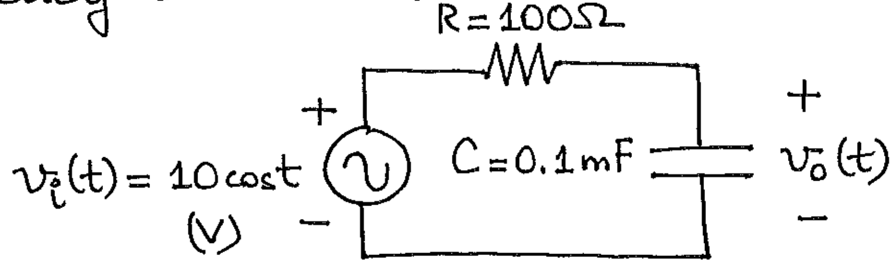
Review of Sinusoidal (AC) Steady-State Circuits

Example # 1 For the circuit shown, find the steady-state output voltage $v_o(t)$.

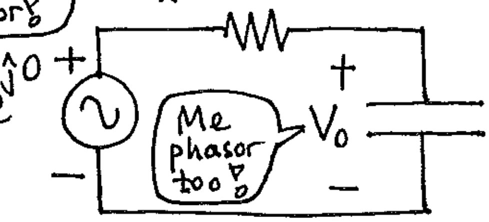


Solution: Beam the circuit into the imaginary phasor domain:

I'm the impedance of the resistor! $Z_R = R = 100\Omega$

I'm the impedance of the capacitor! $Z_C = -\frac{\hat{j}}{\omega C} = -\hat{j}\,10^4\,\Omega$

I'm the phasor voltage of the input signal! $V_i = 10e^{\hat{j}0}$ (V)



Using VDP, we can write:

$$V_o = \frac{Z_2}{Z_1 + Z_2} V_i = \underbrace{\left(\frac{-\frac{\hat{j}}{\omega C}}{R - \frac{\hat{j}}{\omega C}}\right)}_{\text{Frequency response } H(\hat{j}\omega)!} V_i = \frac{-\hat{j}\,10^4}{100 - \hat{j}\,10^4}(10)$$

Note that $H(\hat{j}\omega) = H(s)\big|_{s=\hat{j}\omega}$!

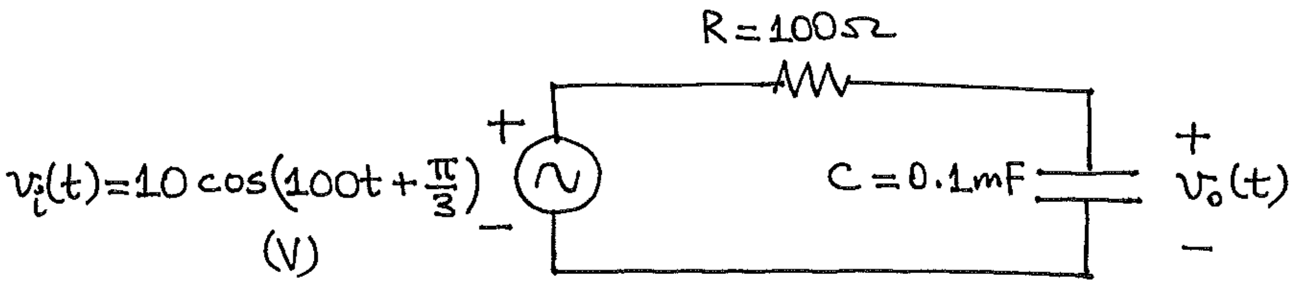
$$= \frac{10^4 e^{-\hat{j}\frac{\pi}{2}}}{\sqrt{10^4 + 10^8}\, e^{-\hat{j}\tan^{-1}(100)}}(10) = \frac{10^5}{\sqrt{10^4 + 10^8}}\, e^{\hat{j}\left(\tan^{-1}(100) - \frac{\pi}{2}\right)}$$

