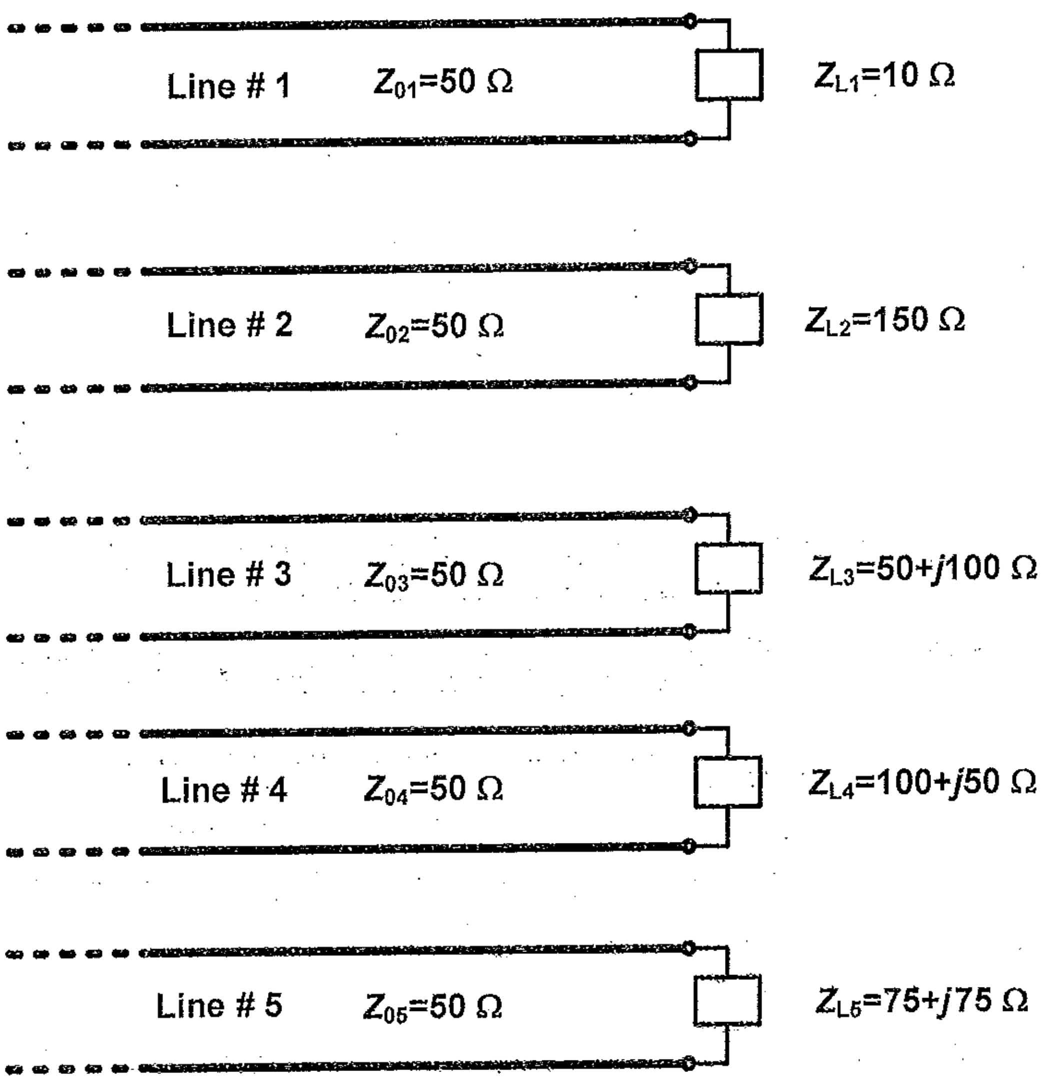
University of Portland
School of Engineering

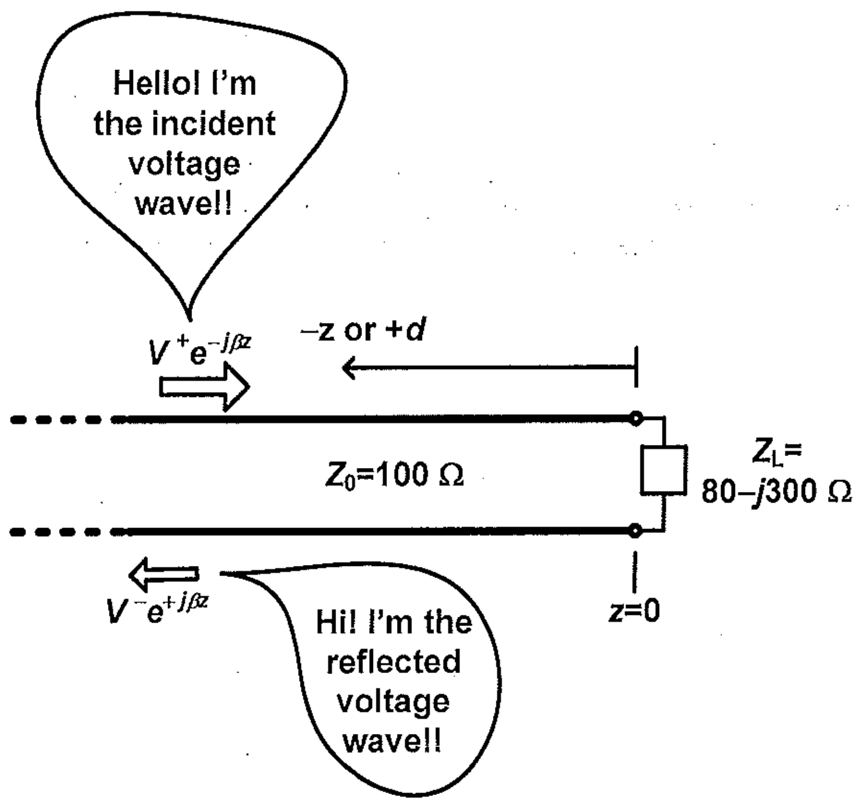
EE 301
Spring 2006
A.Inan

Homework # 6
(Assigned: March 24, 2006)
Sinusoidal Steady-State (SSS) Waves on Transmission Lines-II
(Due: Friday, March 31, 2006, 11:25a.m.)

