University of Portland
School of Engineering

EE 262-Signals & Systems-3 cr. hrs.
Spring 2012

Midterm Exam # 2
(Prepared by Professor A. S. Inan)

(Friday, March 30, 2012)

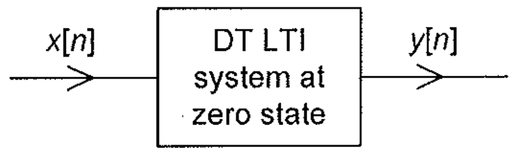
Name: SOLUTIONS ☺

Signature: SOLUTIONS ☺

> ***"Honesty is the best policy."***
> **Aesop (~ 620B.C. –?)**
>
> ***"An honest mind possesses a kingdom."***
> **Lucius Annaeus Seneca (4B.C.–65A.D.)**
>
> ***"Honest people are the true winners of the universe."***
> **Anonymous**

Select and do any 4 of the 5 problems assigned during class time. The fifth problem is a take-home problem due class time on Monday, April 3, 2012.

(1) (Total: 20 points). **DT LTI system.** The unit impulse of a DT LTI system is provided below as shown. An input signal $x[n] = 3\delta[n-1] - 2\delta[n-3]$ is applied to this system.

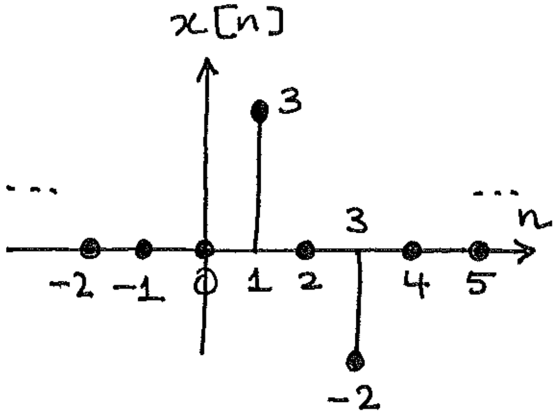


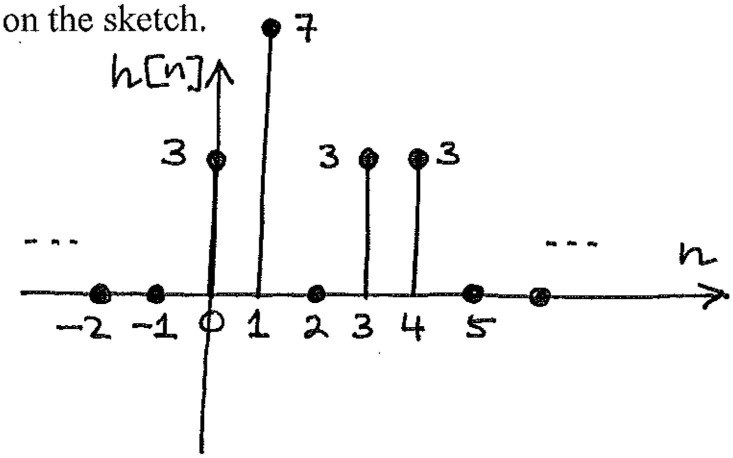
I can also be expressed in terms of impulses?

$h[n] = 3u[n] + 4\delta[n-1] - 3\delta[n-2] - 3u[n-5]$

$= 3\delta[n] + 7\delta[n-1] + 3\delta[n-3] + 3\delta[n-4]$

(a) (8 points) Sketch both signals $x[n]$ and $h[n]$. Provide all the pertinent values on the sketch.

