4/20/2005

University of Portland
School of Engineering

EE 301-Electromagnetic Fields-3 cr. hrs.
Spring 2005

Midterm Exam # 2
Sinusoidal Steady-State Waves on Transmission Lines
(Prepared by Professor A. S. Inan)

(Monday, April 11, 2005)
(Closed Book Exam; 1 Formula Sheet Allowed)
(Total Time: 55 mins.)

Name: SOLUTIONS! ☺

Go get it! Hurry up!

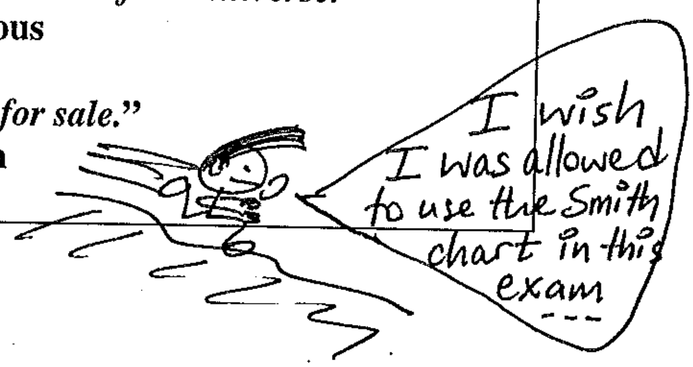
Signature: Solutions ☺

"Honesty is the best policy."
Aesop (~ 620B.C. –?)

"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

"Honest people are the true winners of the universe."
Anonymous

"Honesty is not for sale."
A. Inan



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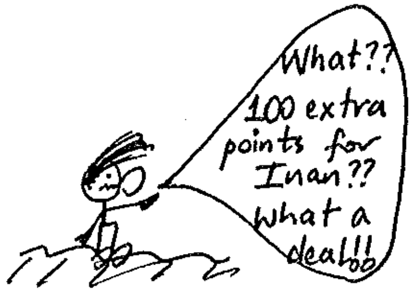
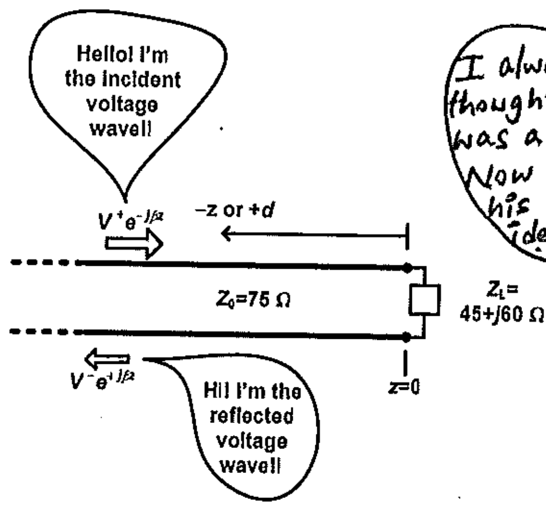
This table will be used by Inan for grading!

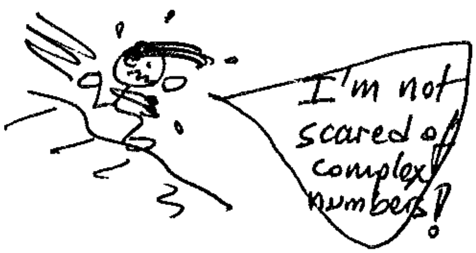
INAN'S GRADE: 200/100



Problem #	Points gained
#1	40
#2	30
#3	30
Total	100 + 100 extra credit to Inan for his solutions!

(1) (*15 mins., <u>Total:</u> 40 points*) **A lossless transmission line terminated with a complex impedance.** A $75\,\Omega$ transmission line is terminated with an inductive load impedance given by $Z_L = 45 + j60\,\Omega$, as shown.





(a) (*10 points*) Calculate the load reflection coefficient, Γ_L. (Provide your answer in polar form.) Show your work!

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{45 + j60 - 75}{45 + j60 + 75}$$

$$= \frac{-30 + j60}{120 + j60} = \frac{-1 + j2}{4 + j2}$$

$$= \frac{\cancel{\sqrt{5}}\, e^{j\tan^{-1}(-2) + j\pi}}{2\cancel{\sqrt{5}}\, e^{j\tan^{-1}(0.5)}} = \boxed{\frac{1}{2} e^{j\pi/2}}$$

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(b) (*10 points*) What is the value of the standing wave ratio, S, on the line?