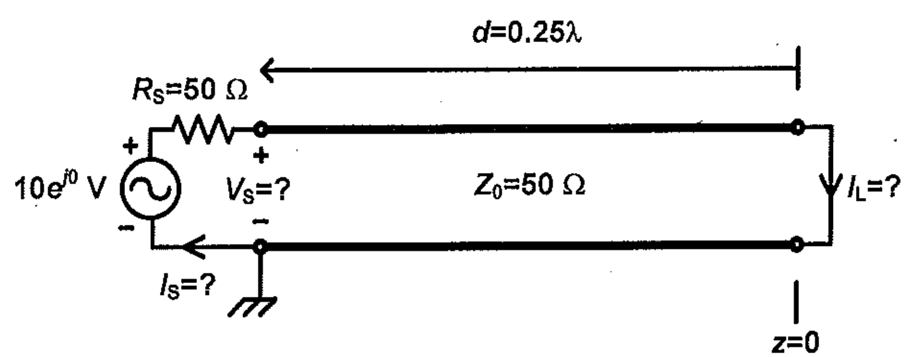
(1) **Standing wave ratio.** Calculate the standing wave ratio S on each line shown below and find out which line has the largest S value.



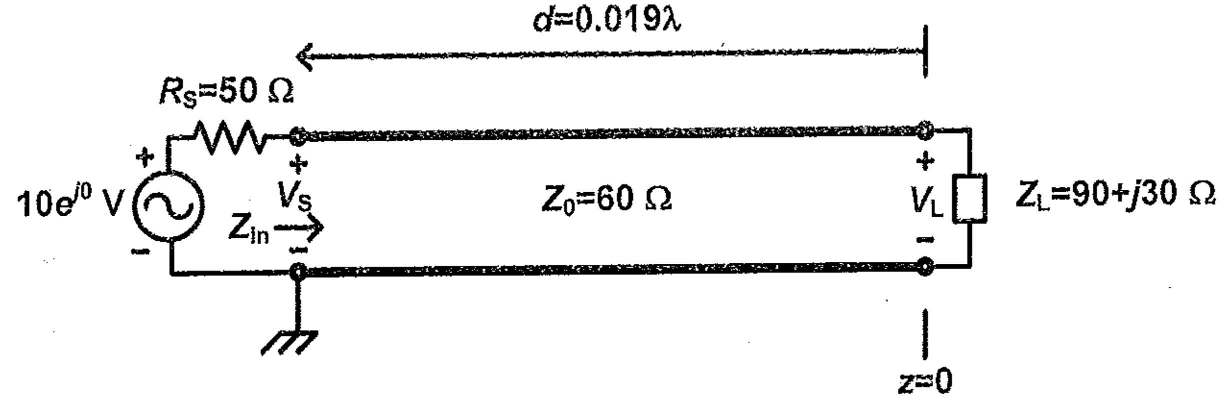
(2) **A lossless transmission line terminated with a complex impedance.** A 100 Ω transmission line is terminated with a capacitive load having a load impedance given by $Z_L = 80 - j300\,\Omega$, as shown. (a) Find the load reflection coefficient Γ_L. (b) Find the standing wave ratio S on the line. (c) Find the nearest V_{max} position with respect to the load. (d) Find the nearest V_{min} position with respect to the load. (e) Find the nearest position with respect to the load where the line impedance is given by $Z_{line} = 100 + jX$. (Note that your answers for parts (c), (d) and (e) will be electrical lengths or positions.)



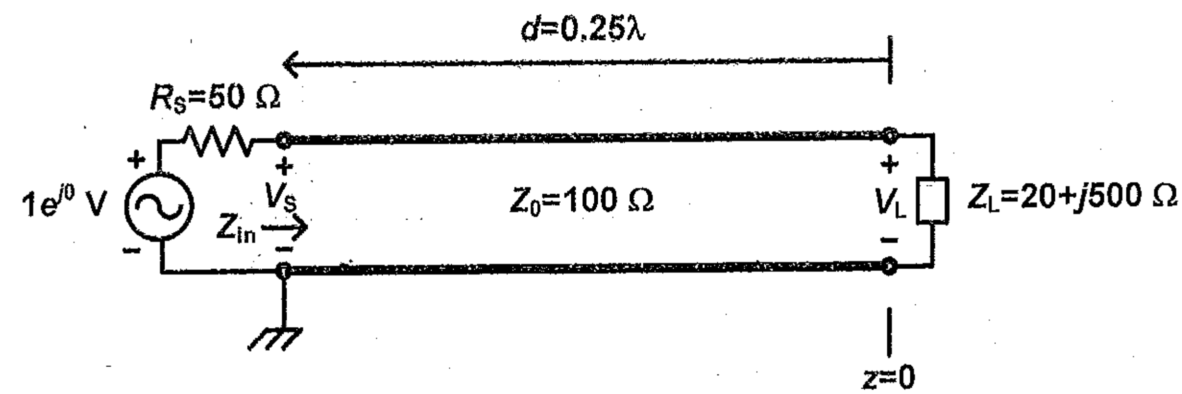
(3) **A lossless transmission line terminated with a short circuit.** Consider a short-circuit terminated 50 Ω quarter-wavelength transmission line excited by a sinusoidal voltage source as shown. Calculate the source-end phasor voltage V_S, source-end phasor current I_S, and load-end phasor current I_L.



