6u(t) + \frac{5}{2}u(t-2) + \frac{1}{2}u(t-5)$$

(b)(12.5 points) Find the complete mathematical expression for the function $y(t)=dx(t)/dt$ and sketch $y(t)$ versus t. Provide all the pertinent values on your sketch.



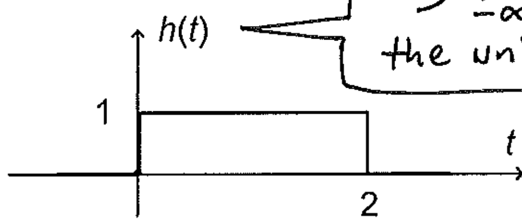
$$x(t) = 5u(t+2) - 6u(t) + u(t-5)$$

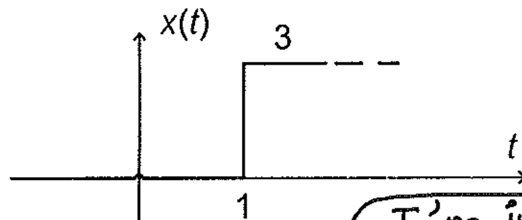
$$y(t) = \frac{dx(t)}{dt} = \boxed{5\delta(t+2) - 6\delta(t) + \delta(t-5)}$$

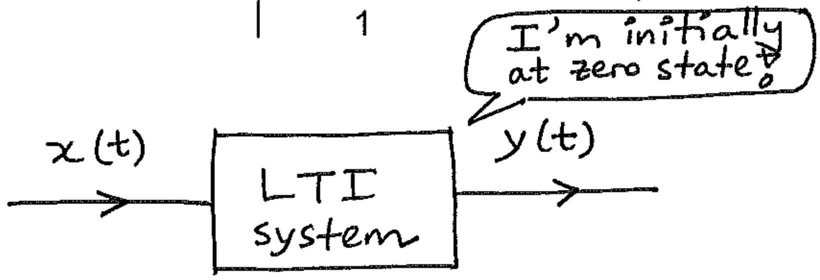


(2) (15 mins., 25 points) **LTI system.** The impulse response $h(t)$ of an LTI system is given as sketched below. Find the complete mathematical expression and sketch the output response $y(t)$ of this system due to the input signal $x(t)$.

My $\int_{-\infty}^{t}$ integral yields the unit-step response $y_s(t)$!





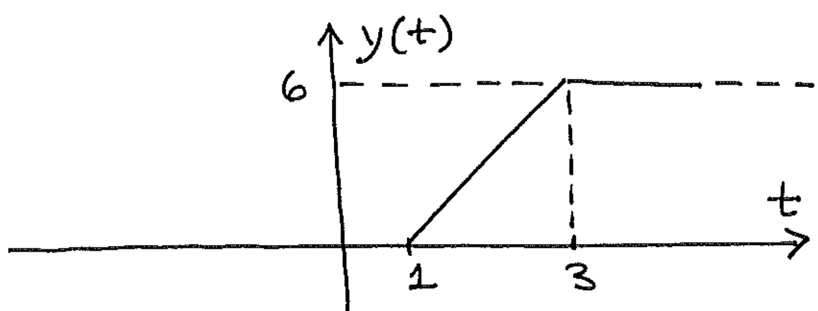


For $x(t) = 3u(t-1) \rightarrow y(t) = 3y_s(t-1)$

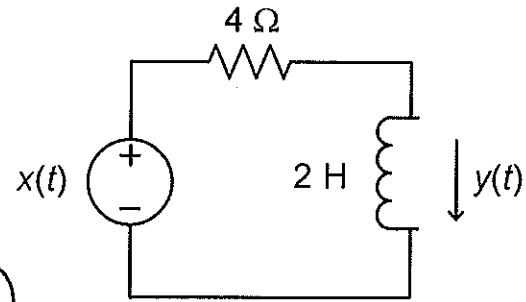
$$y_s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau = \begin{cases} 0, & t \leq 0 \\ t, & 0 \leq t \leq 2 \\ 2, & t \geq 2 \end{cases}$$

$$= r(t) - r(t-2)$$

$$\therefore\ y(t) = 3y_s(t-1) = \boxed{3r(t-1) - 3r(t-3)}$$



(3)(10 mins., 25 points) **First-order electric circuit.** Find and sketch the unit-step and impulse responses of the first-order electric circuit shown.