$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.5}{1-0.5} = \boxed{3}$$

(c) (*10 points*) Find the percentage time-average incident power that is absorbed by the load.

% time-average incident power delivered to the load can be calculated as

$$(1-|\Gamma_L|^2)\times 100 = (1-(0.5)^2)\times 100$$

$$= \boxed{75\%}$$

(d) (*10 points*) Find the V_{max} and V_{min} positions nearest to the load. Provide your answers as electrical lengths.

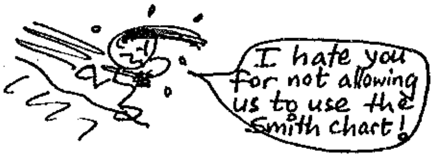
$$\phi_L = \frac{\pi}{2}$$

$\frac{2\pi}{\lambda}$

$$\phi_L + 2\beta z_{max} = 0 \rightarrow z_{max}/\lambda = \boxed{-0.125}$$

$$\phi_L + 2\beta z_{min} = -\pi \rightarrow z_{min}/\lambda = \boxed{-0.375}$$

Note that as expected, $|z_{max} - z_{min}|/\lambda = 0.25$



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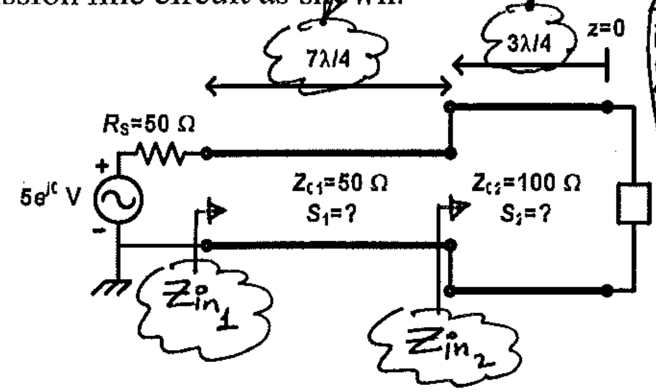
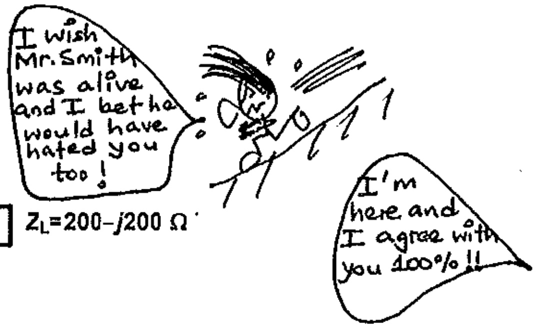
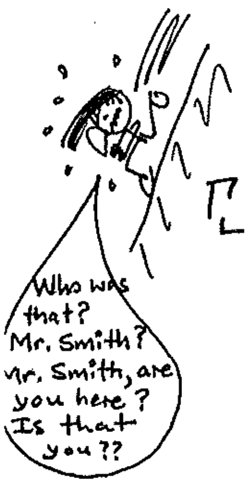
(2) (*15 mins.*, *Total: 30 points*) **Two cascaded transmission lines.** Consider the transmission line circuit as shown.

$\lambda/4$?





(a) (*15 points*) Find the standing wave ratio on each line. Show your work!

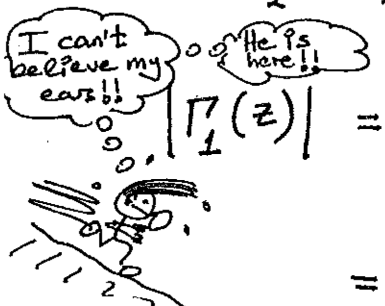


$$\Gamma_L = \frac{Z_L - Z_{02}}{Z_L + Z_{02}} = \frac{200 - \hat{j}200 - 100}{200 - \hat{j}200 + 100} = \frac{10\cancel{0} - \hat{j}20\cancel{0}}{30\cancel{0} - \hat{j}20\cancel{0}}$$

$$= \frac{\sqrt{5}\, e^{\hat{j}\tan^{-1}(-2)}}{\sqrt{13}\, e^{\hat{j}\tan^{-1}(-2/3)}} \cong 0.62\, e^{-\hat{j}29.7^\circ}$$