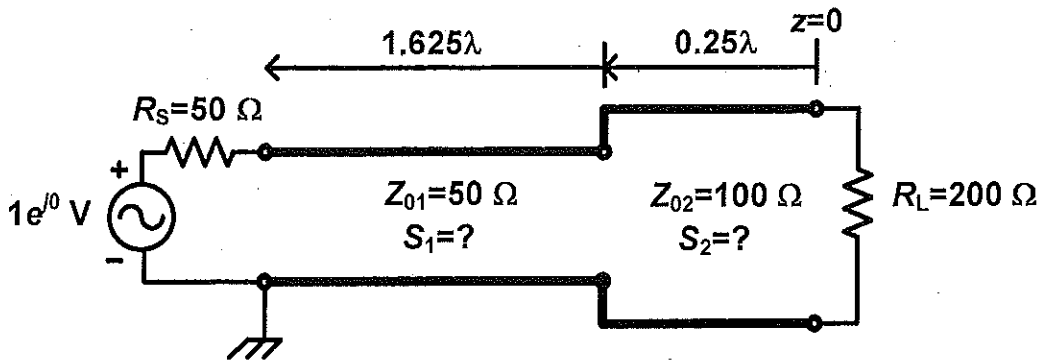
(4) **A lossless transmission line terminated with an inductive load.** A 60 Ω transmission line is terminated with an inductive load having a load impedance given by $Z_L = 90 + j30\,\Omega$, as shown. (a) Find the load reflection coefficient Γ_L. (b) Find the standing wave ratio S on the line. (c) Find the input impedance Z_{in}. (d) Find the phasor voltages V_S and V_L. (e) Find the time-average power delivered to the load.



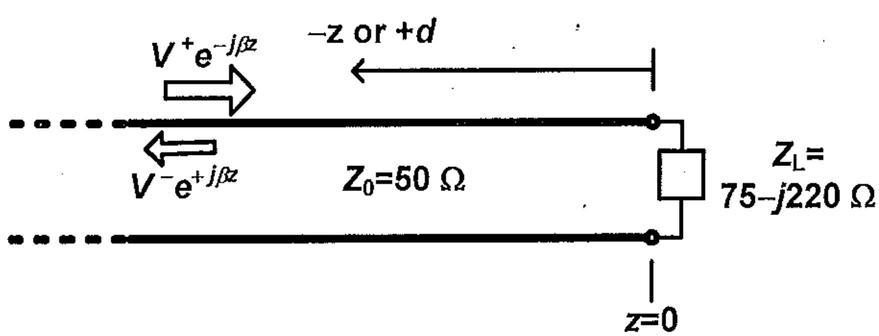
(5) **A lossless transmission line terminated with a complex impedance.** A 100 Ω transmission line is terminated with an inductive load having a load impedance given by $Z_L = 20 + j500\,\Omega$, as shown. (a) Find the load reflection coefficient Γ_L. (b) Find the standing wave ratio S on the line. (c) Find the total time-average power supplied by the sinusoidal voltage source and how this power splits between the source resistance and the load impedance. (d) Repeat part (c) for $d = 0.5\lambda$.



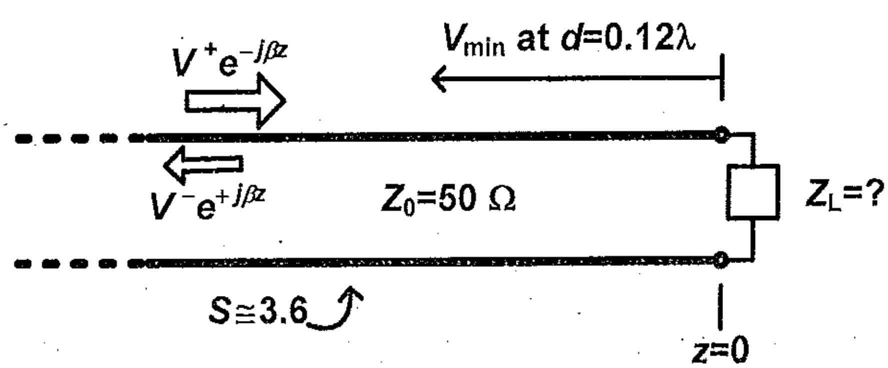
(6) **Cascaded transmission lines.** For the two cascaded transmission lines shown, (a) find the standing wave ratio on each line; and (b) the time-average power delivered to the load resistance.



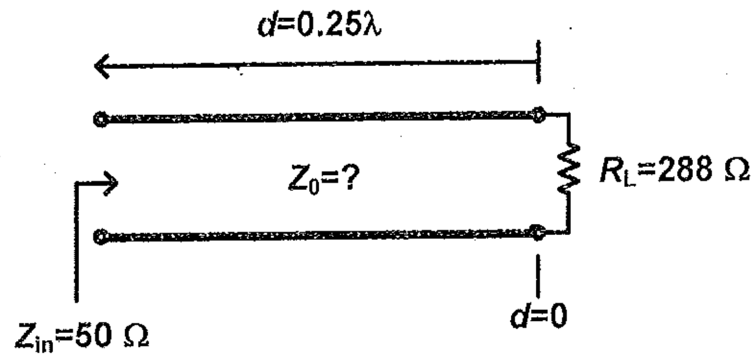
(7) **Resistive line impedance.** A 50 Ω transmission line is terminated with a capacitive load $Z_L = 75 - j220\,\Omega$, as shown. (a) Find the load reflection coefficient Γ_L and the standing wave ratio S on the line. (b) What percentage of the incident power reflects back from the load? (c) Find the two nearest positions on the line with respect to the load where the line impedance is purely real. (d) Calculate the line impedances at the positions found in part (c).



