Univerδity ☺f P☺rtland
δch☺☺l ☺f Engineering

EE 262-δignalδ & δyδtemδ-3 cr. hrδ.
δpring 2012

Midterm Exam # 3
(Prepared by Profeδδor A. δ. Inan)

(Friday, April 20, 2012)

Name: SOLUTIONS ☺

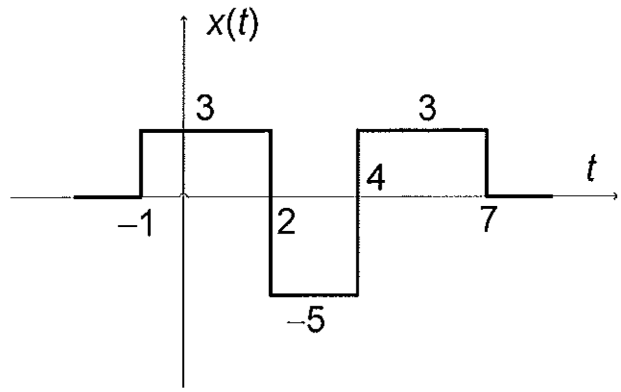
δignature: Solutions ☺

"Honesty is the best policy."
Aesop (~ 620B.C.–?)

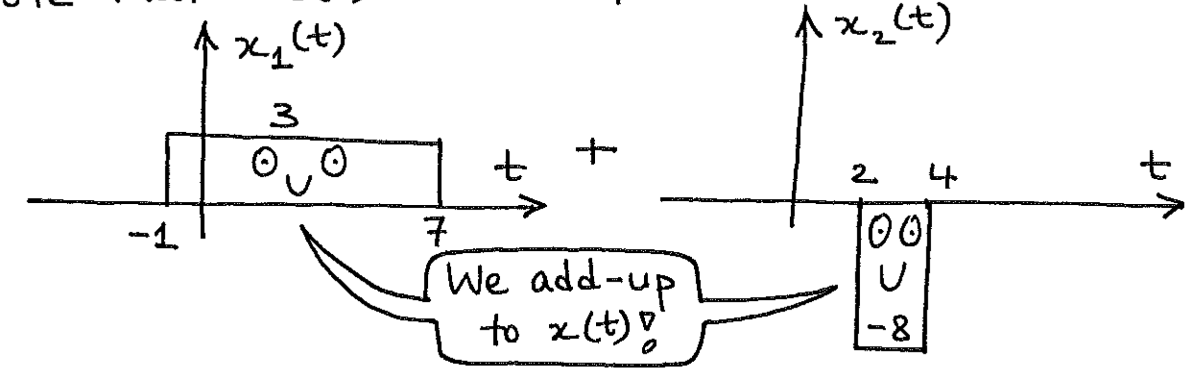
"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

"Honest people are the true winners of the universe."
Anonymous

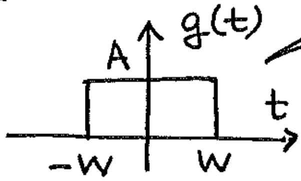
(1)(25 points) Find the Fourier transform of the signal shown below. (Please present your work step by step and simplify your expressions whenever possible.)



Note that $x(t)$ can be split as $x(t) = x_1(t) + x_2(t)$.



Using Fourier pair



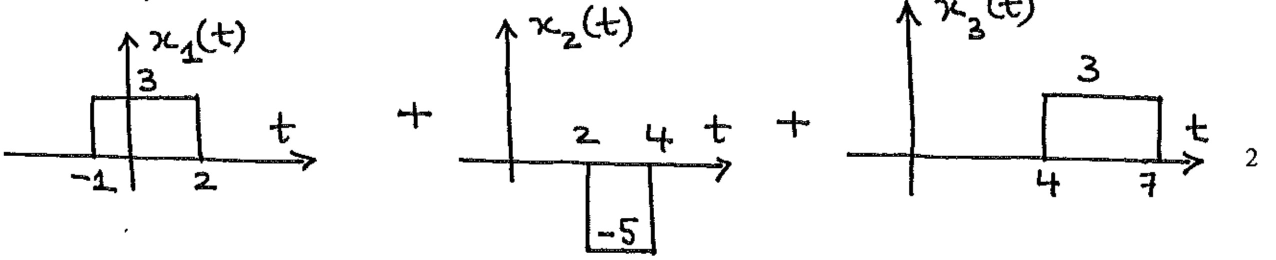
We are a well-known Fourier pair!

