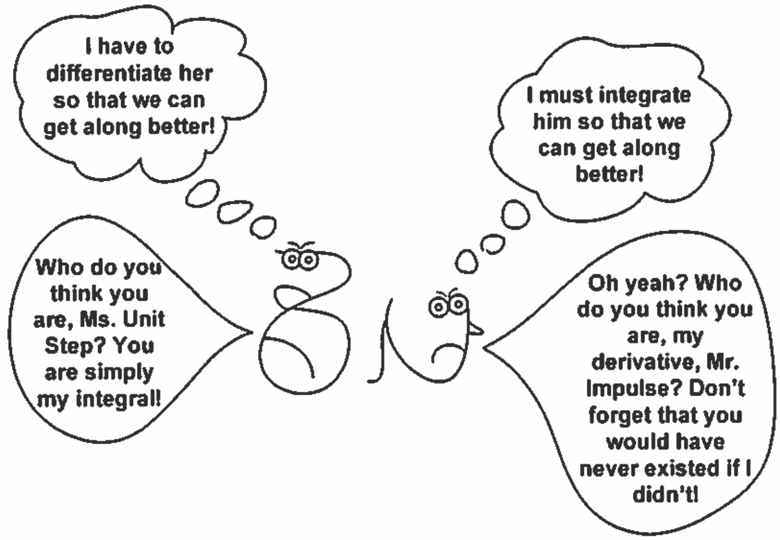



# *University of Portland*
# *School of Engineering*

## EE 262-δignals & δystems-3 cr. hrs.
## Spring 2015

## Midterm Exam # 1

(Prepared by Professor A. S. Inan)

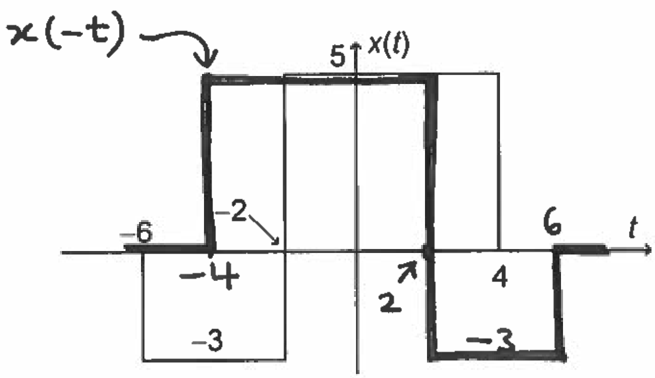
(Monday, February 23, 2015)
(Closed Book Exam, One formula sheet allowed.)
(Total Time: 55 mins.)

**Name:** SOLUTIONS ! ☺

***Signature:*** ☺

(Any 6 problems in-class, the other 6 problems take-home due this Wednesday, February 25, 2015)

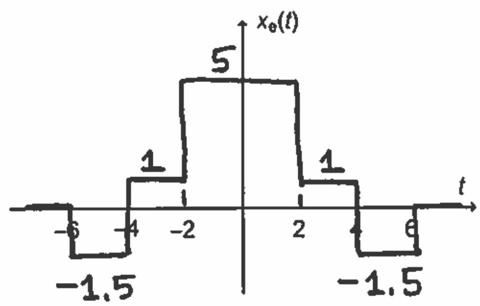
(1) (<u>Total:</u> 20 points) **Signals.** Consider a continuous-time signal, $x(t)$, as shown.

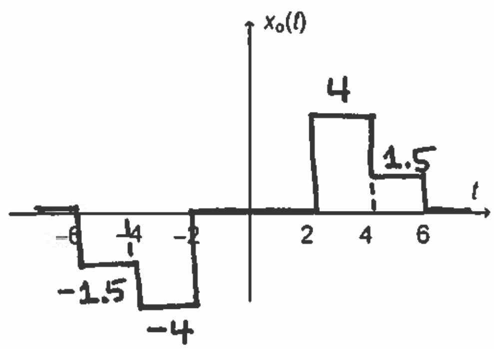



$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

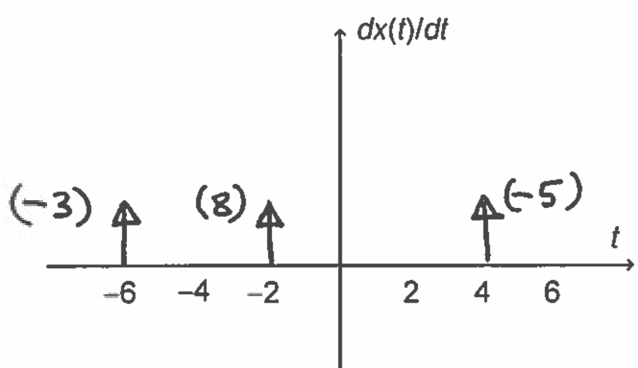
$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

(a) (10 points) **Even and odd parts.** Sketch the even and odd parts of $x(t)$. Provide all the pertinent values on your sketch.




