University ☺f P☺rtland
Sch☺☺l ☺f Engineering

EE 262-δignals & δystems-3 cr. hrs.
Spring 2016

Midterm Exam # 2
(Prepared by Professor A. S. Inan)

(Friday, April 8, 2016)
(Closed Book Exam, formula sheets allowed.)
(Total Time: 55 mins.)
(Any 7 of 10 problems in-class, other 3 take-home!)

Name: SOLUTIONS! ☺

Signature: Solutions! ☺

(1)(10 points) **Unilateral Laplace transform.** Find the unilateral Laplace transform of the signal given by

$$x(t) = 4e^{3-2t}u(t-1) = 4e\,e^{2-2t}\,u(t-1)$$
$$= 4e\,e^{-2(t-1)}u(t-1)$$

$$4e\,e^{-2t}u(t) \leftrightarrow 4e/(s+2)$$

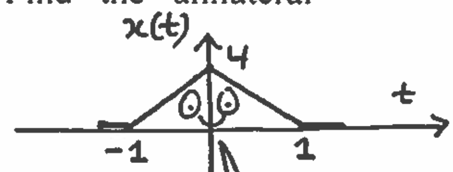
Thanks to time shift property of Laplace xform ☺

$$x(t) = 4e\,e^{-2(t-1)}u(t-1) \leftrightarrow \boxed{X(s) = \frac{4e^{1-s}}{s+2}}$$

(2)(10 points) **Unilateral Laplace transform.** Find the unilateral Laplace transform of the signal $x(t)$ given by

$$x(t) = 4r(t+1) - 8r(t) + 4r(t-1)$$

(Hint: Sketch the signal.)



Unilateral Laplace xform ignores my variation for t<0 ☺

For unilateral Laplace transform:

$$x(t) = 4u(t) - 4r(t) + 4r(t-1)$$

$$\boxed{X(s) = \frac{4}{s} - \frac{4}{s^2} + \frac{4e^{-s}}{s^2}}$$

(3)(10 points) **Inverse Laplace transform.** Find the inverse Laplace transform of the signal given by

Split me using long division ☺

$$X(s) = \frac{3s^2 + 16s + 81}{s^2 + 6s + 25} \rightarrow$$

Long division:

$$\begin{array}{r|l} & 3 \\ s^2+6s+25 & 3s^2+16s+81 \\ & \mp 3s^2 \mp 18s \mp 75 \\ \hline & -2s+6 \end{array}$$

$$X(s) = 3 + \frac{-2s+6}{\underbrace{(s^2+6s+25)}_{(s+3)^2+16}} = 3 - \frac{2(s+3)}{(s+3)^2+16} + \frac{12}{(s+3)^2+16}$$

My roots are complex ☺

$$\boxed{\therefore\ x(t) = 3\delta(t) - 2e^{-3t}\cos(4t)u(t) + 3e^{-3t}\sin(4t)u(t)}$$

(4) (Total: 10 points) **Unilateral Laplace transform.** Given the Laplace transform pair:

$$x(t) \leftrightarrow X(s) = \frac{3s}{s^2+4}$$

Find the unilateral Laplace transform of the following signals:

(a) (5 points) $f(t) = x(3t-6)$ $= x(3(t-2))$

Apply the time-scaling property!

$$x(3t) \leftrightarrow \frac{1}{3}X\left(\frac{s}{3}\right) = \frac{1}{\cancel{3}}\frac{\cancel{3}(s/3)}{(s/3)^2+4}$$

$$f(t) = x(3(t-2)) \leftrightarrow F(s) = \frac{se^{-2s}}{3\left[(s/3)^2+4\right]} = \boxed{\frac{3se^{-2s}}{s^2+36}}$$

Apply time-shift property!

