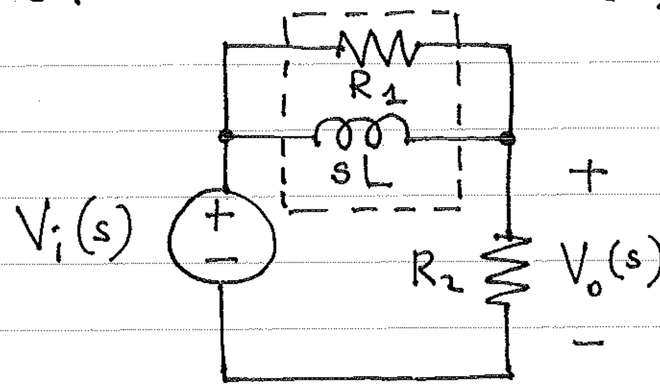
EE 262 SPRING 2014 FINAL EXAM:

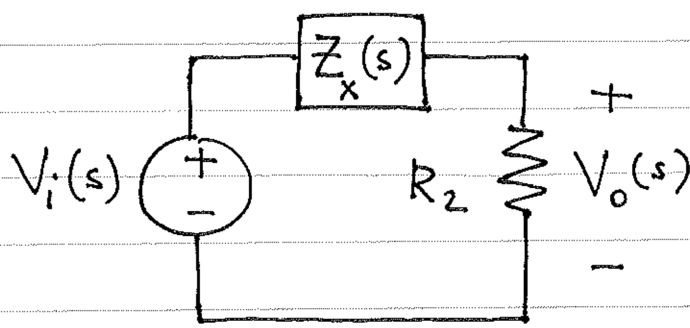
SOLUTION TO PROBLEM (5):



$$Z_x(s) = \frac{sR_1L}{sL+R_1}$$

I'm combined impedance of R_1 & sL!

$\Downarrow$



$$V_o(s) = \frac{R_2}{\dfrac{sR_1L}{sL+R_1} + R_2} V_i(s)$$

$$\rightarrow H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2(sL+R_1)}{sL(R_1+R_2)+R_1R_2}$$

I'm the transfer function!

$$= \frac{R_2\cancel{L}\,(s+R_1/L)}{\cancel{L}(R_1+R_2)\left(s+\dfrac{R_1R_2}{(R_1+R_2)L}\right)}$$

Substituting the values of the elements:

$$H(s) = \frac{3\,(s + 10^4\pi)}{4\,(s + 7500\pi)}$$

To find the frequency response, substitute s with $\hat{j}\omega$:

$$H(\hat{j}\omega) = \frac{3\,(\hat{j}\omega + 10^4\pi)}{4\,(\hat{j}\omega + 7500\pi)}$$

I'm the frequency response

$$= \underbrace{\frac{3}{4}\sqrt{\frac{\omega^2 + (10^4\pi)^2}{\omega^2 + (7500\pi)^2}}}_{|H(\hat{j}\omega)|}\; e^{\hat{j}\underbrace{\left[\tan^{-1}\frac{\omega}{10^4\pi} - \tan^{-1}\frac{\omega}{7500\pi}\right]}_{\angle H(\hat{j}\omega)}}$$

The first three terms (harmonics) of the Fourier series of the input signal $v_i(t)$ are given by:

$$v_i(t) = \frac{20}{\pi}\sin(\overbrace{3000\pi t}^{\omega_0}) - \frac{10}{\pi}\sin(\overbrace{6000\pi t}^{2\omega_0})$$

$$+ \frac{20}{3\pi}\sin(\underbrace{9000\pi t}_{3\omega_0})$$

The first three terms (harmonics) of the Fourier series of the output signal $v_o(t)$ can be found as follows:

$$V_o(t) = \frac{20}{\pi}\left|H(j\omega_o)\right| \sin\left(3000\pi t + \angle H(j\omega_o)\right)$$

$$- \frac{10}{\pi}\left|H(j2\omega_o)\right| \sin\left(6000\pi t + \angle H(j2\omega_o)\right)$$

$$+ \frac{20}{3\pi}\left|H(j3\omega_o)\right| \sin\left(9000\pi t + \angle H(j3\omega_o)\right)$$

$\boxed{3000\pi}$

$$\left|H(j\omega_o)\right| = \frac{3}{4}\sqrt{\frac{(3000\cancel{\pi})^2 + (10^4\cancel{\pi})^2}{(3000\cancel{\pi})^2 + (7500\cancel{\pi})^2}} \cong 0.9694$$

$$\left|H(j2\omega_o)\right| = \frac{3}{4}\sqrt{\frac{(6000\cancel{\pi})^2 + (10^4\cancel{\pi})^2}{(6000\cancel{\pi})^2 + (7500\cancel{\pi})^2}} \cong 0.9106$$

$$\left|H(j3\omega_o)\right| = \frac{3}{4}\sqrt{\frac{(9000\cancel{\pi})^2 + (10^4\cancel{\pi})^2}{(9000\cancel{\pi})^2 + (7500\cancel{\pi})^2}} \cong 0.8613$$

$$\angle H(j\omega_o) = \underbrace{\tan^{-1}\left(\frac{3000\cancel{\pi}}{10^4\cancel{\pi}}\right)}_{\sim 16.7^\circ} - \underbrace{\tan^{-1}\left(\frac{3000\cancel{\pi}}{7500\cancel{\pi}}\right)}_{\sim 21.8^\circ} \cong -5.1^\circ$$

$$\angle H(j2\omega_0) = \underbrace{\tan^{-1}\left(\frac{6000\cancel{\pi}}{10^4\cancel{\pi}}\right)}_{\sim 30.96^\circ} - \underbrace{\tan^{-1}\left(\frac{6000\cancel{\pi}}{7500\cancel{\pi}}\right)}_{\sim 38.66^\circ} \cong -7.696^\circ$$

$$\angle H(j3\omega_0) = \underbrace{\tan^{-1}\left(\frac{9000\cancel{\pi}}{10^4\cancel{\pi}}\right)}_{\sim 41.99^\circ} - \underbrace{\tan^{-1}\left(\frac{9000\cancel{\pi}}{7500\cancel{\pi}}\right)}_{\sim 50.19^\circ} \cong -8.207^\circ$$

∴ The first three terms of the Fourier series of the output signal $V_o(t)$ can be written as:

$$\boxed{\begin{aligned} V_o(t) \cong\ & 6.171 \sin(3000\pi t - 5.1^\circ) \\ & - 2.899 \sin(6000\pi t - 7.696^\circ) \\ & + 1.828 \sin(9000\pi t - 8.207^\circ) \end{aligned}}$$