$$\simeq 9.9995\, e^{-\hat{j}0.57294^\circ} \text{ (V)}$$

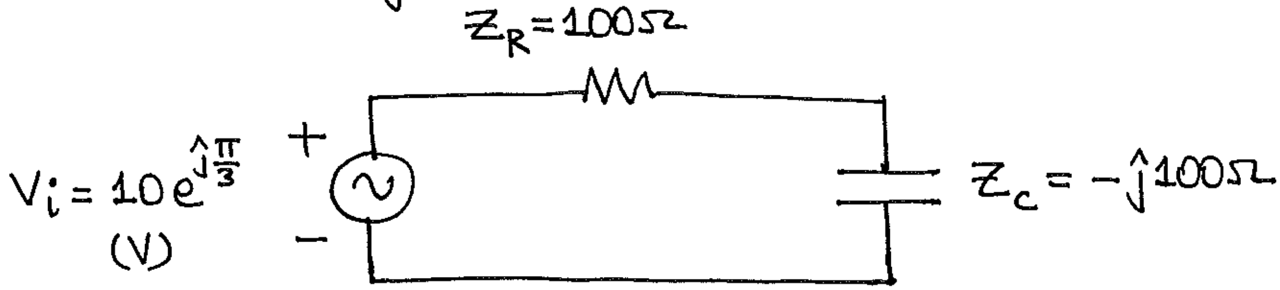
Converting phasor voltage V_o back into time domain:

$$\boxed{v_o(t) \simeq 9.9995 \cos(t - 0.57294^\circ) \text{ (V)}}$$

Example #2 For the circuit shown, find the steady-state voltage $v_o(t)$.



Solution: Following the same steps:



VDP: $$V_o = H(\hat{j}100)\,V_i = \frac{-\hat{j}100}{100-\hat{j}100}\left(10e^{\hat{j}\frac{\pi}{3}}\right)$$

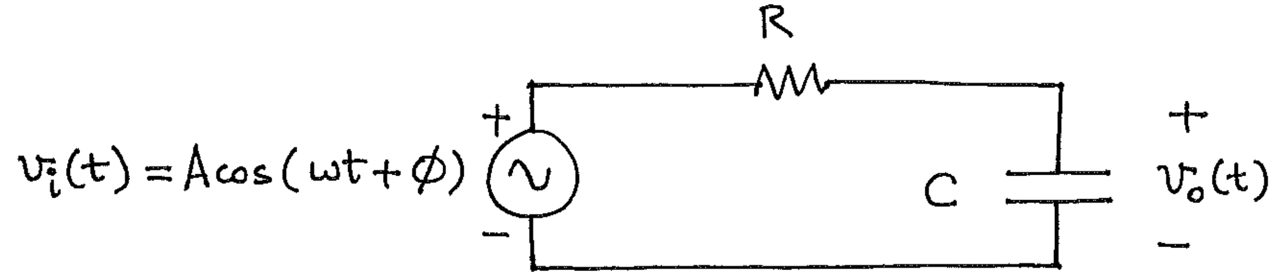
$$= \frac{\cancel{100}e^{-\hat{j}\frac{\pi}{2}}}{\cancel{100}\sqrt{2}\,e^{-\hat{j}\frac{\pi}{4}}}\left(10e^{\hat{j}\frac{\pi}{3}}\right)$$

$$= \frac{10}{\sqrt{2}}e^{\hat{j}(-\frac{\pi}{2}+\frac{\pi}{3}+\frac{\pi}{4})} = \frac{10}{\sqrt{2}}e^{\hat{j}\frac{\pi}{12}} \simeq 7.0711e^{\hat{j}15^\circ} \text{ (V)}$$

