UNIVERSITY ☺F P☺RTLAND
Sch☺☺l ☺f Engineering

EE 301
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EE 301/Handout # 10
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Transmission Lines: The Real and Imaginary Parts of Z_{in}

The following steps lead to the real and imaginary parts of the input impedance Z_{in} of a lossless transmission line with characteristic impedance Z_0, length l and terminated with a load impedance $Z_L = R_L + jX_L$:

$$Z_{in} = R_{in} + jX_{in} = Z_0 \frac{Z_L + jZ_0 \overbrace{\tan(\beta l)}^{T}}{Z_0 + jZ_L \underbrace{\tan(\beta l)}_{T}} = Z_0 \frac{R_L + jX_L + jZ_0 T}{Z_0 + j(R_L + jX_L)T}$$

$$= Z_0 \frac{R_L + j(X_L + Z_0 T)}{(Z_0 - X_L T) + jR_L T} \times \frac{(Z_0 - X_L T) - jR_L T}{(Z_0 - X_L T) - jR_L T}$$

$$= Z_0 \frac{[R_L(Z_0 - X_L T) + R_L T(X_L + Z_0 T)] + j[(X_L + Z_0 T)(Z_0 - X_L T) - R_L^2 T]}{(Z_0 - X_L T)^2 + (R_L T)^2}$$

$$= \underbrace{Z_0 \frac{R_L Z_0 (1 + T^2)}{(Z_0 - X_L T)^2 + (R_L T)^2}}_{\mathrm{Re}\{Z_{in}\} = R_{in}} + \underbrace{jZ_0 \frac{X_L Z_0 + (Z_0^2 - R_L^2 - X_L^2)T - Z_0 X_L T^2}{(Z_0 - X_L T)^2 + (R_L T)^2}}_{\mathrm{Im}\{Z_{in}\} = X_{in}}$$

Please check my steps one more time to make sure I did it correctly.