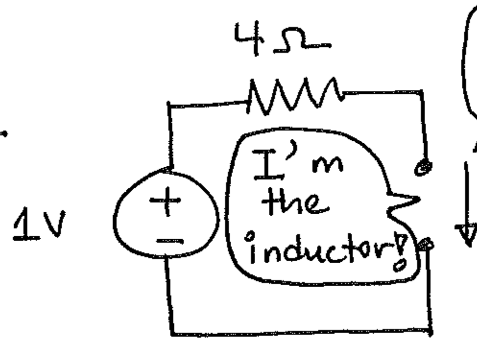


$$\tau = \frac{L}{R} = \frac{2}{4} = \frac{1}{2}\,s$$

I'm the unit-step response!

$$y_s(t) = \left[y_s(0^+)e^{-t/\tau} + y_s(\infty)(1-e^{-t/\tau})\right]u(t)$$

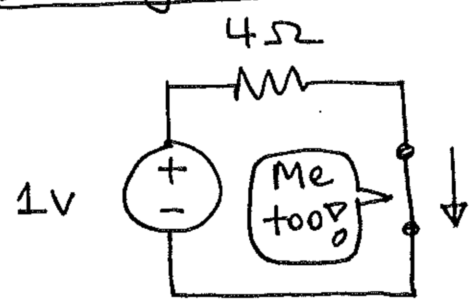
At $t=0^+$:



I don't let my current to jump!

$y_s(0^+) = 0$

At $t=\infty$ (steady state):



$y_s(\infty) = \frac{1}{4}$ A

$$\therefore\ y_s(t) = \frac{1}{4}(1-e^{-2t})u(t)$$

Apply the sampling property of $\delta(t)$!

$$\therefore\ h(t) = \frac{dy_s(t)}{dt} = \frac{1}{2}e^{-2t}u(t) + \frac{1}{4}\underbrace{(1-\cancel{e^{-2t}}^{\,1})}_{=0}\delta(t)$$

I'm the impulse response!

$$= \boxed{\frac{1}{2}e^{-2t}u(t)}$$

(4)(15 mins., Total: 25 points) **Properties of a discrete-time system.** The input $x[n]$ and the output $y[n]$ relationship of a discrete-time system is given by $y[n] = \sum_{k=-2}^{2} x[n-k]$.

Note that $y[n] = x[n+2] + x[n+1] + x[n] + x[n-1] + x[n-2]$

(a) (3 points) Is this system memory-less? (Provide a clearly stated justification for your answer.)

No since $y[n]$ does not only depend on $x[n]$.

(b) (3 points) Is this system causal? (Clear justification required!)

No since $y[n]$ also depends on future values of $x[n]$ (such as $x[n+1]$, $x[n+2]$).

(c) (3 points) Is this system BIBO stable? (Justification required!)

Yes. If $|x[n]|$ is bounded by M (that is, $|x[n]| \leq M$), then, $|y[n]|$ will be bounded by 5M.

(d) (3 points) Is this system invertible? (Justification required!)

Yes since the transformation from x to y values is one-to-one.

(e) (4 points) Is this system linear? (Justification required!)

$$y_1[n] = \sum_{-2}^{2} x_1[n-k]$$

$$y_2[n] = \sum_{-2}^{2} x_2[n-k]$$

$$y_3[n] = \sum_{-2}^{2} (a x_1[n-k] + b x_2[n-k])$$

$$= a\sum_{-2}^{2} x_1[n-k] + b\sum_{-2}^{2} x_2[n-k] = a y_1[n] + b y_2[n]$$

$\therefore$ Linear.

(f) (4 points) Is this system time invariant? (Justification required!)

$$x_1[n] \longrightarrow y_1[n] = \sum_{k=-2}^{2} x_1[n-k]$$

$$x_2[n] = x_1[n-n_0]$$

$$\rightarrow y_2[n] = \sum_{k=-2}^{2} x_2[n-k]$$

$$= \sum_{k=-2}^{2} x_1[n-n_0-k]$$

$$= y_1[n-n_0]$$

$\therefore$ Time - invariant.

——— FINITO ———