(8) **Unknown load impedance.** If the standing wave ratio and the voltage minimum position on a 50 Ω transmission line terminated with an unknown load impedance are measured to be $S \cong 3.6$ and $d = 0.12\lambda$, determine the value of the unknown load impedance.



(9) Unknown characteristic impedance. If the input impedance of a quarter-wavelength long lossless transmission line terminated with a resistive load R_L=288 Ω is measured to be Z_{in}=50 Ω, find the characteristic impedance, Z_0, of the line.



<u>Please use the following guidelines for your homework solutions:</u>

1) On the first sheet, at the top, indicate that this is <u>EE 301/Spring 2006/HW #6 Solutions</u> and provide <u>your name</u> somewhere on that sheet where the grader can easily see it.
2) Solve each problem on a separate sheet unless the solution is very short.
3) Do not use the back of the sheets unless you have to.
4) Staple your solutions in the above order before you turn them in.

Please turn in your homework solutions on time. The solutions (or answers) for each homework assignment will be provided as a separate handout on the due date. **<u>Late homework solutions will not be accepted!</u>**

<u>An important reminder note:</u>

<u>EE 301-Midterm # 2 is scheduled to be given on Friday, April 7, 2006</u>

(It will be in-class closed-book exam. One formula sheet will be allowed.)

EE 301
SPRING 2006
A. INAN

4/12/2006

SOLUTIONS TO HOMEWORK # 6

Lets goooo!

#1

$$\Gamma_{L_1} = \frac{Z_{L_1} - Z_{01}}{Z_{L_1} + Z_{01}} = \frac{10-50}{10+50} = -\frac{2}{3} = \frac{2}{3} e^{\hat{j}\pi}$$

$$S_1 = \frac{1+|\Gamma_{L_1}|}{1-|\Gamma_{L_1}|} = \frac{1+2/3}{1-2/3} = \boxed{5}$$

$$\Gamma_{L_2} = \frac{150-50}{150+50} = \frac{1}{2}$$

$$S_2 = \frac{1+|\Gamma_{L_2}|}{1-|\Gamma_{L_2}|} = \frac{1+1/2}{1-1/2} = \boxed{3}$$

$$\Gamma_{L_3} = \frac{\cancel{50}+\hat{j}100\cancel{-50}}{50+\hat{j}100+50} = \frac{\hat{j}100}{100+\hat{j}100} = \frac{\hat{j}}{1+\hat{j}} \cdot \frac{(1-\hat{j})}{(1-\hat{j})}$$

$$= \frac{1+\hat{j}}{2} = \frac{1}{\sqrt{2}} e^{\hat{j}\pi/4}$$

$$S_3 = \frac{1+|\Gamma_{L_3}|}{1-|\Gamma_{L_3}|} = \frac{1+1/\sqrt{2}}{1-1/\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \cong \boxed{5.83}$$

$$\Gamma_{L_4} = \frac{100+\hat{j}50-50}{100+\hat{j}50+50} = \frac{50(1+\hat{j})}{50(3+\hat{j})} \cdot \frac{(3-\hat{j})}{(3-\hat{j})}$$

$$= \frac{4+\hat{j}2}{10} = \frac{2+\hat{j}}{5} = \frac{1}{\sqrt{5}} e^{\hat{j}\tan^{-1}(1/2)} \cong 0.447 e^{\hat{j}26.6^\circ}$$

$$S_4 = \frac{1+|\Gamma_{L4}|}{1-|\Gamma_{L4}|} = \frac{1+1/\sqrt{5}}{1-1/\sqrt{5}}$$

$$= \frac{\sqrt{5}+1}{\sqrt{5}-1} \cong \boxed{2.62}$$

$$\Gamma_{L5} = \frac{75+\hat{j}75-50}{75+\hat{j}75+50} = \frac{25+\hat{j}75}{125+\hat{j}75} = \frac{1+\hat{j}3}{5+\hat{j}3}$$

$$= \frac{(1+\hat{j}3)(5-\hat{j}3)}{5^2+3^2} = \frac{14+\hat{j}12}{34} = \frac{7+\hat{j}6}{17}$$

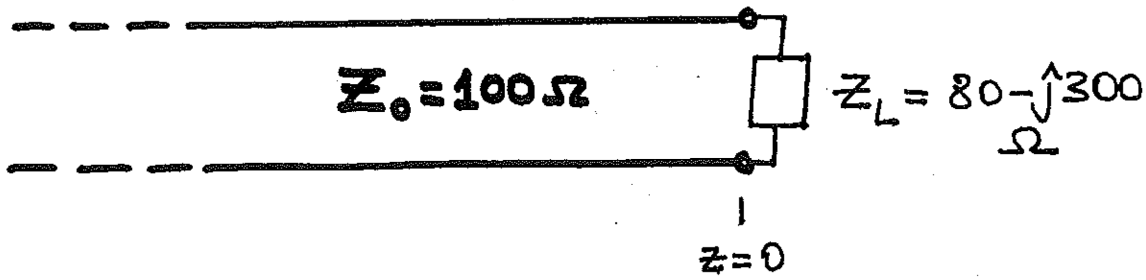
$$= \sqrt{\frac{5}{17}}\, e^{\hat{j}\tan^{-1}(6/7)} \cong 0.542\, e^{\hat{j}40.6^\circ}$$

$$S_5 = \frac{1+|\Gamma_{L5}|}{1-|\Gamma_{L5}|} = \frac{1+\sqrt{5/17}}{1-\sqrt{5/17}}$$

$$= \frac{\sqrt{17}+\sqrt{5}}{\sqrt{17}-\sqrt{5}} \cong \boxed{3.37}$$

$\therefore$ Line # 3 has the largest S value which is $S \cong 5.83$.