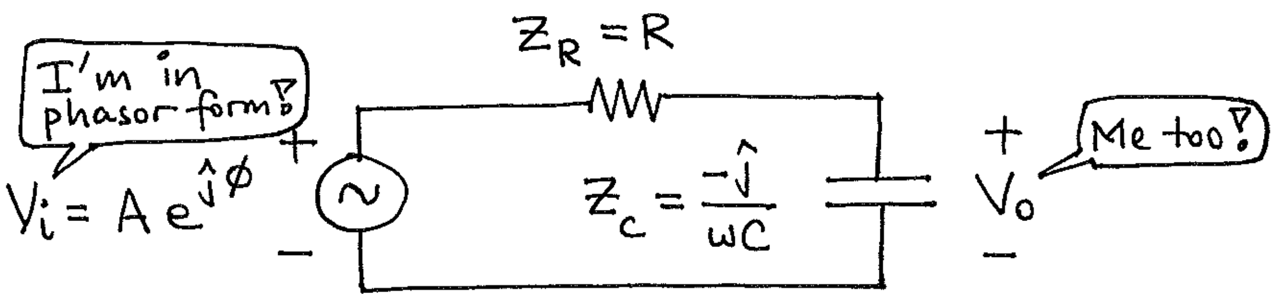
or π/12

$$\boxed{\therefore\ v_o(t) \simeq 7.0711\cos(100t+15^\circ) \text{ (V)}}$$

Example #3 For the circuit shown, find the steady-state $v_o(t)$.



Solution: Beam the circuit into phasor domain:



VDP:

$$V_o = \underbrace{\left(\frac{-\frac{\hat{j}}{\omega C}}{R - \frac{\hat{j}}{\omega C}}\right)}_{\text{Note that this is } H(\hat{j}\omega)\text{!}}\left(A e^{\hat{j}\phi}\right)$$

$$= \frac{\frac{1}{\omega C} e^{-\hat{j}\pi/2}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}\, e^{\hat{j}\tan^{-1}(1/\omega RC)}}\left(A e^{\hat{j}\phi}\right)$$

$$= \underbrace{\frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}}_{|H(\hat{j}\omega)|}\, e^{\hat{j}\overbrace{\left(\tan^{-1}\left(\frac{1}{\omega RC}\right) - \frac{\pi}{2}\right)}^{\angle H(\hat{j}\omega)}} A e^{\hat{j}\phi}$$

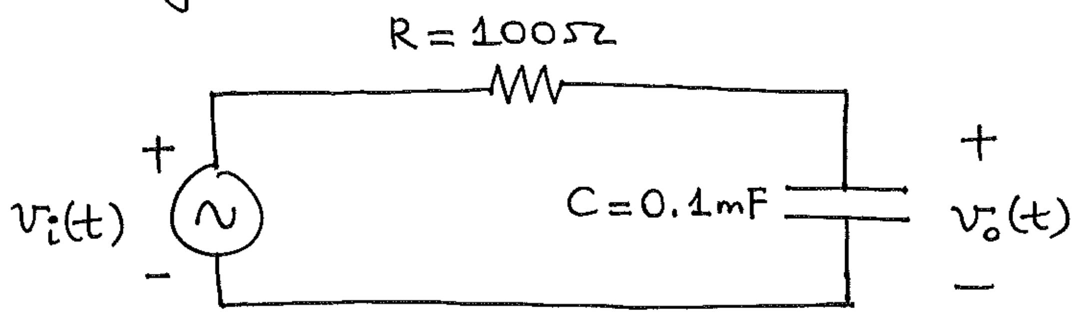
I'm the magnitude of the frequency response!

I'm the angle of the frequency response!

$$= |H(\hat{j}\omega)|\, A\, e^{\hat{j}(\phi + \angle H(\hat{j}\omega))}$$

$$\boxed{\therefore\ v_o(t) = |H(\hat{j}\omega)|\, A\cos(\omega t + \phi + \angle H(\hat{j}\omega))}$$

Example #4 For the circuit shown, find the steady-state output voltage $v_o(t)$.