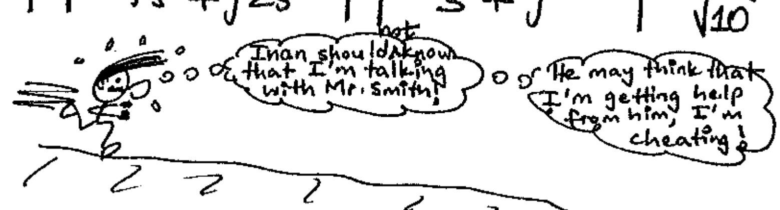
$$\therefore S_2 = \frac{1 + |\Gamma_2(z)|}{1 - |\Gamma_2(z)|} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \cong \frac{1 + 0.62}{1 - 0.62} \cong \boxed{4.27}$$

$$Z_{in_2} = \frac{Z_{02}^2}{Z_L} = \frac{(100)^2}{200 - \hat{j}200} = \frac{50}{1 - \hat{j}} = 25 + \hat{j}25\ \Omega$$

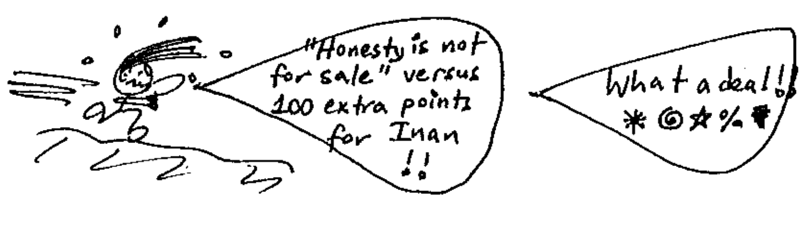


$$|\Gamma_1(z)| = \left|\frac{25 + \hat{j}25 - 50}{25 + \hat{j}25 + 50}\right| = \left|\frac{-25 + \hat{j}25}{75 + \hat{j}25}\right| = \left|\frac{-1 + \hat{j}}{3 + \hat{j}}\right| = \frac{\sqrt{2}}{\sqrt{10}}$$

$$= \frac{1}{\sqrt{5}} \cong 0.447$$



$$\therefore S_1 = \frac{1 + |\Gamma_1(z)|}{1 - |\Gamma_1(z)|} \cong \frac{1 + 0.447}{1 - 0.447} \cong \boxed{2.62}$$



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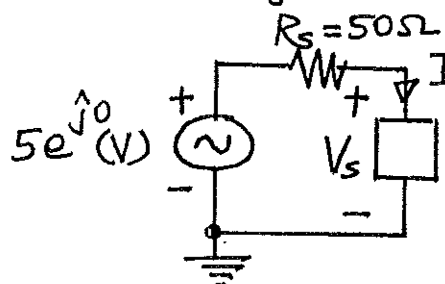
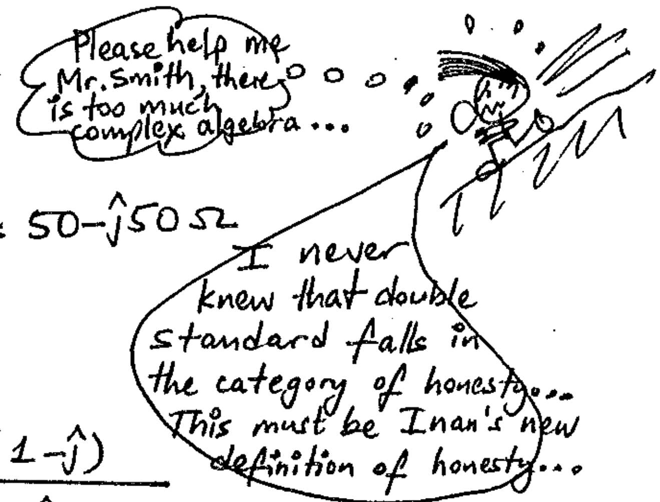
(b) (*15 points*) Find the time-average power delivered to the load impedance.

$$Z_{in_1} = \frac{Z_{01}^2}{Z_{in_2}} = \frac{(50)^2}{25+\hat{j}25} = \frac{100}{(1+\hat{j})}\cdot\frac{(1-\hat{j})}{(1-\hat{j})}$$

$$= 50-\hat{j}50\ \Omega$$





$$V_s = \frac{50-\hat{j}50}{50+50-\hat{j}50}(5) = \frac{5(1-\hat{j})}{(2-\hat{j})}$$