$$\longleftrightarrow \quad G(\omega) = \frac{2A\sin(W\omega)}{\omega}$$

along with linearity and time-shift properties of Fourier transform:

$$X(\omega) = X_1(\omega) + X_2(\omega) = \boxed{\frac{6\sin(4\omega)e^{-j3\omega}}{\omega} - \frac{16\sin(\omega)e^{-j3\omega}}{\omega}}$$

If $x(t)$ is split as $x(t) = x_1(t) + x_2(t) + x_3(t)$



$$X(\omega) = X_1(\omega) + X_2(\omega) + X_3(\omega) = \boxed{\frac{6\sin(3\omega/2)e^{-j\omega/2}}{\omega} - \frac{10\sin(\omega)e^{-j3\omega}}{\omega} + \frac{6\sin(3\omega/2)e^{-j11\omega/2}}{\omega}}$$

(2) (25 points). Find the Fourier transform (FT) of the signal given by

$$x(t) = \frac{d}{dt}\left((2t+3)e^{-2t}\cos(2t)u(t-1)\right)$$

Please provide all your steps!

$$e^{-2t}u(t) \longleftrightarrow 1/(2+\hat{j}\omega)$$

$$e^{-2}e^{-2(t-1)}u(t-1) \overset{\text{Time}}{\underset{\text{shift}}{\longleftrightarrow}} e^{-2}e^{-\hat{j}\omega}/(2+\hat{j}\omega)$$

$$\cos(2t)e^{-2}e^{-2(t-1)}u(t-1) \overset{\text{Freq.}}{\underset{\text{shift property}}{\longleftrightarrow}} \frac{1}{2e^{2}}\underbrace{\left[\frac{e^{-\hat{j}(\omega-2)}}{2+\hat{j}(\omega-2)} + \frac{e^{-\hat{j}(\omega+2)}}{2+\hat{j}(\omega+2)}\right]}_{G(\omega)}$$

I'm $(e^{\hat{j}2t}+e^{-\hat{j}2t})/2$ according to Euler!

$$(2t+3)\cos(2t)e^{-2}e^{-2(t-1)}u(t-1)$$

$$\overset{\text{Multiplication}}{\underset{\text{by } t \text{ property}}{\longleftrightarrow}} \frac{1}{e^{2}}\hat{j}\frac{dG(\omega)}{d\omega} + \frac{3}{2e^{2}}G(\omega)$$

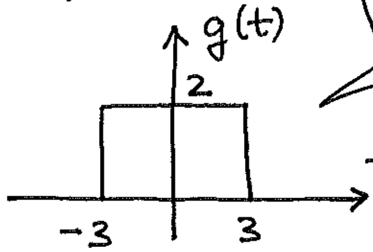
$$\frac{d}{dt}\left((2t+3)\cos(2t)e^{-2}e^{-2(t-1)}u(t-1)\right)$$

$$\overset{\text{Differentiation}}{\underset{\text{property}}{\longleftrightarrow}} \hat{j}\omega\left\{\frac{1}{e^{2}}\hat{j}\frac{dG(\omega)}{d\omega} + \frac{3}{2e^{2}}G(\omega)\right\}$$

$$\boxed{x(t) \longleftrightarrow -\frac{1}{e^{2}}\omega\frac{dG(\omega)}{d\omega} + \frac{\hat{j}3\omega}{2e^{2}} \quad \text{where } G(\omega) = \frac{e^{-\hat{j}(\omega-2)}}{2+\hat{j}(\omega-2)} + \frac{e^{-\hat{j}(\omega+2)}}{2+\hat{j}(\omega+2)}}$$

(3) (25 points) Find the inverse Fourier transform of $X(\omega) = \frac{4j\sin^2(3\omega)}{\omega}$ and sketch $x(t)$ as a function of time. Provide all the pertinent values on your sketch!