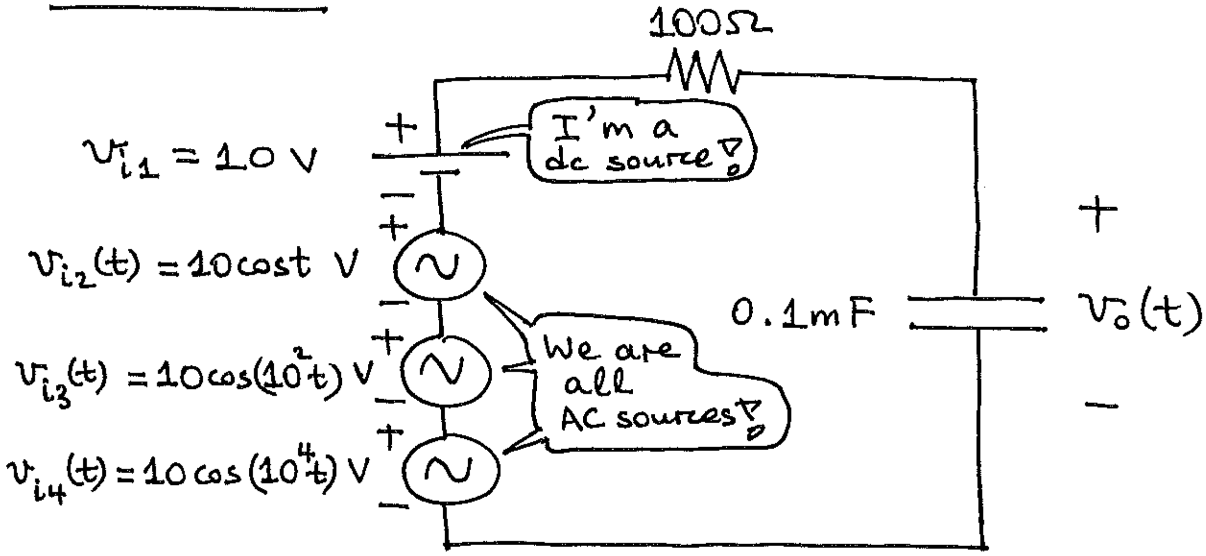


$$v_i(t) = 10 + 10\cos t + 10\cos(100t) + 10\cos(10^4 t)\ (\mathrm{V})$$

I represent a periodic signal!

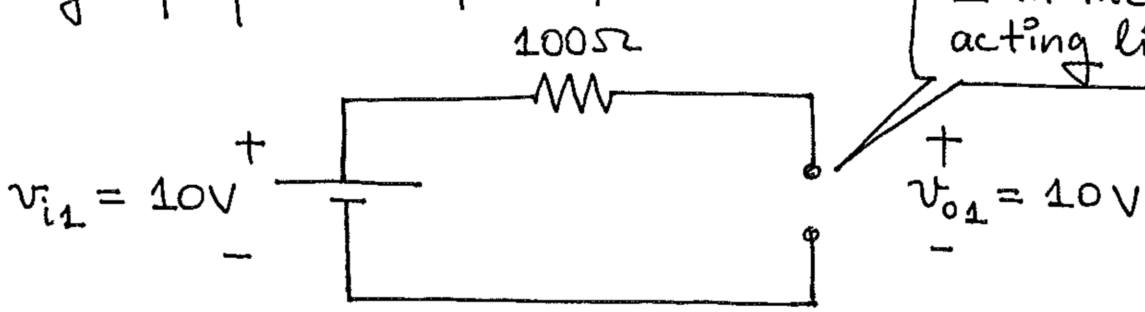
Indeed, I am its Fourier series!