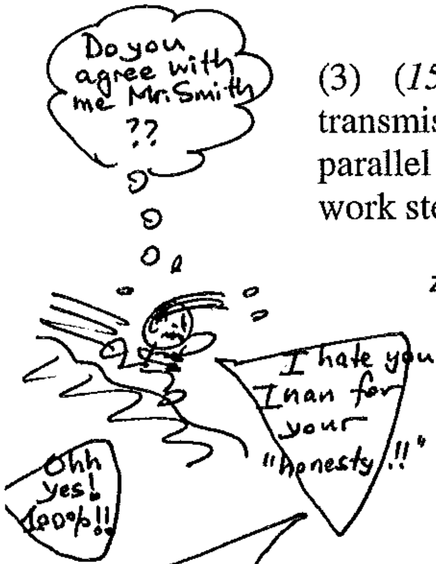
$$= (1-\hat{j})(2+\hat{j}) = 3-\hat{j} \cong 3.16e^{-\hat{j}18.4^\circ}\ (\text{V})$$

$$I_s = \frac{V_s}{Z_{in_1}} = \frac{3-\hat{j}}{50-\hat{j}50} = \frac{(3-\hat{j})(1+\hat{j})}{50(1-\hat{j})(1+\hat{j})}$$

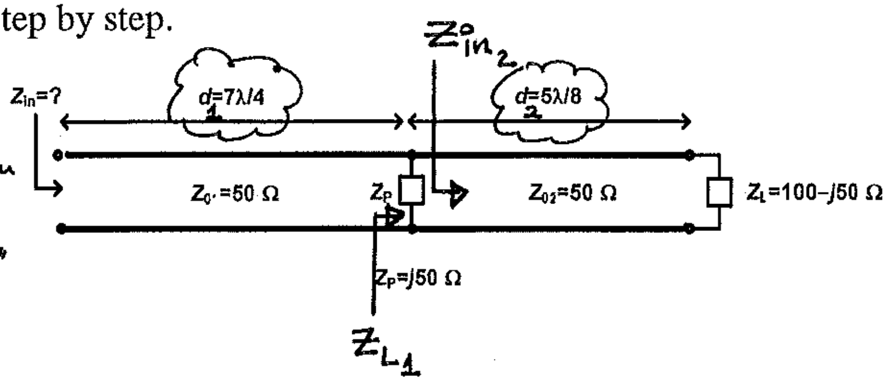
$$= \frac{4+\hat{j}2}{100} = \frac{2+\hat{j}}{50} \cong 44.7e^{\hat{j}26.6^\circ}\ (\text{mA})$$

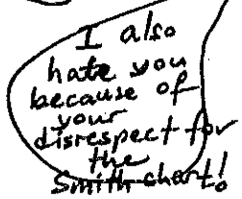
$$\therefore P_{Z_{in_1}} = P_L = \frac{1}{2}|V_s|\,|I_s|\cos(\angle V_s - \angle I_s)$$

$$\cong \frac{1}{2}(3.16)(44.7)\cos(-18.4^\circ - 26.6^\circ) \cong \boxed{50\text{ mW}}$$



(3) (*15 mins., 30 points*) **Input impedance.** Consider the transmission line circuit as shown where Z_P impedance represents a parallel lumped element. Find the input impedance Z_{in}. Show your work step by step.





$$Z_{in_2} = Z_{02}\frac{Z_L+\hat{j}Z_{02}\tan(\beta d_2)}{Z_{02}+\hat{j}Z_L\tan(\beta d_2)} = (50)\frac{(100-\hat{j}50)+\hat{j}50\overset{1}{\cancel{\tan\left(\frac{2\pi}{\lambda}\cdot\frac{5\lambda}{8}\right)}}}{50+\hat{j}(100-\hat{j}50)\underset{1}{\cancel{\tan\left(\frac{2\pi}{\lambda}\cdot\frac{5\lambda}{8}\right)}}}$$

$$= (50)\frac{100}{100+\hat{j}100} = \frac{50(1-\hat{j})}{(1+\hat{j})(1-\hat{j})} = 25-\hat{j}25\,\Omega$$

$$Z_{L_1} = \frac{Z_P\cdot Z_{in_2}}{Z_P+Z_{in_2}} = \frac{(\hat{j}50)(25-\hat{j}25)}{\hat{j}50+25-\hat{j}25}$$

$$= \frac{\hat{j}50(1-\hat{j})}{(1+\hat{j})}\cdot\frac{(1-\hat{j})}{(1-\hat{j})} = \frac{(\hat{j}50)(-\hat{j}2)}{2} = 50\,\Omega\,!$$

$\therefore$ Regardless of the length of d_1, $\boxed{Z_{in} = 50\,\Omega}$