Happy 543rd Birthday Copernicus!

Nicolaus Copernicus
(19-2-1473—24-5-1*543*)

University ☺f P☺rtland
Sch☺☺l ☺f Engineering

EE 262-δignals & δystems-3 cr. hrs.
Spring 2016

Midterm Exam # 1
(Prepared by Professor A. S. Inan)

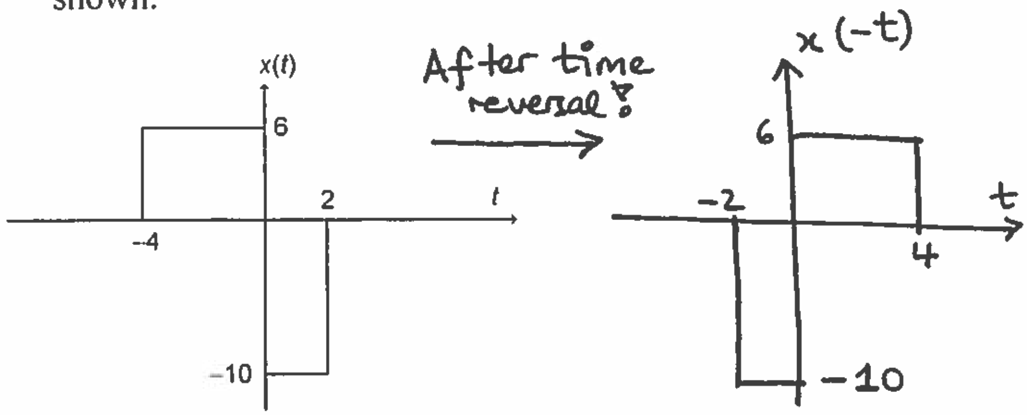
(Friday, February 19, 2016)
(Closed Book Exam, One formula sheet allowed.)
(Total Time: 55 mins.)

Name: SOLUTIONS ! ☺

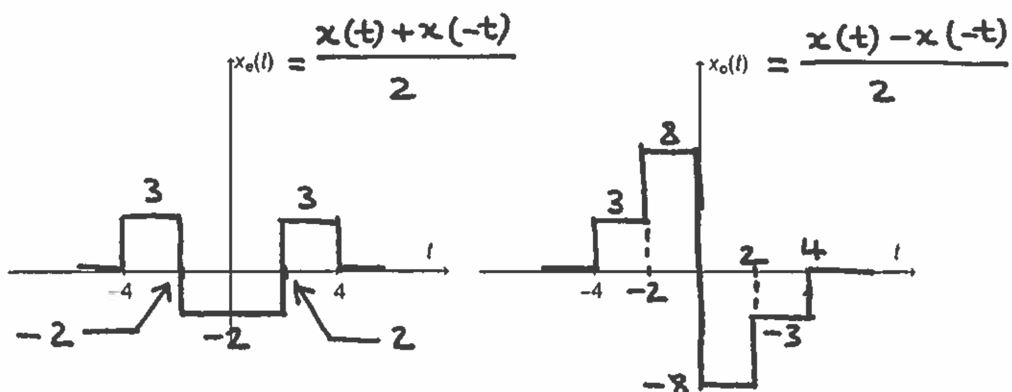
Signature: Solution ☺

(Any 7 problems in-class, the other ~~5 problems~~ take-home due Monday, February 22, 2016)

(1) (Total: 10 points) **Signals.** Consider a continuous-time signal, $x(t)$, as shown.

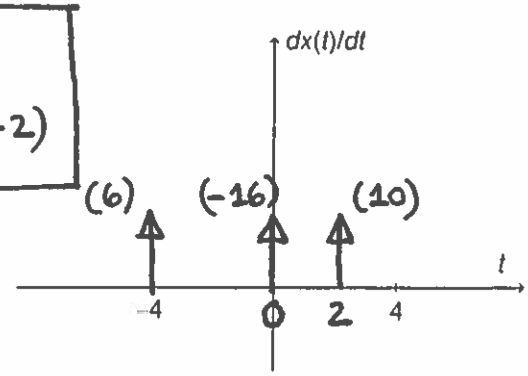


(a) (5 points) **Even and odd parts.** Sketch the even and odd parts of $x(t)$. Provide all the pertinent values on your sketch.



(b) (5 points) **Derivative of a signal.** Find the complete mathematical expression for the function $y(t) = dx(t)/dt$ and sketch $y(t)$ versus t. Provide all the pertinent values on your sketch.