Solution: Redraw the circuit as follows:

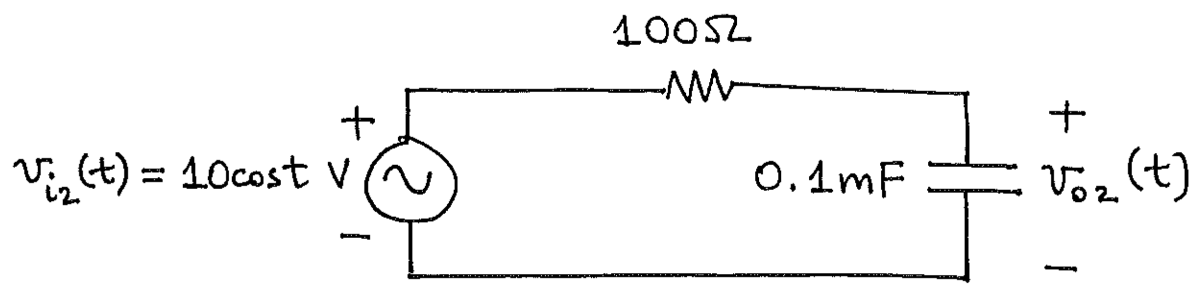


Using superposition principle:

I'm the capacitor acting like an open!



(Note that from Example #1, at $\omega = 0$, $H(j0) = 1$.)



Using the phasor-domain circuit, we find $v_{o2}(t)$ to be

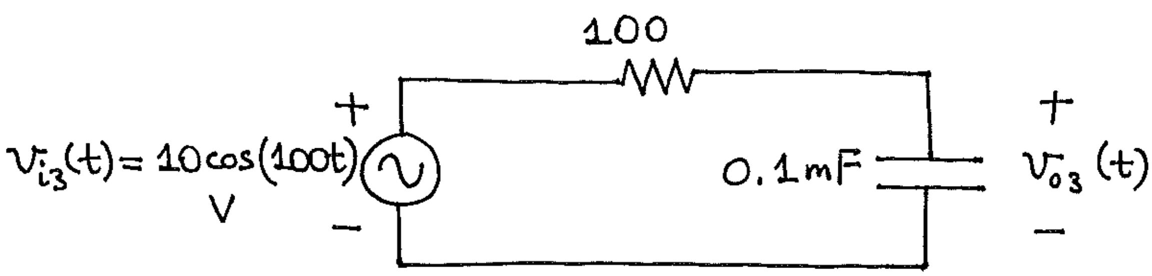
$$v_{o2}(t) = |H(\hat{j})|(10)\cos\left(t + \angle H(\hat{j})\right)$$

From Example #3!

From Example #3!

$$= \underbrace{\frac{1}{\sqrt{1+10^{-4}}}}_{|H(\hat{j})|}(10)\cos\left(t + \underbrace{\tan^{-1}(100) - \pi/2}_{\angle H(\hat{j})}\right)$$

$$\simeq 9.9995\cos(t - 0.57294^\circ)\ (\mathrm{V}) \leftarrow \left(\begin{array}{c}\text{Check}\\ \text{Example \#1}\end{array}\right)$$



Following similar steps:

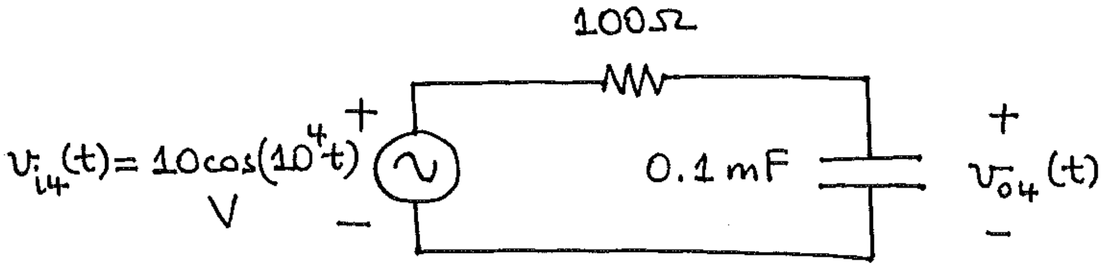
$$v_{o3}(t) = |H(\hat{j}100)|\,(10)\cos\left(100t + \angle H(\hat{j}100)\right)$$