#2



(a)
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(80-\hat{j}300) - 100}{(80-\hat{j}300) + 100}$$

$$= \frac{-20 - \hat{j}300}{180 - \hat{j}300} = \frac{-1-\hat{j}15}{9-\hat{j}15}$$

$$= \frac{\sqrt{226}\; e^{\hat{j}\tan^{-1}(-15/-1) + \hat{j}\pi}}{\sqrt{306}\; e^{\hat{j}\tan^{-1}(-15/9)}}$$

$$= \sqrt{\frac{113}{153}}\; e^{\hat{j}\tan^{-1}(15) + \hat{j}\pi - \hat{j}\tan^{-1}(-15/9)}$$

$$\cong \boxed{0.859\, e^{-\hat{j}34.8^\circ}}$$

(b)
$$S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \cong \frac{1 + 0.859}{1 - 0.859} \cong \boxed{13.2}$$

(c)
$$-34.8^\circ \times \frac{\pi}{180^\circ} + 4\pi\left(\frac{z_{max}}{\lambda}\right) = -2\pi$$

$$\rightarrow z_{max} \cong \boxed{-0.4517\,\lambda}$$

(d) $z_{min} = z_{max} + 0.25\lambda \cong \boxed{-0.2017\lambda}$

(e) $Re\{Z_{in}\} = R_{in} = 100\,\Omega$

From Inan's Z_{in} handout!

$$\frac{R_L Z_0^2 (1+T^2)}{(Z_0 - X_L T)^2 + R_L^2 T^2} = \frac{80(100)^2(1+T^2)}{(100-(-300)T)^2 + (80)^2 T^2} = 100$$

$$8{,}0\cancel{0}\cancel{0}(1+T^2) = (10\cancel{0} + 30\cancel{0}T)^2 + 6{,}4\cancel{0}\cancel{0}T^2$$

$$80 + 80T^2 = 100 + 600T + 900T^2 + 64T^2$$

$$884T^2 + 600T + 20 = 0$$

$$221T^2 + 150T + 5 = 0$$

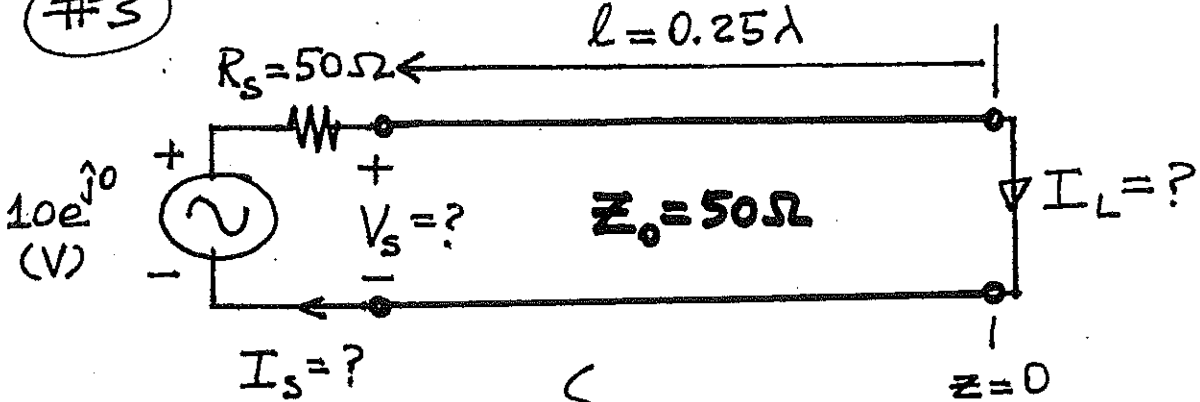
$$T_1, T_2 = \frac{-75 \pm \sqrt{(75)^2 - 5(221)}}{221} \cong \begin{matrix} -3.515 \times 10^{-2} \\ -6.436 \times 10^{-1} \end{matrix}$$

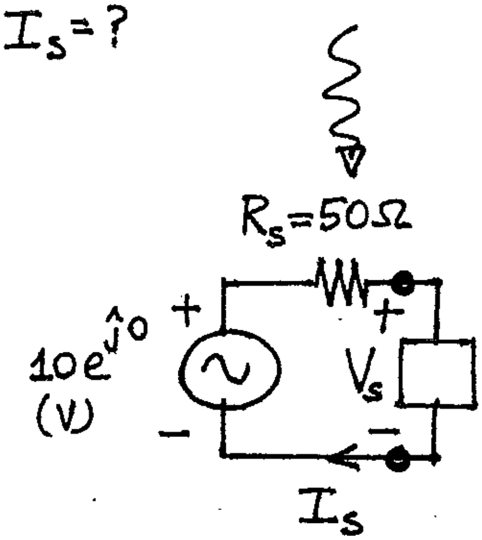
Since $T = \tan(\beta l) = \tan\left(2\pi \frac{l}{\lambda}\right)$, we have

$$l_1, l_2 \cong 0.4944,\ 0.4090$$

$\therefore$ The nearest position with respect to the load where $Z_{line} = 100 + jX$ is $\boxed{l_{nearest} \cong 0.409\lambda.}$