(b) (5 points) $g(t) = x(t) * \frac{dx(t)}{dt}$

Thanks to derivative & convolution properties of Laplace xform!

$$G(s) = X(s)\left[sX(s) - x(0^-)\right] = sX^2(s) = \boxed{\frac{9s^3}{(s^2+4)^2}}$$

since $x(0^-) = 3\cos(2t)u(t)\Big|_{t=0^-} = 0$

(5) (10 points) **Applications of Laplace transform to solve differential equations.** Find $y(t)$ for $t>0$ of the differential equation with the specified input signal and the initial condition:

My Laplace pair is $2/(s+1)$!

$$\frac{dy(t)}{dt} + 3y(t) = 5x(t), \quad x(t) = 2e^{-t}u(t), \quad y(0^-) = -4$$

$$\xrightarrow{\mathcal{L}} sY(s) - \underbrace{y(0^-)}_{-4} + 3Y(s) = 5X(s) = \frac{10}{s+1}$$

$$\rightarrow Y(s)(s+3) = \frac{10}{s+1} - 4 = \frac{-4s+6}{s+1}$$

$$\rightarrow Y(s) = \frac{-2(2s-3)}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$

Applying Partial Fraction Expansion!

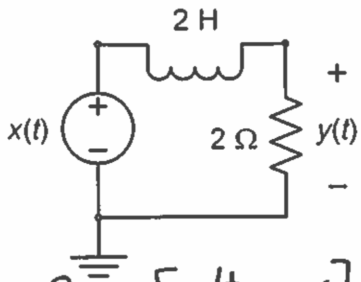
$$A = \left[(s+1)Y(s)\right]\Big|_{s=-1} = \frac{-2(2s-3)}{s+3}\Big|_{s=-1} = 5$$

$$B = \left[(s+3)Y(s)\right]\Big|_{s=-3} = \frac{-2(2s-3)}{s+1}\Big|_{s=-3} = -9$$

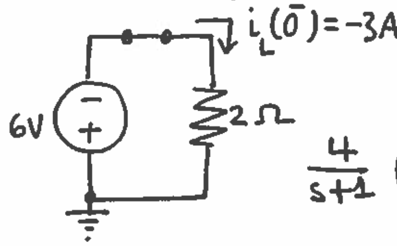
$$\therefore \boxed{y(t) = 5e^{-t}u(t) - 9e^{-3t}u(t)}$$

I'm the initial condition of the circuit

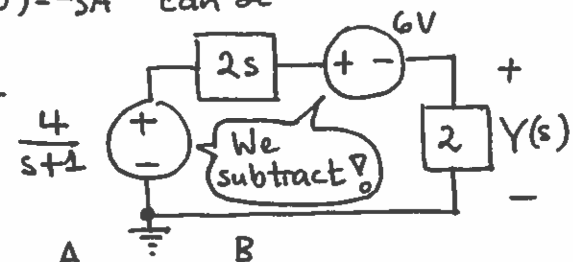
(6)(10 points) (10 points) **Application of Laplace transform to electric circuits.** For the electric circuit shown, the input voltage signal $x(t)$ is given by $x(t) = 4e^{-t}u(t) - 6u(-t)$ V. Use Laplace-domain circuit to find the output voltage signal $y(t)$ for $t \geq 0$.



For $t=0^-$ (ss):



Laplace-domain circuit can be drawn as:

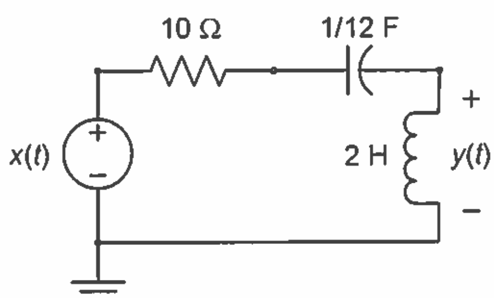


$$Y(s) = \frac{2}{2s+2}\left[\frac{4}{s+1} - 6\right] = \frac{-6s-2}{(s+1)^2} = \frac{A}{(s+1)^2} + \frac{B}{(s+1)}$$

