Practice Problems on Unilateral Laplace Transform and its Applications

(Tuesday, April 9, 2013)

(1) Determine the Laplace transform of each of the following signals:

(a) $x(t)=5te^{-2t}u(t)$ (b) $y(t)=4\sin(10t-\pi/3)u(t)$ (c) $z(t)=6e^{4-2t}u(t-2)$

(d) $f(t)=3e^{-3t}u(t-3)$ (e) $g(t)=8te^{-t}u(t)$ (f) $h(t)=10te^{-2t}\cos(4t)u(t)$

(g) $m(t)=2(t+3)u(t-2)$ (h) $n(t)=3e^{-4t}\cos(5t)u(t)$ (i) $p(t)=2r(t)-2r(t-1)-2u(t-1)$

(2) Find the Laplace transform of each of the following signals:

(a) $x(t)=2e^{-3t}\delta(t)$ (b) $y(t)=10\cos(t\pi/2+\pi/6)\delta(t-2)$ (c) $f(t)=2\delta(t-3)$

(d) $g(t)=4e^{-2t}\cos(3t-\pi/4)u(t)$ (e) $h(t)=t^2[u(t)-u(t-2)]$ (f) $m(t)=\dfrac{d^2}{dt^2}\left[6e^{-2t}u(t)\right]$

(3) For a Laplace pair $x(t)\leftrightarrow X(s)$, determine $x(0^+)$ and $x(\infty)$ given $X(s)$ to be:

(a) $X(s)=\dfrac{2s^2+15s+12}{s^3+5s^2+4s}$ (b) $X(s)=\dfrac{10+4e^{-2s}}{s(s+2)^2}$ (c) $X(s)=\dfrac{5s^2+24}{(s+1)(s^2+4s+16)}$

(d) $X(s)=\dfrac{4(s+6)}{s^2+4s+16}$ (e) $X(s)=\dfrac{(2s^2-3)e^{-3s}}{s(s+3)^2}$

(4) Determine the inverse Laplace transform of each of the following signals:

(a) $X(s)=\dfrac{4}{(s+1)(s+2)}$ (b) $Y(s)=\dfrac{10e^{-s}}{s^2(s+3)}$ (c) $Z(s)=\dfrac{2s^2+3s-4}{s^3+4s^2+4s}$

(d) $F(s)=\dfrac{2(s+3)}{s^3+3s^2+2s}$ (e) $G(s)=\dfrac{2(s^2-5)}{s(s+1)^2}$ (f) $H(s)=\dfrac{3s^2}{(s+2)^3}$

(5) Determine the inverse Laplace transform of each of the following signals:

(a) $X(s)=\dfrac{5s^2-3s+7}{(s+1)(s^2+4)}$ (b) $Y(s)=\dfrac{s^3+8s^2+21s+34}{(s+1)^2(s^2+4s+8)}$ (c) $F(s)=\dfrac{8s(e^{-2s}-4)}{(s^2+16)}$

(d) $F(s)=\dfrac{-3(s-9)}{(s+1)(s^2+9)}$ (e) $F(s)=\dfrac{d}{ds}\left[\dfrac{2s^2-4}{s^2+1}\right]$ (f) $G(s)=e^{-2s}\dfrac{d}{ds}\left[\dfrac{2s-3}{(s+1)^2}\right]$

(g) $F(s)=s\dfrac{d^2}{ds^2}\left[\dfrac{2(s+4)}{s^2+4}\right]$

(6) Given the Laplace transform pair $x(t)\leftrightarrow X(s)$, express the Laplace transform of the following signals in terms of $X(s)$:

(a) $y(t)=4x(3t)$ (b) $z(t)=4tx(t-2)$ (c) $f(t)=4x(3t-2)$ (d) $g(t)=4e^{-2t}x(3t-2)$

(d) $h(t)=4x(3t)*x(t-2)$ (e) $m(t)=4x(t)*\dfrac{dx(t)}{dt}$ (f) $n(t)=\displaystyle\int_0^t x(2\tau)d\tau$

(7) Given the Laplace transform pair $x(t) \leftrightarrow X(s)$, express the inverse Laplace transform of each Laplace-domain function below in terms of $x(t)$:

(a) $Y(s) = (s-1)X(s)$ (b) $Z(s) = X(4s)$ (c) $P(s) = 3X(s+2)$ (d) $M(s) = \frac{d}{ds}\left[e^{-2s}X(s)\right]$

(e) $N(s) = \frac{2X(s)}{s^2}$