University ☺f P☺rtland
Sch☺☺l ☺f Engineering

EE 262-δignals & δystems-3 cr. hrs.
Spring 2015

Midterm Exam # 2

(Prepared by Professor A. S. Inan)

(Friday, April 10, 2015)
(Closed Book Exam, 3 formula sheets allowed.)
(Total Time: 55 mins.)
(Any 5 of 10 problems in-class, other 5 take-home!)

Name: SOLUTIONS ☺ ☺

Signature: Solutions ☺

(1)(10 points) **Unilateral Laplace transform.** Find the unilateral Laplace transform of the signal given by

$$x(t) = \underbrace{2tu(t-1)}_{x_1(t)} * \underbrace{6e^{-t}u(t-2)}_{x_2(t)}$$

(Note that this is a convolution problem. Provide your answer in its simplest form.)

Thanks to convolution property of Laplace xform!

Thanks to time shift property of Laplace xform!

$u(t) \leftrightarrow 1/s$

$r(t) = tu(t) \leftrightarrow 1/s^2$

$(t-1)u(t-1) + u(t-1) \leftrightarrow \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}$

$e^{-t}u(t) \leftrightarrow 1/(s+1)$

$e^{-2}e^{-(t-2)}u(t-2) \leftrightarrow e^{-2}e^{-2s}/(s+1)$

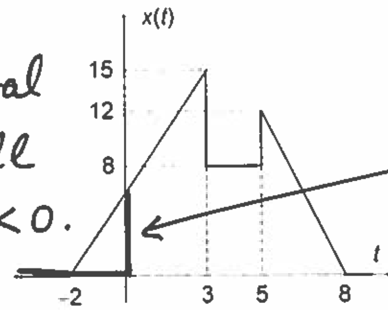
$$X(s) = X_1(s)X_2(s)$$

$$= 2\left(\frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}\right) 6\left(\frac{e^{-2}e^{-2s}}{s+1}\right)$$

$$= \boxed{\frac{12e^{-3s}}{e^2 s^2}}$$

(2)(10 points) **Unilateral Laplace transform.** Find the unilateral Laplace transform of the signal $x(t)$ as shown.

Note that Unilateral Laplace xform will ignore $x(t)$ for $t<0$.



This is how the signal $x(t)$ will be interpreted with respect to Unilateral Laplace xform!

For Unilateral Laplace xform, $x(t)$ can be expressed as

$$x(t) = 6u(t) + 3r(t) - 3r(t-3) - 7u(t-3) + 4u(t-5) - 4r(t-5) + 4r(t-8)$$

$$\therefore \boxed{X(s) = \frac{6}{s} + \frac{3}{s^2} - \frac{3e^{-3s}}{s^2} - \frac{7e^{-3s}}{s} + \frac{4e^{-5s}}{s} - \frac{4e^{-5s}}{s^2} + \frac{4e^{-8s}}{s^2}}$$

(3)(10 points) **Inverse Laplace transform.** Find the inverse Laplace transform of the signal given by

$$X(s) = \underbrace{5e^{-3(s-2)}}_{5e^{6}e^{-3s}} \frac{d}{ds}\left(\frac{4}{(s+1)^2}\right)$$

Apply time shift property of Laplace xform!

$4/s^2 \leftrightarrow 4r(t)$ or $4tu(t)$

$4/(s+1)^2 \leftrightarrow 4te^{-t}u(t)$

$\frac{d}{ds}(4/(s+1)^2) \leftrightarrow -4t^2e^{-t}u(t)$

$$5e^{6}e^{-3s}\frac{d}{ds}\left(4/(s+1)^2\right)$$

$$-5e^{6}\left[4(t-3)^2 e^{-(t-3)} u(t-3)\right]$$

$$\therefore \boxed{x(t) = -20e^{6}(t-3)^2 e^{-(t-3)} u(t-3)}$$

$$
\begin{array}{r|l}
 & 2 \\
\hline
s^2+6s+18 & 2s^2+18s+9 \\
 & 2s^2+12s+36 \\
\hline
 & 6s-27
\end{array}
$$

Long division!

(4)(10 points) **Inverse Laplace transform.** Find the inverse Laplace transform of the signal given by

$$X(s)=\frac{2s^2+18s+9}{s^2+6s+18} = 2 + \frac{6s-27}{(s^2+6s+18)}$$

Use completing square technique!

$$X(s) = 2 + \frac{6(s+3)}{(s+3)^2+9} - \frac{45}{(s+3)^2+9}$$

$$\therefore \quad \boxed{x(t) = 2\delta(t) + 6e^{-3t}\cos(3t)u(t) - 15e^{-3t}\sin(3t)u(t)}$$

(5)(10 points) **Unilateral Laplace transform.** Given $x(t) \leftrightarrow X(s)$ unilateral Laplace transform pair and given the signal $y(t)$ to be $y(t)=2e^{-2t+2}x\left(\frac{t}{2}-2\right)$, express $Y(s)$ in terms of $X(s)$.

$$x\left(\frac{t}{2}\right) \leftrightarrow 2X(2s)$$

Time scaling property of Laplace xform!

$$x\left(\frac{t-4}{2}\right) \leftrightarrow 2e^{-4s}X(2s)$$

Time shift property of Laplace xform!