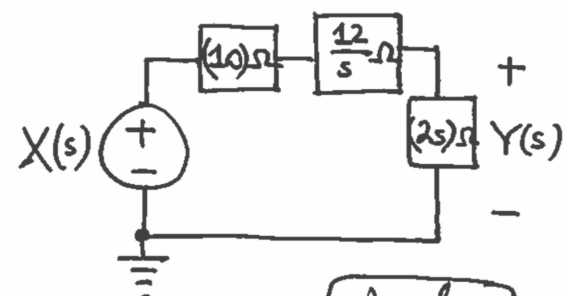
$$A = \left[(s+1)^2 Y(s)\right]_{s=-1} = \left[-6s-2\right]_{s=-1} = 4; \quad B = \left[\frac{d}{ds}(-6s-2)\right]_{s=-1} = -6$$

$$\therefore \boxed{y(t) = 4te^{-t}u(t) - 6e^{-t}u(t)}$$

(7)(10 points) **Transfer function and impulse response.** Find the transfer function $H(s)$ and the impulse response $h(t)$ of the electric circuit shown.



Laplace-domain circuit →



Using VDP

$$Y(s) = \frac{2s}{10 + \frac{12}{s} + 2s} X(s) = \frac{2s^2}{2s^2 + 10s + 12} X(s)$$

Apply PFE

$$\rightarrow H(s) = Y(s)/X(s) = \boxed{\frac{s^2}{s^2+5s+6}} = 1 - \frac{(5s+6)}{\underbrace{s^2+5s+6}_{(s+2)(s+3)}}$$

I result from long division

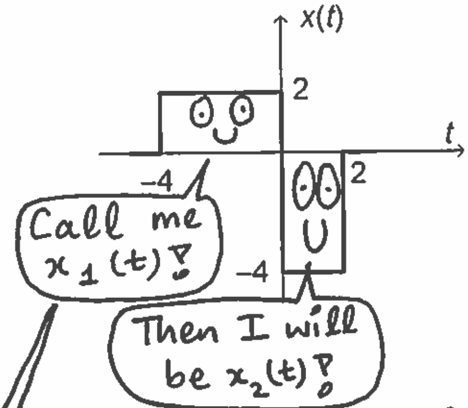
$$= 1 + \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = -\left.\frac{(5s+6)}{(s+3)}\right|_{s=-2} = 4; \quad B = -\left.\frac{(5s+6)}{(s+2)}\right|_{s=-3} = \frac{9}{-1} = -9$$

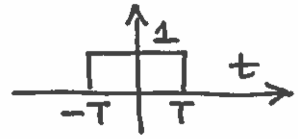
My sign is minus

$$\therefore h(t) = \mathcal{L}^{-1}\{H(s)\} = \boxed{\delta(t) + 4e^{-2t}u(t) - 9e^{-3t}u(t)}$$

(8) (10 points) **Fourier transform.** Find the Fourier transform of the signal shown.



Use the Fourier transform pair:



$$\longleftrightarrow \frac{2\sin(\omega T)}{\omega}$$

Using time-shift property:

$$x_1(t) \leftrightarrow \hat{X}_1(\omega) = 4e^{\hat{j}2\omega}\sin(2\omega)/\omega$$

$$x_2(t) \leftrightarrow \hat{X}_2(\omega) = -8e^{-\hat{j}\omega}\sin(\omega)/\omega$$

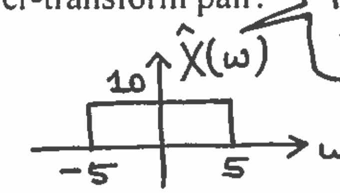
$$\therefore\ x(t) = x_1(t) + x_2(t) \leftrightarrow \hat{X}(\omega) = \hat{X}_1(\omega) + \hat{X}_2(\omega)$$

Add us up to obtain x(t)!

$$\therefore\ \boxed{\hat{X}(\omega) = \frac{4e^{\hat{j}2\omega}\sin(2\omega)}{\omega} - \frac{8e^{-\hat{j}\omega}\sin(\omega)}{\omega}}$$

(9)(10 points) **Fourier transform.** Given the Fourier-transform pair:

$$x(t) \leftrightarrow \hat{X}(\omega) = \begin{cases} 10, & |\omega| < 5 \\ 0, & |\omega| > 5 \end{cases}$$



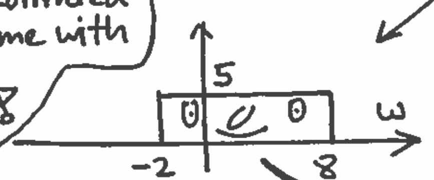
I'm the FT of x(t)!

