University ☺f P☺rtland
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EE 271–Electrical Circuits Laboratory
Fall 2016

<u>Lab Experiment #8:</u>
<u>Transient Response of First-Order *RL* and Second-Order *RLC* Circuits</u>

Transient Response of First-Order *RL* and Second-Order *RLC* Circuits

I. Objective

In this experiment, the students will make measurements and observations on the transient step response of simple *RL* and *RLC* circuits.

II. Procedure

PART 1: Step Excitation of First-Order *RL* Circuits

Pre-lab Assignment 1.a: A first-order inductive circuit is excited by a periodic pulse train as shown in Fig. 1. The values of the elements are given by R_1=1 kΩ and L_{coil}=15 mH respectively. Assuming both the source and the inductor to be ideal (i.e., R_S = 0 and R_{coil} = 0), calculate the time constant (designate it with $\tau_{pre\text{-}lab}$ or τ_p) of this circuit. Approximately how long does it take for the inductor of this circuit to fully charge or discharge under pulse excitations? (Fully charge or discharge means the time it takes for the inductor current to reach approximately 99% of its final value.)

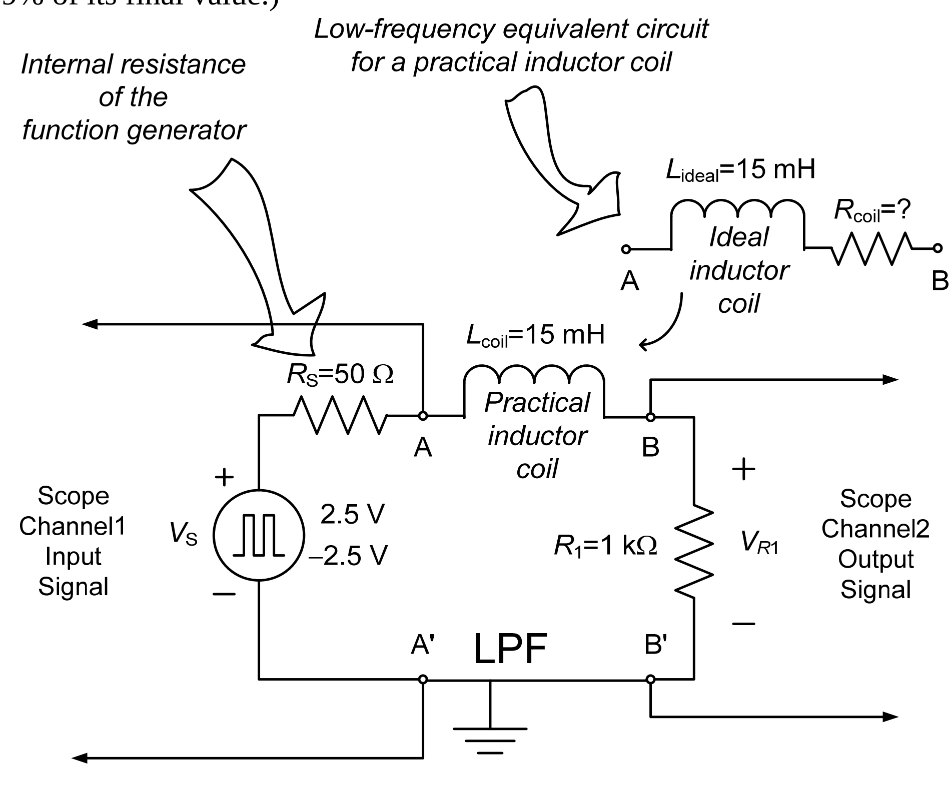

Figure 1. First-order *RL* circuit connected like a Low-Pass Filter (LPF). (Note that terminals A′ and B′ are the same.)

Lab Experiment 1.a: Construct the first-order *RL* circuit shown in Fig. 1 using R_1=1 kΩ and L_{coil}=15 mH. Do the following:

- Using the digital *LCR* meter, measure and record the actual values of R_1 and L_{coil} you use to build your circuit. Note that a practical inductor coil is not an ideal inductor. For low-frequency applications, a practical inductor can be represented in terms of an equivalent circuit model which consists of an ideal inductor with value L_{ideal}=15 mH in series with the internal resistance R_{coil} of the inductor (see Fig. 1). Using the *LCR* meter, measure and record the internal resistance, R_{coil}, of the inductor coil.
- Also, assume the internal source resistance of the function generator R_S to be 50 Ω.
- Use the actual element values measured to recalculate the time constant of this circuit (designate this time constant as τ_{actual} or τ_a).
- Next, use the function generator available on your bench to supply the periodic rectangular pulse train to the circuit. Set the function generator to provide the rectangular pulse train represented with the source voltage $V_S(t)$ which oscillates between −2.5 V and 2.5 V with frequency of oscillation $f = 1/T = 1$ kHz. Use the two channels of the oscilloscope to monitor the source voltage $V_S(t)$ and the resistor voltage $V_{R1}(t)$ across the resistor simultaneously.
- Measure the approximate value of the time constant τ of the circuit from the $V_{R1}(t)$ waveform (call this time constant $\tau_{measured}$ or τ_m). Note that over each T/2 time interval during which the source voltage $V_S(t)$ is either −2.5 V or 2.5 V, assuming t=0 to be the starting time of each one of these T/2 intervals, the resistor voltage $V_{R1}(t)$ varies with respect to time as $V_{R1}(t) = V_{R1}(0^+)e^{-t/\tau_m} + V_{R1}(\infty)(1-e^{-t/\tau_m})$ where $V_{R1}(0^+)$ is its initial value and $V_{R1}(\infty)$ is its final value. So, for example, the resistor voltage at $t = \tau_m$ is approximately given by $V_{R1}(t=\tau_m) \cong 0.368V_{R1}(0^+) + 0.632V_{R1}(\infty)$. Refer to the middle portion of the $V_{R1}(t)$ sketch shown in Fig. 2 for which $V_{R1}(0^+)$ = –2.5 V initial and $V_{R1}(\infty)$ = 2.5 V final voltage values are indicated. Substituting these values yield $V_{R1}(\tau_m) \approx 660$ mV. Using this portion of the $V_{R1}(t)$ waveform seen on the oscilloscope display, measure and record the approximate value of the time constant τ_m using the $V_{R1}(\tau_m)$ voltage point on the $V_{R1}(t)$ waveform.
- Calculate the percentage error in the τ_m value measured using

$$\%\text{ error in } \tau_m \text{ value} = \left|\frac{\tau_a - \tau_m}{\tau_a}\right| \times 100$$

- Compare T/2 (or 1/(2f)) with ~5τ_m and comment on the two waveforms ($V_S(t)$ and $V_{R1}(t)$) observed simultaneously on the scope. (Hint: Does the inductor have enough time to fully charge over the time interval T/2?)

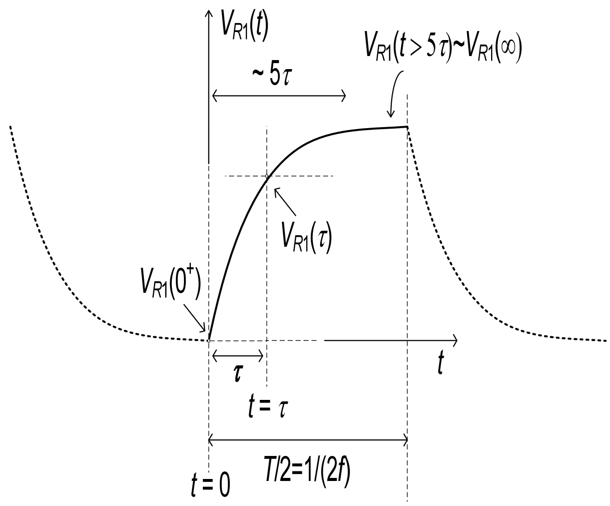

Figure 2. The resistor voltage $V_{R1}(t)$ versus time t.

Lab Experiment 1.b: Repeat Lab Experiment 1.a for the following source frequencies: f = 5 kHz, 30 kHz, 50 kHz, and 100 kHz. Observe $V_S(t)$ and $V_{R1}(t)$ waveforms simultaneously for each case. Sketch and label the waveforms. Explain what happens.

Lab Experiment 1.c: Switch the places of the 15 mH inductor and 1 kΩ resistor as shown in Fig. 3. Use the two oscilloscope channels to observe the voltage waveforms $V_S(t)$ and $V_{L1}(t)$ simultaneously at the same frequencies used in Lab Experiments 1.a and 1.b. Sketch and label the waveforms. Explain what this circuit does and why.

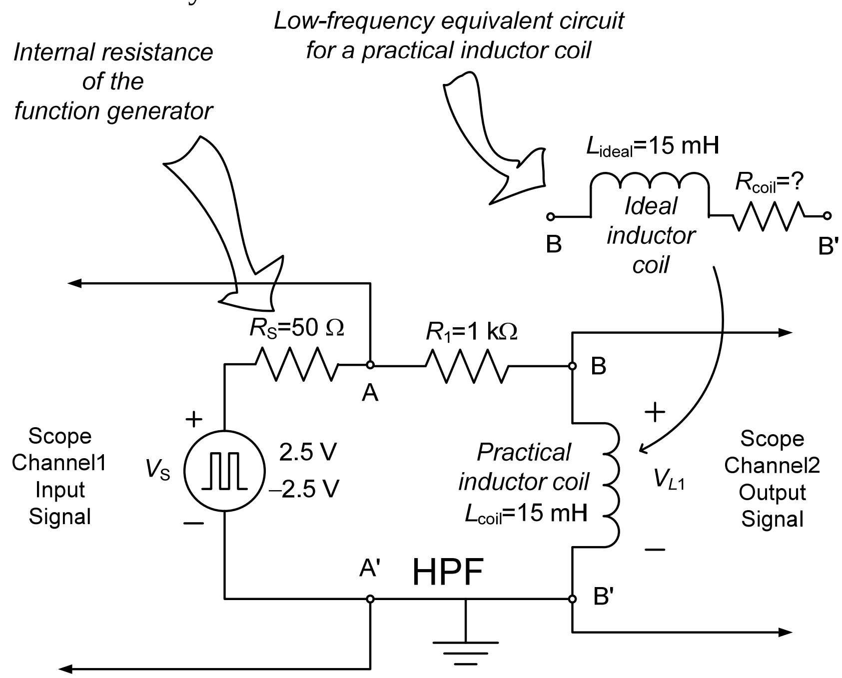

Figure 3. First-order RL circuit connected like a High-Pass Filter (HPF).