Use frequency shift property!

$$\underbrace{2e^{2}e^{-2t}x\left(\frac{t-4}{2}\right)}_{y(t)} \leftrightarrow \boxed{\underbrace{4e^{2}e^{-4(s+2)}X(2(s+2))}_{Y(s)}}$$

(6)(10 points) **Initial and final values.** Determine the initial and final values of $x(t)$ if the unilateral Laplace transform of $x(t)$ is given by

$$X(s)=\frac{9s^2+20s+16}{s^3+4s^2+4s}$$

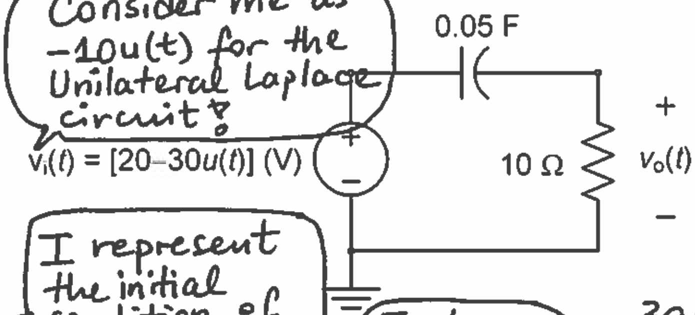
$$x(0^+) = \lim_{s\to\infty} \frac{\cancel{s}(9s^2+20s+16)}{\cancel{s}(s^2+4s+4)} = \boxed{9}$$

$$x(\infty) = \lim_{s\to 0} \frac{\cancel{s}(9s^2+20s+16)}{\cancel{s}(s^2+4s+4)} = \boxed{4}$$

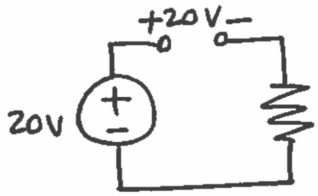
That is $V_i(s) = -\frac{10}{s}$!

(7)(10 points) **Application of Laplace transform to electric circuits.** For the electric circuit shown, use Laplace transform to find the output voltage signal $v_o(t)$ for $t > 0$.

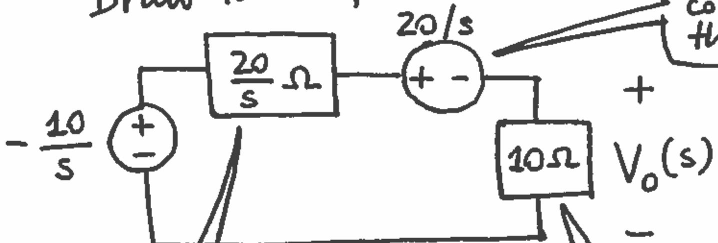
Consider me as $-10u(t)$ for the Unilateral Laplace circuit!



At $t = 0^-$ (ss):



Draw the Laplace circuit:



I represent the initial condition of the capacitor!

Find me using VDP!

$$\to V_o(s) = -\frac{10}{10+\frac{20}{s}}\left(\overbrace{\frac{10}{s}+\frac{20}{s}}^{30/s}\right)$$

$$= \frac{-30}{s+2}$$

I'm the impedance of the capacitor!

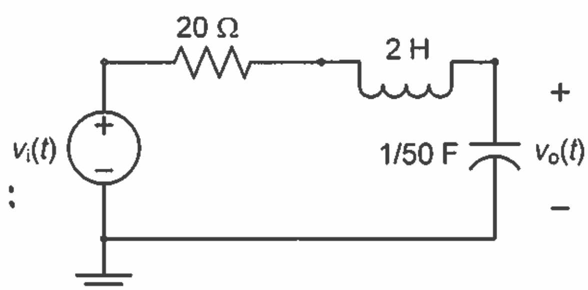
I'm the impedance of the resistor!

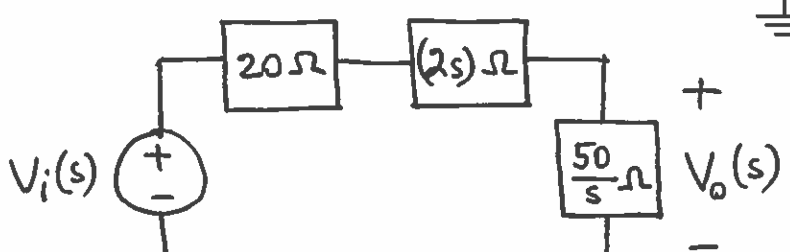
$$\therefore \boxed{v_o(t) = -30e^{-2t}u(t)\ \text{V}}$$

(8)(10 points) **Transfer function and impulse response.** Find the transfer function and the impulse response of the electric circuit shown.