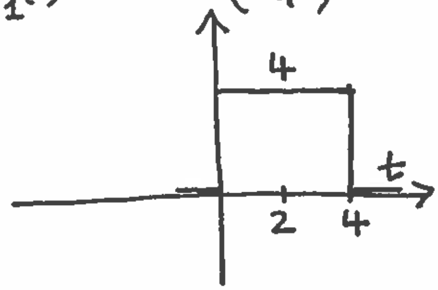
$$\boxed{y(t) = 6\,\delta(t+4) - 16\,\delta(t) + 10\,\delta(t-2)}$$

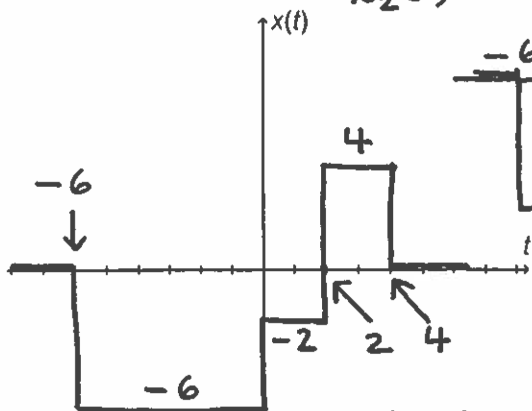


(2) (10 points) **Rectangular pulse signal.** Sketch the rectangular pulse signal given by

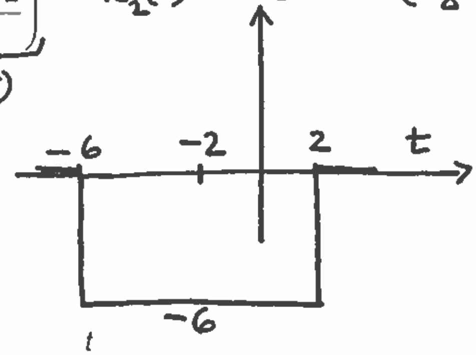
$$x(t) = \underbrace{4\operatorname{rect}\left(\frac{t-2}{4}\right)}^{x_1(t)} \underbrace{-6\operatorname{rect}\left(\frac{t+2}{8}\right)}_{x_2(t)}$$

$x_1(t) = 4\operatorname{rect}\left(\frac{t-2}{4}\right)$





$x_2(t) = -6\operatorname{rect}\left(\frac{t+2}{8}\right)$



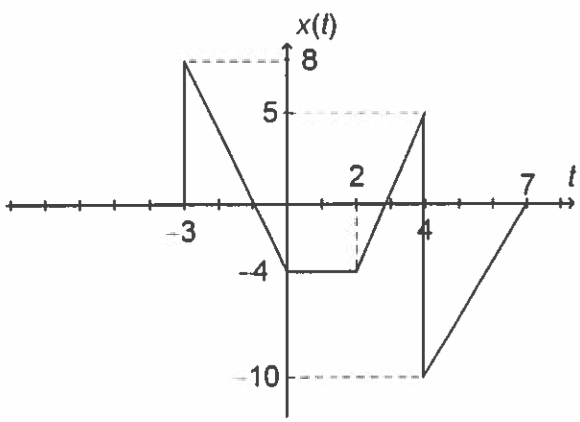
(3) (10 points) **Period of a signal.** Determine the period of the signal given by $x(t) = \underbrace{3\cos(6\pi t - 2\pi/5)}_{x_1(t)} + \underbrace{8\sin(15\pi t + 5\pi/8)}_{x_2(t)}$.

Period of $x_1(t) \rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{6\pi} = \frac{1}{3}$ s

Period of $x_2(t) \rightarrow T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{15\pi} = \frac{2}{15}$ s

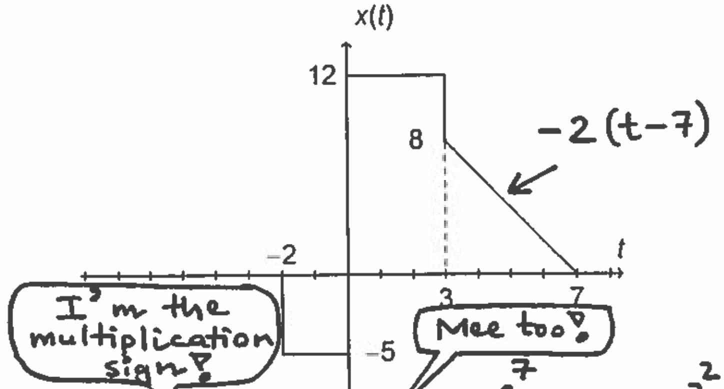
LCM of T_1 & T_2 yield $T = \boxed{\frac{2}{3}\text{ s}}$