We are a well-known Fourier pair!



$$\longleftrightarrow \quad G(\omega) = \frac{4\sin(3\omega)}{\omega}$$

Thanks to Euler!

$$g(t) = 2u(t+3) - 2u(t-3)$$

Note that $X(\omega) = G(\omega)\,j\sin(3\omega) = G(\omega)\left[\frac{e^{j3\omega} - e^{-j3\omega}}{2}\right]$

$$= \frac{1}{2}G(\omega)e^{j3\omega} - \frac{1}{2}G(\omega)e^{-j3\omega}$$

Using linearity and time-shift properties of Fourier transform:

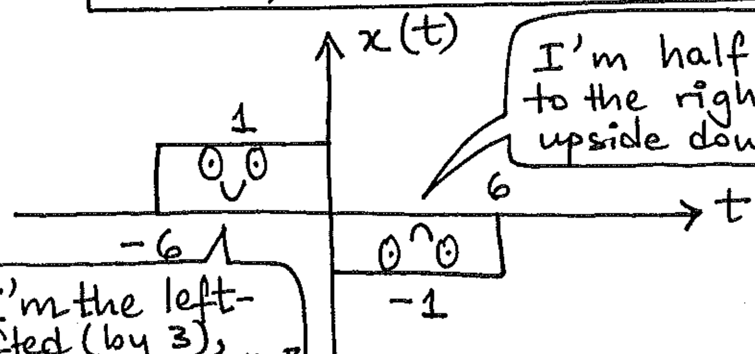
$$\frac{1}{2}g(t+3) \longleftrightarrow \frac{1}{2}G(\omega)e^{j3\omega}$$

$$\frac{1}{2}g(t-3) \longleftrightarrow \frac{1}{2}G(\omega)e^{-j3\omega}$$

$$\therefore x(t) = \frac{1}{2}g(t+3) - \frac{1}{2}g(t-3)$$

$$= u(t+6) - u(t) - u(t) + u(t-6)$$

$$= \boxed{u(t+6) - 2u(t) + u(t-6)}$$



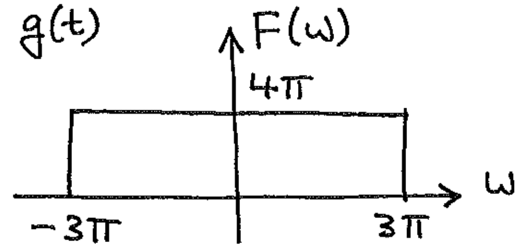
I'm half of g(t) shifted to the right by 3 and turned upside down!

I'm the left-shifted (by 3), half version of g(t)!

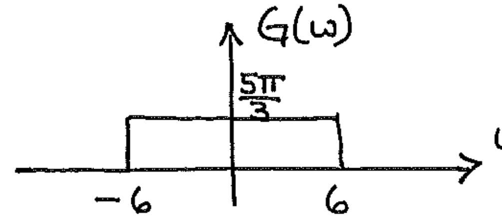
(4) (25 points) Find the Fourier transform of the signal given by

$$x(t) = \underbrace{\left[\frac{4\sin(3\pi t)}{t}\right]}_{f(t)}\underbrace{\left[\frac{5\sin(6t)}{3t}\right]}_{g(t)}$$

$$f(t) = \frac{4\sin(3\pi t)}{t} \longleftrightarrow$$



$$g(t) = \frac{5\sin(6t)}{3t} \longleftrightarrow$$



Time-domain multiplication property of Fourier transform states:

I'm the convolution symbol !

$$x(t) = f(t)\,g(t) \longleftrightarrow X(\omega) = \frac{1}{2\pi}\left\{F(\omega) * G(\omega)\right\}$$

$$\therefore\ X(\omega) = \frac{1}{2\pi}\left\{\left[4\pi u(\omega+3\pi) - 4\pi u(\omega-3\pi)\right] * \left[\frac{5\pi}{3}u(\omega+6) - \frac{5\pi}{3}u(\omega-6)\right]\right\}$$

$$= \boxed{\frac{10\pi}{3}r(\omega+3\pi+6) - \frac{10\pi}{3}r(\omega+3\pi-6) - \frac{10\pi}{3}r(\omega-3\pi+6) + \frac{10\pi}{3}r(\omega-3\pi-6)}$$