Use the result of Example #3 to calculate us!

$$|H(\hat{j}100)| = \frac{1}{\sqrt{1+(10^{4})(10^{-4})}} = \frac{1}{\sqrt{2}} \simeq 0.70711$$

$$\angle H(\hat{j}100) = \tan^{-1}(1) - \frac{\pi}{2} = -\frac{\pi}{4}$$

$$\therefore\ v_{o3}(t) \simeq 7.0711\cos\left(100t - \frac{\pi}{4}\right)\ (\mathrm{V})$$



$$v_{o4}(t) = |H(j10^4)|(10)\cos(10^4 t + \angle H(j10^4))$$

Using the results of Example # 3 :

$$|H(j10^4)| = \frac{1}{\sqrt{1+(10^8)(10^{-4})}} \simeq 0.0099995$$

$$\angle H(j10^4) = \tan^{-1}(10^{-2}) - \frac{\pi}{2} \simeq -89.427^\circ$$

$$\therefore\ v_{o4}(t) \simeq \underbrace{0.099995}_{\sim 0.1}\cos(10^4 t - 89.427^\circ)\ \ (V)$$

$$\boxed{\therefore\ v_o(t) \simeq 10 + 10\cos(t - 0.573^\circ) + 7.07\cos(100t - 45^\circ) + 0.1\cos(10^4 t - 89.4^\circ)\ \ (V)}$$

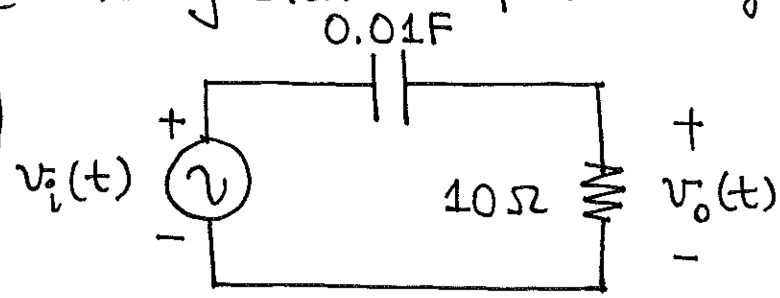
I'm the Fourier series of the output signal!

Exercise # 1 For the circuit shown, given:

$$v_i(t) = 10 + 10\sin t + 10\sin(10t) + 10\sin(100t)$$

Find the steady-state output voltage $v_o(t)$.

I'm the Fourier series of a periodic signal!

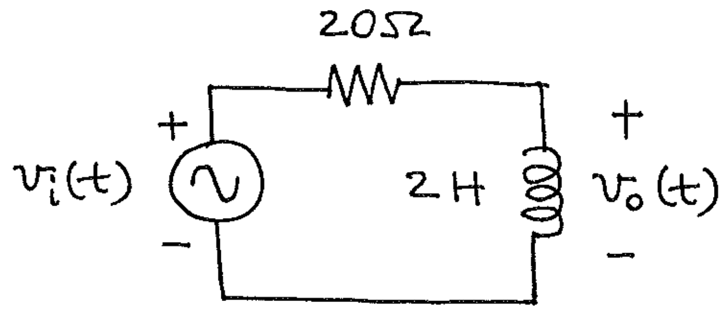


Exercise # 2 For the circuit shown, given:

I'm also a Fourier series ♡

$$v_i(t) = 10 + 6\cos\left(10t - \frac{\pi}{4}\right) - 8\sin\left(100t + \frac{\pi}{3}\right)$$

Find the steady-state output voltage $v_o(t)$.



Exercise # 3 Repeat the previous exercise for the circuit shown below.