(4) (10 points) **Singularity functions.** Express the signal $x(t)$ sketched below in terms of singularity functions.



$$\boxed{\begin{aligned} x(t) = {} & 8u(t+3) - 4r(t+3) + 4r(t) + \tfrac{9}{2}r(t-2) \\ & - 15u(t-4) - \tfrac{7}{6}r(t-4) - \tfrac{10}{3}r(t-7) \end{aligned}}$$

(5) (10 points) **Energy of a signal.** Determine the total energy of the signal $x(t)$ shown below.



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = (-5)^2 \times 2 + (12)^2 \times 3 + \int_3^7 \underbrace{\left[-2(t-7)\right]^2}_{4(t-7)^2} dt = 50 + 432 + \left[\frac{4}{3}(t-7)^3\right]_3^7$$

$$= 482 + \frac{256}{3} = \boxed{\frac{1702}{3} \simeq 567.3}$$

(6) (10 points) **Causal system?** The input-output relationship of the system shown below is described as $y(t) = x(3-t)$. Is this system causal? (Provide a meaningful justification.)

No, it is $\boxed{\text{noncausal system}}$

since $y(0) = \underbrace{x(3)}_{\text{future value of } x(t)!}$

(7) (10 points) **Linear system?** The input-output relationship of the system shown below is described as $y(t) = 3x(t)x(t-1)$. Is this system linear? (Provide a meaningful justification.)

For a system to be linear, both additivity and scalibility properties must be satisfied.

Additivity: $y_1(t) = 3x_1(t)x_1(t-1)$

$y_2(t) = 3x_2(t)x_2(t-1)$

$y_3(t) = 3\underbrace{[x_1(t)+x_2(t)]}_{x_3(t)}\underbrace{[x_1(t-1)+x_2(t-1)]}_{x_3(t-1)} \neq y_1(t)+y_2(t)$

$\therefore$ Additivity is NOT satisfied $\therefore$ No, it is a $\boxed{\text{nonlinear system}}$

Scalibility: $y_1(t) = 3x_1(t)x_1(t-1)$

$y_2(t) = 3\underbrace{c\,x_1(t)}_{x_2(t)}\underbrace{c\,x_1(t-1)}_{x_2(t-1)} = 3c^2 x_1(t)x_1(t-1) \neq c\,y_1(t)$ $\therefore$ Scalibility is NOT satisfied.

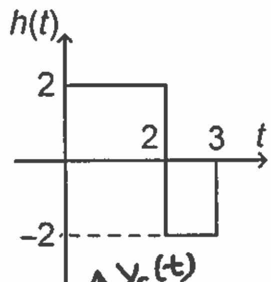
(8) (10 points) **Time-invariant system?** The input-output relationship of the system shown below is described as $y(t)=e^{(3-t)}x(t+1)$. Is this system time invariant? (Provide a meaningful justification.)

$$y_1(t) = e^{(3-t)}x_1(t+1)$$

$$y_2(t) = e^{(3-t)}\underbrace{x_1(t-t_0+1)}_{x_2(t+1)} \neq y_1(t-t_0) = e^{3-(t-t_0)}x_1(t-t_0+1)$$

$\therefore$ No, it is a $\boxed{\text{time-variant system}}$

(9) (10 points) **LTI system.** The impulse response of an LTI system is given as shown. Find and sketch the unit-step response of this system.

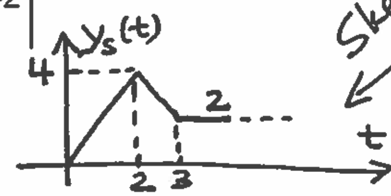


$$y_s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

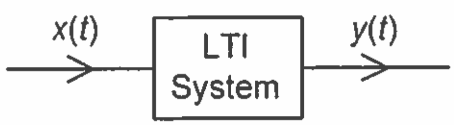
$$= \boxed{\begin{cases} 0, & t \leq 0 \\ 2t, & 0 \leq t \leq 2 \\ 8-2t, & 2 \leq t \leq 3 \\ 2, & t \geq 3 \end{cases}}$$

$$= \boxed{2r(t) - 4r(t-2) + 2r(t-3)}$$

Sketch!