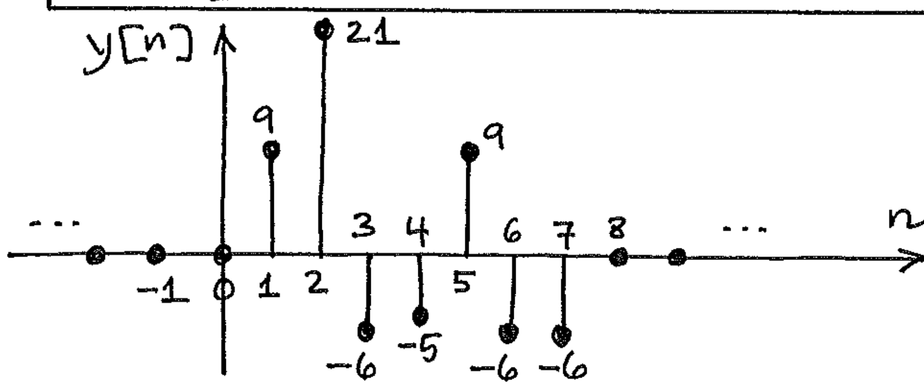




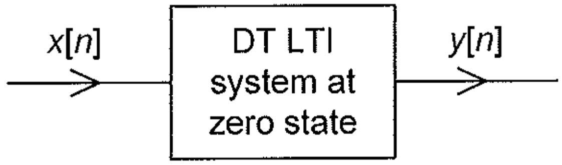
(b) (12 points) Find and sketch the zero-state response $y_{zs}[n]$ of this system.

$y[n] = x[n] * h[n]$

$$\boxed{\begin{aligned} &= 9u[n-1] + 12\delta[n-2] - 9\delta[n-3] - 9u[n-6] \\ &\quad - 6u[n-3] - 8\delta[n-4] + 6\delta[n-5] + 6u[n-8] \\ &= 9\delta[n-1] + 21\delta[n-2] - 6\delta[n-3] - 5\delta[n-4] \\ &\quad + 9\delta[n-5] - 6\delta[n-6] - 6\delta[n-7] \end{aligned}}$$



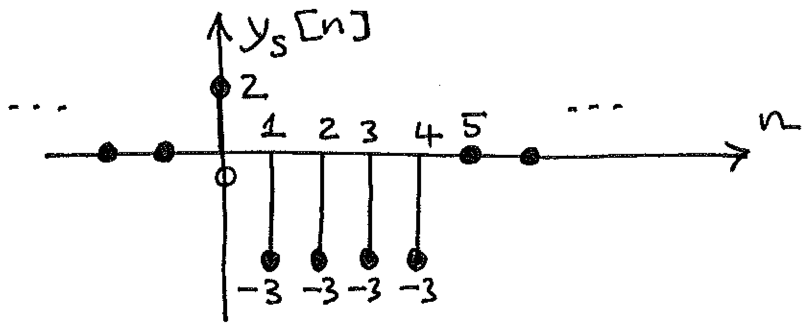
(2)(Total: 20 points) **LTI system.** Consider the following DT LTI system with its impulse response provided as shown.



$$h[n] = 2\delta[n] - 5\delta[n-1] + 3\delta[n-5]$$

(a)(10 points) Find and sketch the unit-step response of this system.

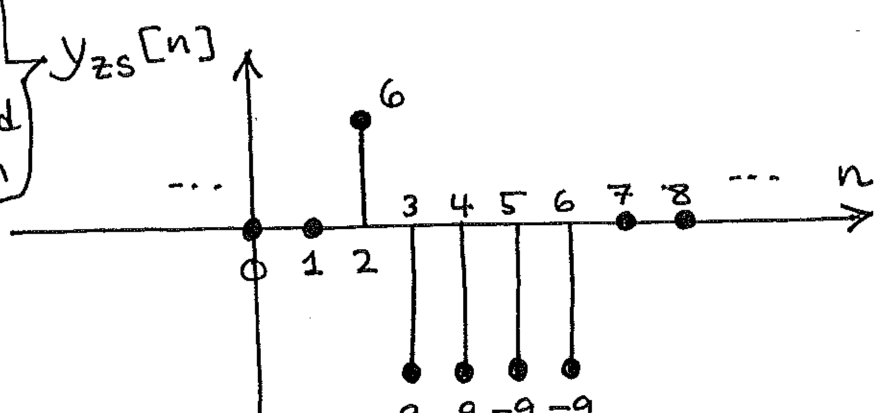
$$y_s[n] = u[n] * h[n]$$
$$= \boxed{2u[n] - 5u[n-1] + 3u[n-5]}$$