PART 2: Step Excitation of Second-Order *RLC* Circuits

Lab Experiment 2: Construct the second-order series *RLC* circuit shown in Fig. 4 using L_{coil}=15 mH and C_1=10 nF. Using the digital *LCR* meter, measure and record the actual value of the capacitor used. Using the actual element values of the circuit measured, do the following:

- Find the characteristic equation of this circuit. Note that the characteristic equation of this second-order series *RLC* circuit is given by
$$s^2 + (R_S + R_{coil})s/L_{coil} + 1/(L_{coil}C_1) = 0$$
- Find the roots of the characteristic equation and verify that the transient response of this circuit will be an under-damped response.
- Calculate the damping frequency f_d of the under-damped response by using the following expression:
$$f_d = \frac{1}{2\pi}\sqrt{\omega_0^2 - \alpha^2} = \frac{1}{2\pi}\sqrt{1/(L_{coil}C_1) - [(R_S + R_{coil})/(2L_{coil})]^2}$$

Next, set the function generator to provide the rectangular pulse train represented with the source voltage $V_S(t)$ which oscillates between –2.5 V and 2.5 V with frequency of oscillation $f = 1/T = 250$ Hz. Use the oscilloscope channels to observe the two voltage waveforms $V_S(t)$ and $V_{C1}(t)$ simultaneously. Sketch and label the waveforms. Explain the difference between the voltage waveform $V_{C1}(t)$ observed in this circuit versus in a first-order *RC* circuit (like the one used in Lab Experiment # 7). Measure the damping frequency f_d of the under-damped oscillations observed in the $V_{C1}(t)$ waveform by measuring the damping period T_d and using $f_d = 1/T_d$. Calculate the percentage error in the measured value of f_d.

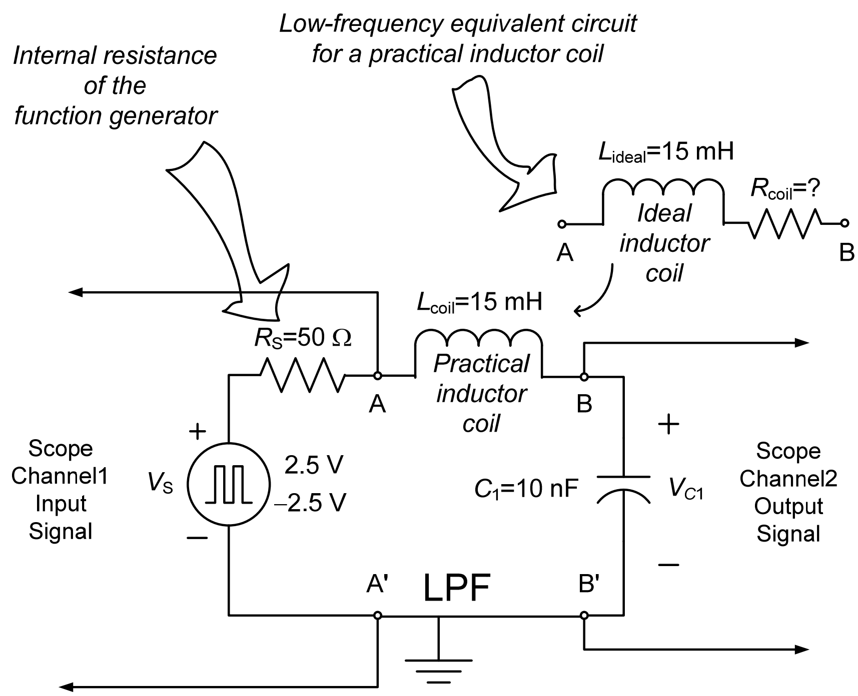

Figure 4. Second-order series *RLC* circuit.

Repeat this experiment at 5 kHz, 10 kHz, and 20 kHz and observe the two voltage waveforms on the oscilloscope simultaneously in each case. Sketch and label the waveforms. Provide an explanation as to what happens to the two waveforms as the source frequency increases.

III. Discussions & Conclusion

In this section, discuss the various aspects of Experiment # 8 and state some conclusions. In your write-up, you should at least address the following questions:

1. What was the objective of this experiment and was the objective achieved?
2. Explain how the output resistance of the function generator affected some of the waveforms observed on the scope and why. Why was this effect not observed in the first-order *RC* experiment (i.e., Experiment # 7)?
3. Did any of your measurements have more than 5% error? What was your maximum % error?
4. What sources of error may have contributed to the differences between the theoretical values and the measured values?
5. Other comments relevant to this experiment.