(b) (10 points) **Derivative of a signal.** Find the complete mathematical expression for the function $y(t) = dx(t)/dt$ and sketch $y(t)$ versus $t$. Provide all the pertinent values on your sketch.

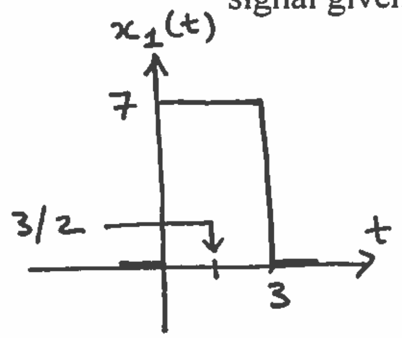


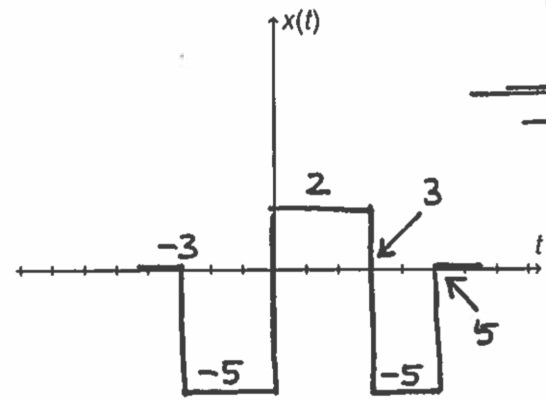

$$\boxed{y(t) = -3\delta(t+6) + 8\delta(t+2) - 5\delta(t-4)}$$

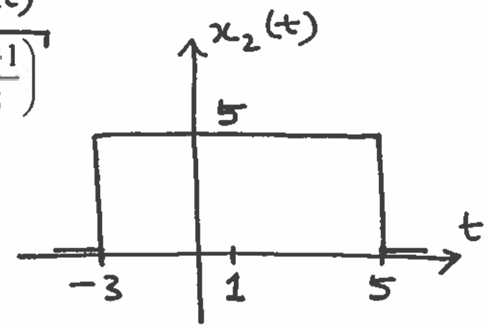
$$x_1(t) = 7\text{rect}\left(\frac{\cancel{2}(t-3/2)}{\cancel{6}\;3}\right)$$

(2) (10 points) **Rectangular pulse signal.** Sketch the rectangular pulse signal given by

$$x(t) = \overbrace{7\text{rect}\left(\frac{2t-3}{6}\right)}^{x_1(t)} - \overbrace{5\text{rect}\left(\frac{t-1}{8}\right)}^{x_2(t)}$$



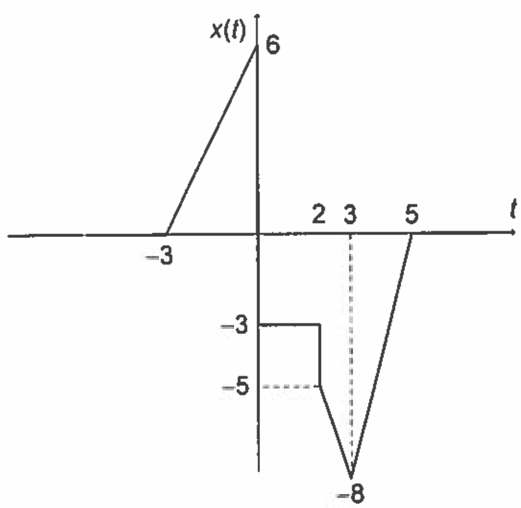

(3) (10 points) **Period of a signal.** Determine the period of the signal given by $x(t) = \underbrace{10\cos(4\pi t - 5\pi/6)}_{x_1(t)} - \underbrace{9\sin(8\pi t/3 + 2\pi/7)}_{x_2(t)}$.

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi} = 0.5\ ;\quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{8\pi/3} = \frac{3}{4} = 0.75$$

Least common multiple !