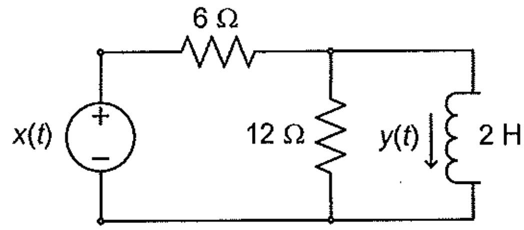
(b)(10 points) Find and sketch the zero-state response of this system due to $x[n] = 3u[n-2]$.

$$y_{zs}[n] = 3y_s[n-2]$$
$$= \boxed{6u[n-2] - 15u[n-3] + 9u[n-7]}$$

I'm shifted right by 2 and amplified by 3 version of $y_s[n]$!



(3)(Total: 20 points) **Electric circuits.** For the electric circuit shown:



$x(t) = u(t)$ and $y(0)$ is assumed to be zero.

I'm the general solution of a first-order circuit!

(a) (10 points) Find the unit-step response $y_s(t)$. Provide your work step by step. (Note that $y_s(t)$ represents the current of the inductor.)

$$y_s(t) = \cancel{y_s(0^+)}^{\,0} e^{-t/\tau} + y_s(\infty)\left(1 - e^{-t/\tau}\right)$$

Inam's students from his electric circuits class know very well when I apply!

$$y_s(0^+) = 0, \quad y_s(\infty) = \frac{1}{6} A \;\&\; \tau = \frac{L}{R_T} = \frac{2}{6 /\!/ 12} = \frac{2}{4} = \frac{1}{2}\,s$$

$$\boxed{\therefore\ y_s(t) = \frac{1}{6}\left(1 - e^{-2t}\right) u(t) \quad (A)}$$

I'm the unit-step response!

Simplify me using sampling property of $\delta(t)$!

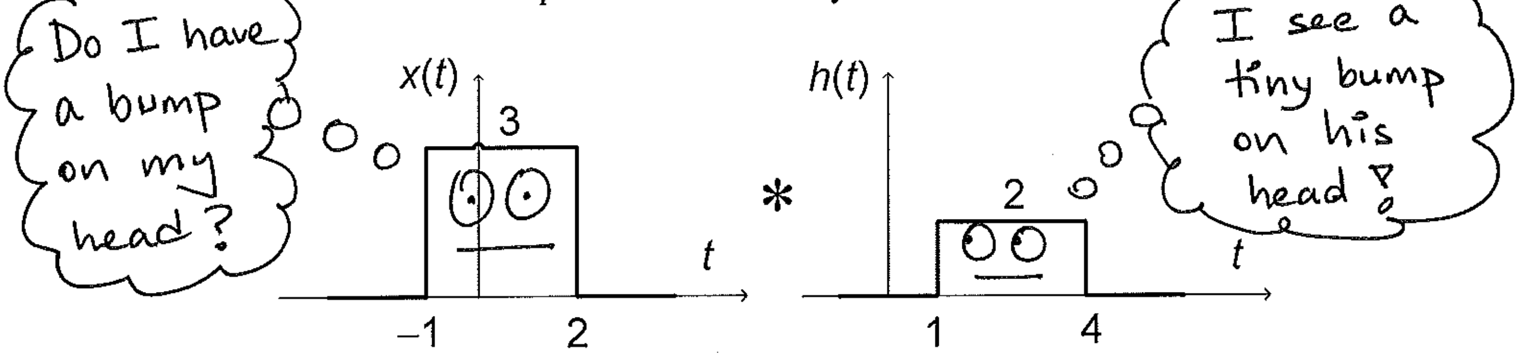
(b) (10 points) Find the impulse response $h(t)$. Show your work.

$$h(t) = \frac{dy_s(t)}{dt} = \frac{1}{3} e^{-2t} u(t) + \frac{1}{6} \underbrace{\left(1 - e^{-2t}\right)}_{0 \text{ at } t=0} \delta(t)$$

I'm the impulse response

$$= \boxed{\frac{1}{3} e^{-2t} u(t) \quad \left(\frac{A}{s}\right)}$$

(4)(20 points) **Convolution integral.** Find and sketch the function $y(t)$ obtained as a result of convolving the two signals shown below. Provide all the pertinent values on your sketch.