(10) (10 points) **LTI system.** The unit-step response of an LTI system is given by $y_{step}(t)=3e^{(5-2t)}u(t)$. If an input signal $x(t)$ given by $x(t)=2\delta(t-1)-3u(t-2)$ is applied to this system, what will be the output signal $y(t)$? (Assume zero-state condition.)



$$h(t) = \frac{dy_s(t)}{dt}$$

$y_{step}(t)=3e^{(5-2t)}u(t)$ $\rightarrow$ $h(t) = -6e^{(5-2t)}u(t) + 3e^{(5-2t)}\delta(t)$

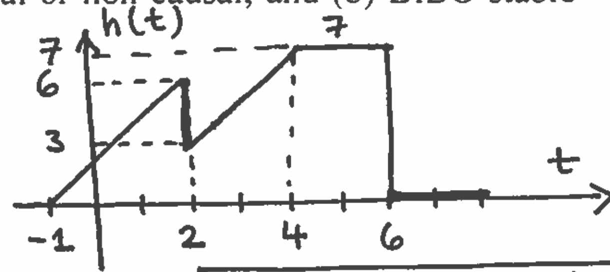
$$= -6e^{(5-2t)}u(t) + 3e^{5}\delta(t)$$

$$\therefore y(t) = 2h(t-1) - 3y_s(t-2)$$

$$= \boxed{\begin{array}{l} -12e^{(5-2(t-1))}u(t-1) + 6e^{5}\delta(t-1) \\ -9e^{(5-2(t-2))}u(t-2) \end{array}}$$

(11) (10 points) **LTI system.** The impulse response of an LTI system is given by $h(t) = 2r(t+1) - 3u(t-2) - 2r(t-4) - 7u(t-6)$. Determine whether this system is (a) causal or non-causal; and (b) BIBO stable or unstable. (Show your work.)

$h(t)=2r(t+1)-3u(t-2)-2r(t-4)-7u(t-6)$

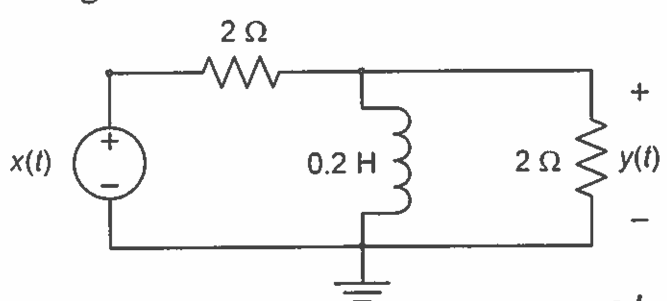


(a) Since $h(t) \neq 0$ for $t<0$ → $\boxed{\text{Noncausal system}}$

(b) $$\int_{-\infty}^{\infty} |h(t)|\, dt = \underbrace{\frac{3\times 6}{2}}_{\text{Area of a triangle}} + \underbrace{\frac{(3+7)\times 2}{2}}_{\text{Area of a trapezoid}} + \underbrace{2\times 7}_{\text{Area of a rectangle}} = 33 < \infty$$

∴ $\boxed{\text{BIBO stable system}}$

(12) (10 points) **Impulse response.** Find the impulse response of the following circuit shown.



$$y_s(t) = \left[y_s(0^+)\, e^{-t/\tau} + y_s(\infty)(1 - e^{-t/\tau})\right] u(t)$$

Let us first find the unit-step response:

$$y_s(0^+) = \frac{2\,\Omega}{(2+2)\,\Omega}(1\text{V}) = \frac{1}{2}\text{V}$$ since acts like

Open circuit!

$$y_s(\infty) = 0$$ since acts like

Short circuit!

$$\tau = \frac{L}{R_T} = \frac{0.2\,\text{H}}{2\,\Omega \,//\, 2\,\Omega} = 0.2\,\text{s}$$

$$\therefore\ y_s(t) = \frac{1}{2} e^{-5t} u(t)$$

$$h(t) = \frac{dy_s(t)}{dt} = -\frac{5}{2} e^{-5t} u(t) + \frac{1}{2} e^{-5t} \delta(t)$$

$$= \boxed{-\frac{5}{2} e^{-5t} u(t) + \frac{1}{2}\delta(t)}$$