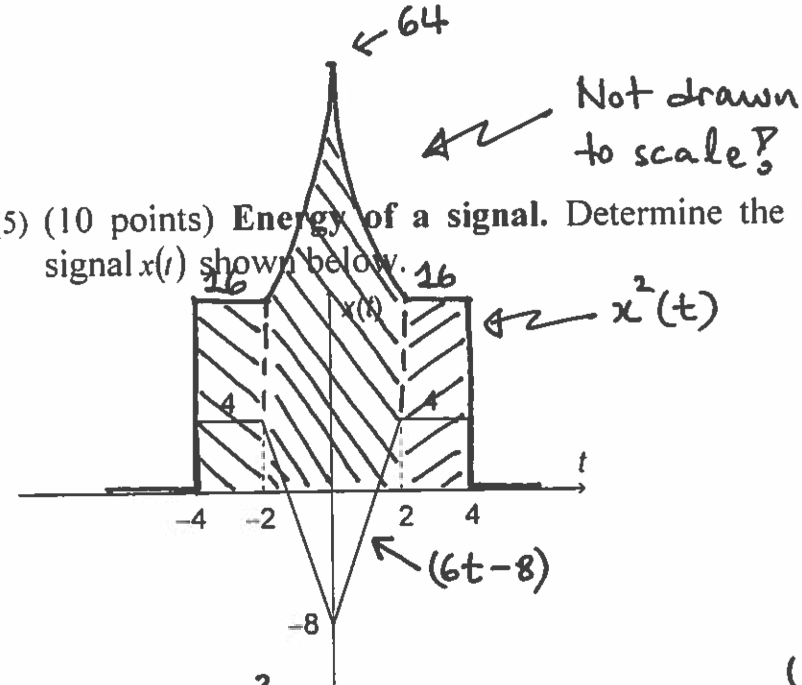
LCM of $T_1$ & $T_2$ is $T = \boxed{1.5\text{ s}}$

(4) (10 points) **Singularity functions.** Express the signal $x(t)$ sketched below in terms of singularity functions.




$$\boxed{\begin{aligned} x(t) = {} & 2r(t+3) - 2r(t) \\ & - 9u(t) - 2u(t-2) \\ & - 3r(t-2) + 7r(t-3) \\ & - 4r(t-5) \end{aligned}}$$

(5) (10 points) **Energy of a signal.** Determine the total energy of the signal $x(t)$ shown below.




$$E = \underbrace{16\times 2}_{\text{Rectangle area on the left}} + 2\int_0^2 (6t-8)^2\,dt + \underbrace{16\times 2}_{\text{Rectangle area on the right}} = 64 + \frac{(6t-8)^3}{9}\Bigg|_0^2 = 64 + \frac{576}{9}$$

$$= \boxed{128}$$

(6) (10 points) **Causal system?** The input-output relationship of the system shown below is described as $y(t) = 4e^{t+2}x(3t-1)$. Is this system causal? (Provide a meaningful justification.)


**Noncausal system** because for example: $y(1)$ depends on $x(2)$ (that is, future value of $x(t)$)

(7) (10 points) **Linear system?** The input-output relationship of the system shown below is described as $y(t) = 2x(t) - 3x(t-1)$. Is this system linear? (Provide a meaningful justification.)


Additivity:

$x_1(t) \to y_1(t) = 2x_1(t) - 3x_1(t-1)$

$x_2(t) \to y_2(t) = 2x_2(t) - 3x_2(t-1)$

$x_3(t) = x_1(t) + x_2(t) \to y_3(t) = 2\underbrace{[x_1(t)+x_2(t)]}_{x_3(t)} - 3\underbrace{[x_1(t-1)+x_2(t-1)]}_{x_3(t-1)}$

$\overset{\text{Yes!}}{=} y_1(t) + y_2(t)$

$\therefore$ $\boxed{\text{Linear system}}$

Scalibility:

$x_3(t) = cx_1(t) \to y_3(t) = 2cx_1(t) - 3cx_1(t-1) \overset{\text{Yes!}}{=} cy_1(t)$

(8) (10 points) **Time-invariant system?** The input-output relationship of the system shown below is described as $y(t) = 3\cos[2tx(t-1)]$. Is this system time invariant? (Provide a meaningful justification.)

$$x_1(t) \rightarrow y_1(t) = 3\cos[2t\,x_1(t-1)]$$

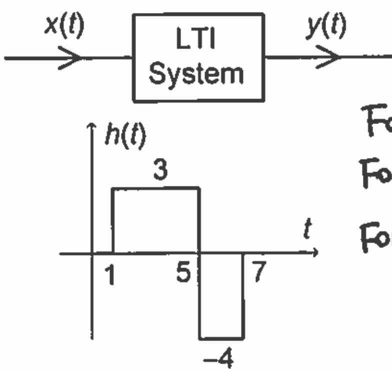
$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t) = 3\cos[2t\,x_2(t-1)] = 3\cos[2t\,x_1(t-t_0-1)]$$

$$\neq y_1(t-t_0)$$

$$= 3\cos[2(t-t_0)x_1(t-t_0-1)]$$

$\therefore$ $\boxed{\text{Time-variant system}}$

(9) (10 points) **LTI system.** The impulse response of an LTI system is given as shown. Find and sketch the unit-step response of this system.