Assume zero initial conditions.

Draw the Laplace circuit:





$$V_o(s) \overset{\text{VDP}}{=} \frac{\frac{50}{s}}{20+2s+\frac{50}{s}}V_i(s)$$

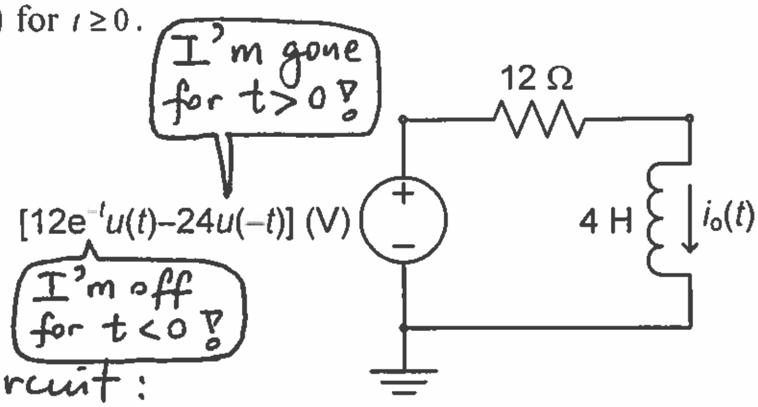
I'm the transfer function!

$$\to H(s) = \frac{V_o(s)}{V_i(s)} = \frac{25}{s^2+10s+25} = \boxed{\frac{25}{(s+5)^2}}$$

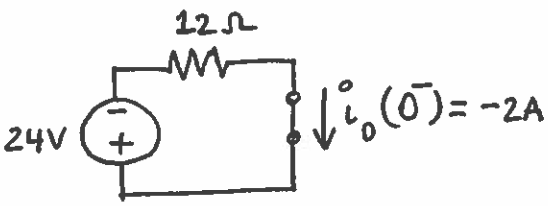
I'm the impulse response

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = 25e^{-5t}t\,u(t) \text{ or } \boxed{25te^{-5t}u(t)}$$

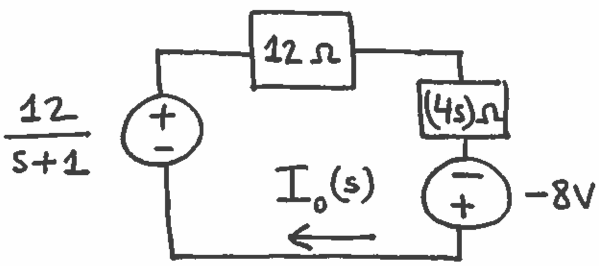
(9)(10 points) **Application of Laplace transform to electric circuits.** For the electric circuit shown, use Laplace transform to find the current signal $i_o(t)$ for $t \geq 0$.



At $t=0^-$ (ss):



Draw the Laplace circuit:



$$I_o(s) = \frac{\frac{12}{s+1} - 8}{12+4s} = \frac{12-8s-8}{4(s+3)(s+1)}$$

$$= \frac{-2s+1}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$

$$A = \left.\frac{-2s+1}{s+3}\right|_{s=-1} = \frac{3}{2} \ \& \ B = \left.\frac{-2s+1}{s+1}\right|_{s=-3} = -\frac{7}{2}$$

$$\therefore \boxed{i_o(t) = \frac{3}{2}e^{-t}u(t) - \frac{7}{2}e^{-3t}u(t)}$$

(10) (10 points) **Applications of Laplace transform to solve differential equations.** Determine the response $y(t)$ for $t \geq 0$ of the differential equation with the specified input signal and the initial conditions:

Use time derivative properties of Laplace to convert us to Laplace domain!

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} = 4x(t), \ x(t) = e^{-2t}u(t), \ y(0^-) = 4, \ \left.\frac{dy(t)}{dt}\right|_{t=0} = 0$$

Apply Laplace xform:

$$s^2Y(s) - 4s + sY(s) - 4 = \frac{1}{s+2}$$

$$Y(s) = \frac{\frac{1}{s+2} + 4s + 4}{s^2+s} = \frac{4s^2+12s+9}{s(s+1)(s+2)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$A = \left.\frac{4s^2+12s+9}{(s+1)(s+2)}\right|_{s=0} = \frac{9}{2}, \ B = \left.\frac{4s^2+12s+9}{s(s+2)}\right|_{s=-1} = -1, \ C = \frac{1}{2}$$

$$\therefore \boxed{y(t) = \frac{9}{2}u(t) - e^{-t}u(t) + \frac{1}{2}e^{-2t}u(t)}$$