#3





$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{0} = \infty \ !$$

$$\therefore \boxed{I_s = 0} \ \& \ \boxed{V_s = 10e^{\hat{j}0} \text{ (V)}}$$

Since $V(\ell) = V^+ e^{\hat{j}\beta\ell} + V^- e^{-\hat{j}\beta\ell}$

$$= V^+ e^{\hat{j}2\pi\ell/\lambda} + V^- e^{-\hat{j}2\pi\ell/\lambda}$$

At $\ell = 0.25\lambda \rightarrow V(\ell = 0.25\lambda) = V^+ e^{\hat{j}\pi/2} + V^- e^{-\hat{j}\pi/2}$

$$= V^+ \hat{j} + V^-(-\hat{j}) = V_s$$

Since $\Gamma_L = -1 \rightarrow V^- = -V^+$, therefore

$$V(\ell = 0.25\lambda) = 2\hat{j}V^+ = V_s = 10e^{\hat{j}0} \rightarrow V^+ = \frac{10e^{\hat{j}0}}{2\hat{j}} = 5e^{-\hat{j}\pi/2} \text{ (V)}$$

The total current $I(\ell)$ is given by

$$I(\ell) = I^{+} e^{\hat{j}\beta\ell} + I^{-} e^{-\hat{j}\beta\ell} = \frac{V^{+}}{Z_0} e^{\hat{j}\beta\ell} - \frac{V^{-}}{Z_0} e^{-\hat{j}\beta\ell}$$

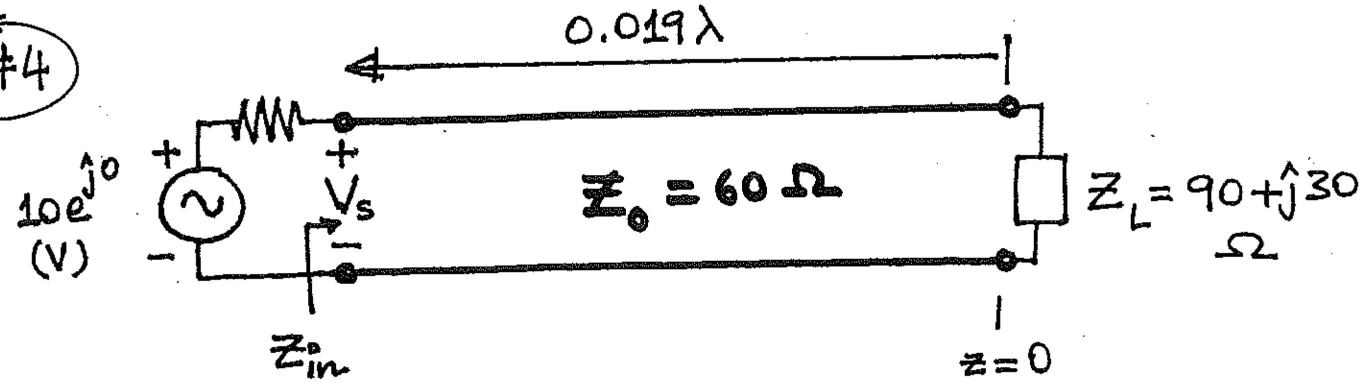
$$= \frac{V^{+}}{Z_0}\left(e^{\hat{j}\beta\ell} + e^{-\hat{j}\beta\ell}\right) = \frac{2V^{+}}{Z_0}\cos(\beta\ell)$$

$$\therefore\ I(\ell) = \frac{2 \times 5 e^{-\hat{j}\pi/2}}{50}\cos(\beta\ell)$$

$$= 0.2\, e^{-\hat{j}\pi/2}\cos(\beta\ell)$$

$$\therefore\ I(\ell = 0) = I_L = 0.2 e^{-\hat{j}\pi/2} = \boxed{-\hat{j}0.2 \text{ A}}$$

#4



(a) $$\Gamma_L = \frac{90+\hat{j}30-60}{90+\hat{j}30+60} = \frac{30+\hat{j}30}{150+\hat{j}30} = \frac{1+\hat{j}}{5+\hat{j}}$$

$$= \frac{\sqrt{2}\, e^{\hat{j}45^\circ}}{\sqrt{26}\, e^{\hat{j}\tan^{-1}(0.2)}} \cong \boxed{0.277 e^{\hat{j}33.7^\circ}}$$

(b) $$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \cong \frac{1+0.277}{1-0.277} \cong \boxed{1.77}$$

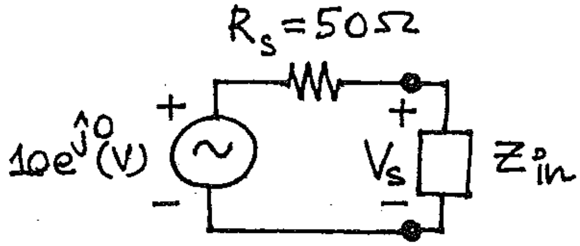
(c) $$Z_{in} = 60\, \frac{(90+\hat{j}30) + \hat{j}60 \tan\left(\frac{2\pi}{\lambda}(0.019\lambda)\right)}{60+\hat{j}(90+\hat{j}30)\tan\left(\frac{2\pi}{\lambda}(0.019\lambda)\right)}$$

$$\cong 60\, \frac{(90+\hat{j}30) + \hat{j}60(0.120)}{60+\hat{j}90(0.120) - 30(0.120)}$$

$$\cong 60\, \frac{90+\hat{j}37.2}{56.4+\hat{j}10.8}$$

$$\cong 102\, e^{\hat{j}11.6^\circ} \cong \boxed{(99.7+\hat{j}20.5)\ \Omega}$$

(d)