$$y_s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

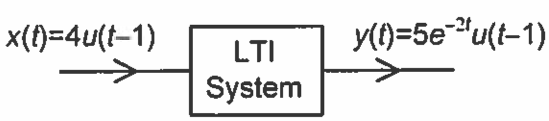
For $t < 1 \rightarrow y_s(t) = 0$

For $1 < t < 5 \rightarrow y_s(t) = \int_1^t 3\,d\tau = 3(t-1)$

For $5 < t < 7 \rightarrow y_s(t) = \int_1^5 3\,d\tau + \int_5^t (-4)\,d\tau = 32 - 4t$
$= 4(8-t)$

For $t > 7 \rightarrow y_s(t) = \int_1^5 3\,d\tau + \int_5^7 (-4)\,d\tau = 12 - 8 = 4$

(10) (10 points) **LTI system.** The zero-state response of an LTI system excited by $x(t) = 4u(t-1)$ is given by $y(t) = 5e^{-2t}u(t-1)$. If an input signal given by $x(t) = 3u(t) - 4\delta(t-2)$ is applied to this system, what will be the output signal $y(t)$?




Initially at zero state

For $x(t) = u(t)$

$$\rightarrow y_s(t) = \frac{5}{4}e^{-2(t+1)}u(t)$$

For $x(t) = \delta(t)$

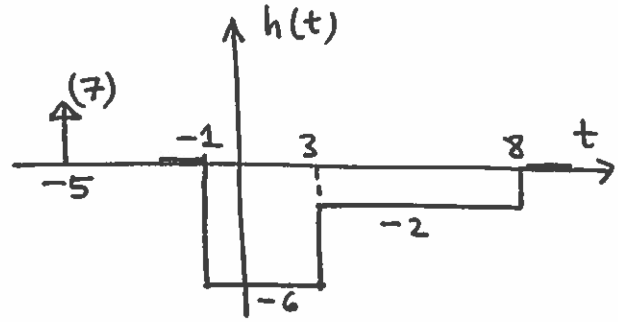
$$\rightarrow h(t) = \frac{dy_s(t)}{dt} = -\frac{5}{2}e^{-2(t+1)}u(t) + \frac{5}{4e^2}\delta(t)$$

For $x(t) = 3u(t) - 4\delta(t-2)$

$$\rightarrow \boxed{y(t) = \frac{15}{4}e^{-2(t+1)}u(t) + 10e^{-2(t-1)}u(t-2) - \frac{5}{e^2}\delta(t-2)}$$

(11) (10 points) **LTI system.** The impulse response of an LTI system is given by $h(t)=7\delta(t+5)-6u(t-1)+4u(t-3)+2u(t-8)$. Determine whether this system is (a) causal or non-causal; and (b) BIBO stable or unstable. (Show your work.)

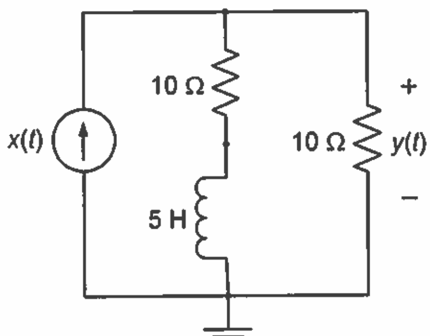

$h(t)=7\delta(t+5)-6u(t-1)+4u(t-3)+2u(t-8)$




Since $h(t) \neq 0$ for $t<0$

$\rightarrow$ $\boxed{\text{Noncausal system}}$

(12) (10 points) **Impulse response.** Find the impulse response of the following circuit shown.




$$x(t)=u(t) \rightarrow y(t)=\left[y(0^+)\,e^{-t/\tau}+y(\infty)\left(1-e^{-t/\tau}\right)\right]u(t)$$

$y(0^+) = 10 \times 1 = 10\text{ V}$ since (inductor) acts like open circuit

$y(\infty) = 10 \times \frac{1}{2} = 5\text{V}$ since (inductor) acts like short circuit

$$\tau = \frac{L}{R_T} = \frac{5\text{H}}{20\Omega} = \frac{1}{4}\text{ s}$$

$$\therefore\ y_s(t) = \left[10e^{-4t}+5\left(1-e^{-4t}\right)\right]u(t)$$

$$h(t)=\frac{dy_s(t)}{dt} = \left(-40e^{-4t}+20e^{-4t}\right)u(t) + \underbrace{\left[10e^{-4t}+5\left(1-e^{-4t}\right)\right]\delta(t)}_{10\delta(t)}$$

$$= \boxed{-20e^{-4t}u(t)+10\delta(t)}$$