Sketch the Fourier transform of $y(t) = x(t)\cos(3t)$. (That is, sketch $\hat{Y}(\omega)$ versus ω.)

Thanks to modulation property!

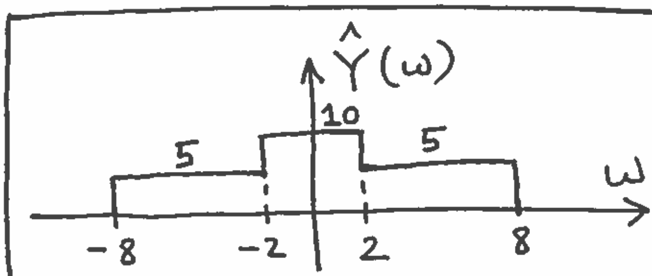
$$y(t) = x(t)\cos(3t) \longleftrightarrow \hat{Y}(\omega) = \underbrace{\tfrac{1}{2}\hat{X}(\omega-3)} + \underbrace{\tfrac{1}{2}\hat{X}(\omega+3)}$$

Use Euler's formula to replace me with $\dfrac{e^{\hat{j}3t}+e^{-\hat{j}3t}}{2}$!



Add us up!

To obtain $\hat{Y}(\omega)$!



(10) (10 points) **Fourier transform.** Given the signal $x(t)$ to be

$$x(t) = \underbrace{\left[\frac{6\sin(3t)}{\pi t}\right]}_{x_1(t)} * \underbrace{\left[\frac{\sin(5t)}{2t}\right]}_{x_2(t)}$$

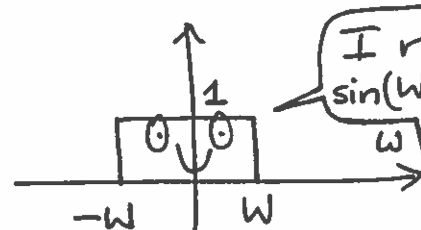
Thanks to convolution property of Fourier transform!

Sketch the Fourier transform of $x(t)$. (That is, sketch $\hat{X}(\omega)$ versus ω.)

$$x(t) = x_1(t) * x_2(t) \longleftrightarrow \hat{X}(\omega) = \hat{X}_1(\omega)\hat{X}_2(\omega)$$

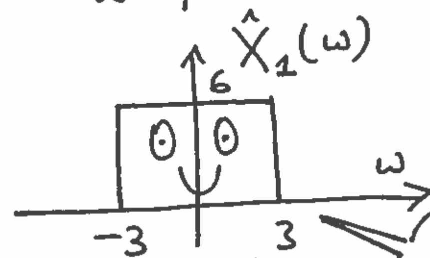
Apply the Fourier transform pair given by

$$\frac{\sin(Wt)}{\pi t} \longleftrightarrow$$

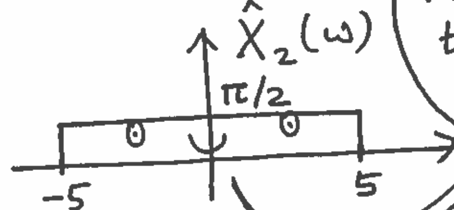


I represent sin(Wt)/(πt) in frequency domain!

$$x_1(t) = \frac{6\sin(3t)}{\pi t} \longleftrightarrow$$



$$x_2(t) = \frac{\sin(5t)}{2t} \longleftrightarrow$$



Multiply us to obtain $\hat{X}(\omega)$!