Using the voltage divider principle, we have

$$V_s = \frac{Z_{in}}{R_s + Z_{in}}\left(10e^{\hat{j}0}\right) \cong \frac{99.7 + \hat{j}20.5}{149.7 + \hat{j}20.5}\left(10e^{\hat{j}0}\right)$$

$$\cong 6.736\, e^{\hat{j}3.82^\circ} \text{ (V)}$$

Since $V(\ell) = V^+ e^{\hat{j}\beta\ell} + V^- e^{-\hat{j}\beta\ell}$

$$= V^+ e^{\hat{j}\frac{2\pi}{\lambda}\cdot\ell}\left(1 + \Gamma_L\, e^{-\hat{j}2\frac{2\pi}{\lambda}\ell}\right)$$

$$V(\ell = 0.019\lambda) \cong V^+ e^{\hat{j}6.84^\circ}\left(1 + 0.277\, e^{\hat{j}33.7^\circ} e^{-\hat{j}13.7^\circ}\right)$$

$$= V_s \cong 6.736\, e^{\hat{j}3.82^\circ}$$

$$\rightarrow V^+ \cong \frac{6.736\, e^{\hat{j}3.82^\circ}}{e^{\hat{j}6.84^\circ}\left(\underbrace{1 + 0.277\, e^{\hat{j}20.0^\circ}}_{\sim 1.26 + \hat{j}0.0949}\right)}$$

$$\cong 5.33\, e^{-\hat{j}7.32^\circ}$$

$$\therefore V(\ell) \cong 5.33\, e^{-\hat{j}7.32^\circ} e^{\hat{j}\frac{2\pi}{\lambda}\ell}\left(1 + 0.277\, e^{\hat{j}33.7^\circ} e^{-\hat{j}2\frac{2\pi}{\lambda}\ell}\right)$$

The load-end voltage can be calculated as

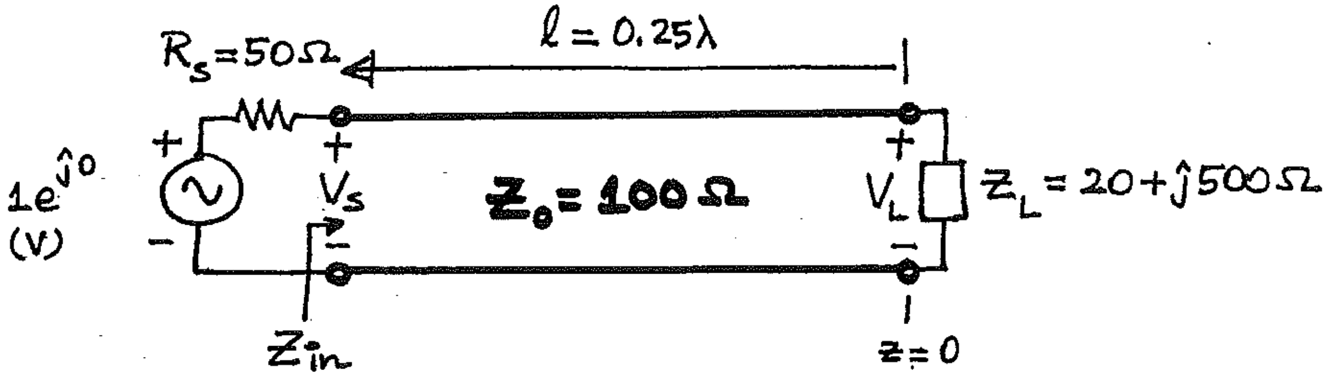
$$V_L = V(\ell = 0) \cong 5.33 e^{-\hat{j}7.32^\circ} \underbrace{\left(1 + 0.277 e^{\hat{j}33.7^\circ}\right)}_{\sim 1.23 + \hat{j}0.154}$$

$$\cong 5.33 e^{-\hat{j}7.32^\circ}\left(1.24 e^{\hat{j}7.125^\circ}\right) \cong \boxed{6.61 e^{-\hat{j}0.198^\circ} (\mathrm{V})}$$

(e) $$P_{Z_{in}} = P_{Z_L} = \frac{1}{2}\frac{|V_s|^2}{|Z_{in}|}\cos(\angle Z_{in})$$

$$\cong \frac{1}{2}\frac{(6.736)^2}{(102)}\cos(11.62^\circ) \cong \boxed{0.218\ \mathrm{W}}$$

#5



(a) $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{20 + \hat{j}500 - 100}{20 + \hat{j}500 + 100}$$