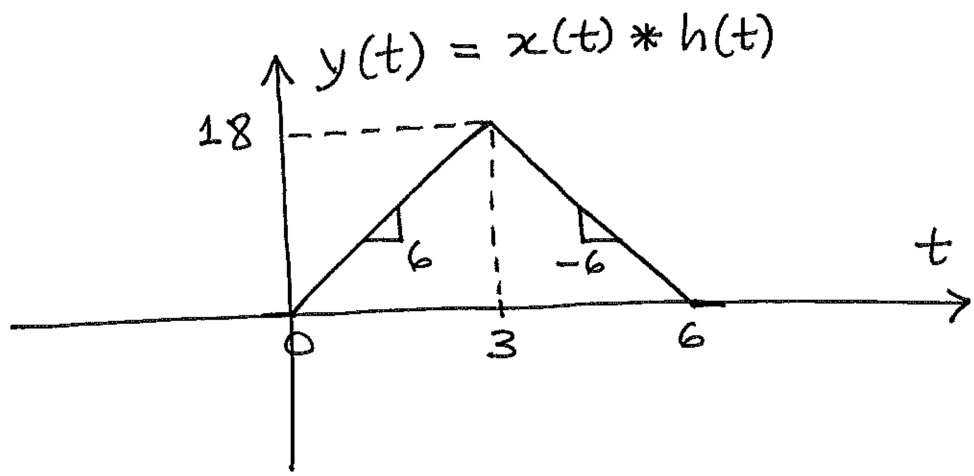
$$x(t) = 3u(t+1) - 3u(t-2)$$

$$h(t) = 2u(t-1) - 2u(t-4)$$

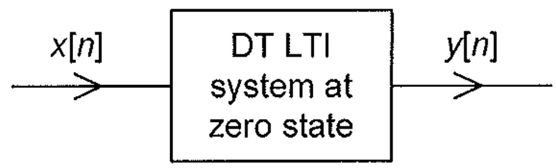
Using $u(t-a) * u(t-b) = r(t-a-b)$, we have

$$x(t) * h(t) = 6r(t) - 6r(t-3) - 6r(t-3) + 6r(t-6)$$

$$= \boxed{6r(t) - 12r(t-3) + 6r(t-6)}$$



(5) (Total: 20 points) **LTI system.** The difference equation governing an LTI system is provided as shown.



$$y[n] - 0.5y[n-1] + 3x[n] = 0$$

(a) (10) Find the impulse response $h[n]$ of this system in its most general form.

$$h[n] - 0.5h[n-1] + 3\delta[n] = 0$$

$$n=0 \rightarrow h[0] - 0.5\cancel{h[-1]}^{\,0} + 3 = 0 \rightarrow h[0] = -3$$

$$n=1 \rightarrow h[1] - 0.5h[0] + 0 = 0 \rightarrow h[1] = -(0.5)(3)$$

$$n=2 \rightarrow h[2] - 0.5h[1] + 0 = 0 \rightarrow h[2] = -(0.5)^2(3)$$

$$n=3 \rightarrow h[3] - 0.5h[2] + 0 = 0 \rightarrow h[3] = -(0.5)^3(3)$$

$$\vdots$$

I'm generalized!

$$\boxed{\therefore\ h[n] = -(0.5)^n(3)u[n]}$$

(b) (10) Find the value of the sample of the unit-step response of this system at $n = 3$. (That is, find $y_s[3]$.)

$$y_s[n] = \sum_{k=0}^{\infty} h[n-k]$$

$$y_s[3] = \sum_{k=0}^{\infty} h[3-k] = h[3] + h[2] + h[1] + h[0]$$

$$= (-3)\left[(0.5)^3 + (0.5)^2 + (0.5) + 1\right]$$

$$= (-3)\left[\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1\right] = (-3)\left(\frac{15}{8}\right) = \boxed{-\frac{45}{8}}$$

$$\cong \boxed{-5.625}$$