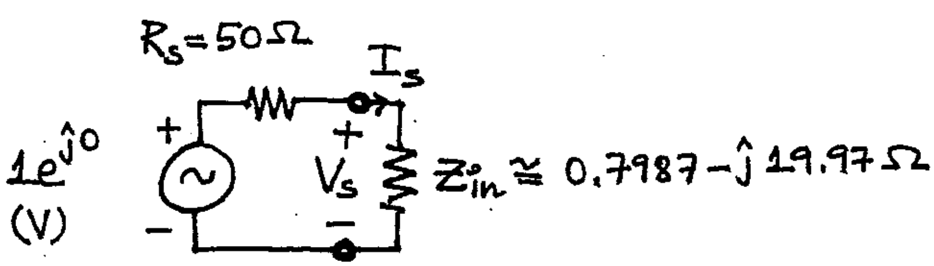
$$= \frac{-8\cancel{0} + \hat{j}50\cancel{0}}{12\cancel{0} + \hat{j}50\cancel{0}} = \frac{-4 + \hat{j}25}{6 + \hat{j}25}$$

$$= \frac{\sqrt{641}\, e^{\hat{j}\tan^{-1}(-25/4) + \hat{j}\pi}}{\sqrt{661}\, e^{\hat{j}\tan^{-1}(25/6)}} \cong \boxed{0.985 e^{\hat{j}22.6^\circ}}$$

(b) $$S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \cong \frac{1 + 0.985}{1 - 0.985} \cong \boxed{130.2}$$

(c) $$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(100)^2}{20 + \hat{j}500} = \frac{500}{1 + \hat{j}25} \cdot \frac{1 - \hat{j}25}{1 - \hat{j}25}$$

$$= \frac{500(1 - \hat{j}25)}{1^2 + (25)^2} \cong 0.7987 - \hat{j}19.97\,\Omega$$



Using Ohm's law & KVL $\Rightarrow$ $I_s = \dfrac{1e^{\hat{j}0}\ (\text{V})}{(50 + 0.7987 - \hat{j}\,19.97)\ \Omega}$

$$\cong 18.32\, e^{\hat{j}21.46°}\ (\text{mA})$$

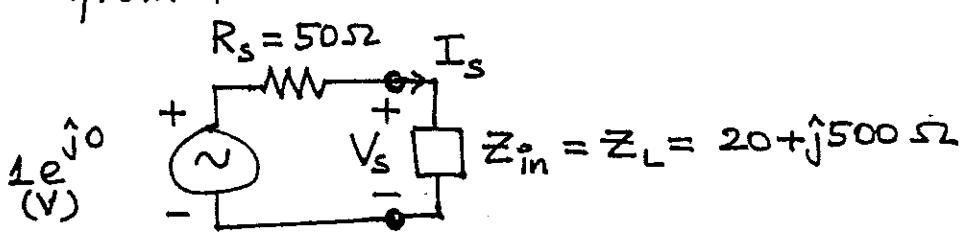
$$\therefore\ P_{R_s} = \frac{1}{2} R_s |I_s|^2 \cong \frac{1}{2}(50)(18.32\text{m})^2 \cong \boxed{8.39\ \text{mW}}$$

$$\&\ P_{R_L} = P_{Z_{in}} = \frac{1}{2} R_{in} |I_s|^2 \cong \frac{1}{2}(0.7987)(18.32\text{m})^2$$

$$\cong \boxed{134\ \mu\text{W}}$$

$$\therefore\ P_{TOTAL} = P_{R_s} + P_{R_L} \cong \boxed{8.52\ \text{mW}}$$

(d) For $\ell = 0.5\lambda$, the Thevenin equivalent circuit seen from the source end will be



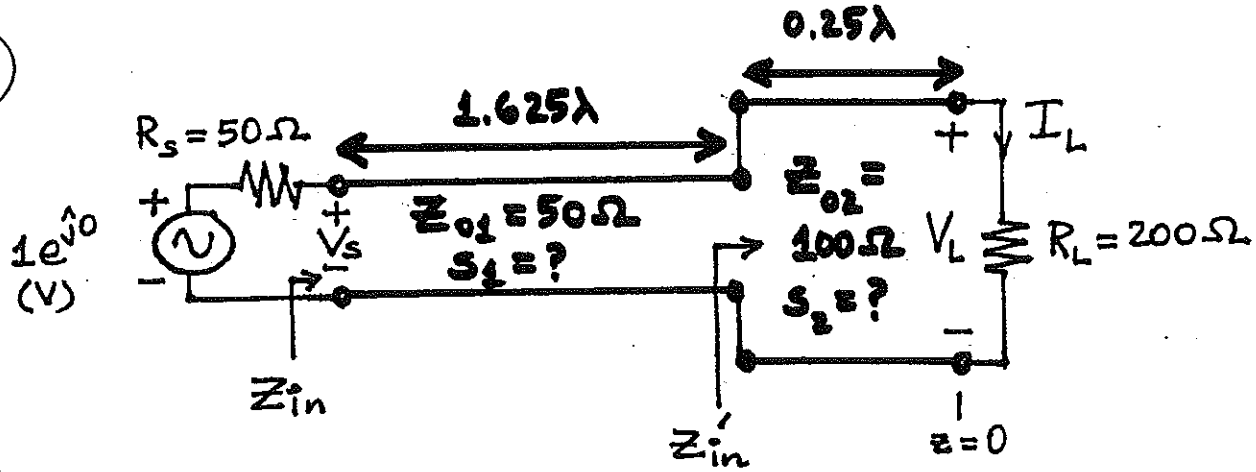
$$I_s = \frac{1e^{\hat{j}0}\ (\text{V})}{(50 + 20 + \hat{j}500)\ \Omega} \cong 1.98\, e^{-\hat{j}82.03°}\ (\text{mA})$$

$$\therefore\ P_{R_s} \cong \frac{1}{2}(50)(1.98\text{m})^2 \cong \boxed{98\ \mu\text{W}}$$

$$\&\ P_{R_L} \cong \frac{1}{2}(20)(1.98\text{m})^2 \cong \boxed{39.2\ \mu\text{W}}$$

$$\therefore\ P_{TOTAL} \cong \boxed{137\ \mu\text{W}}$$

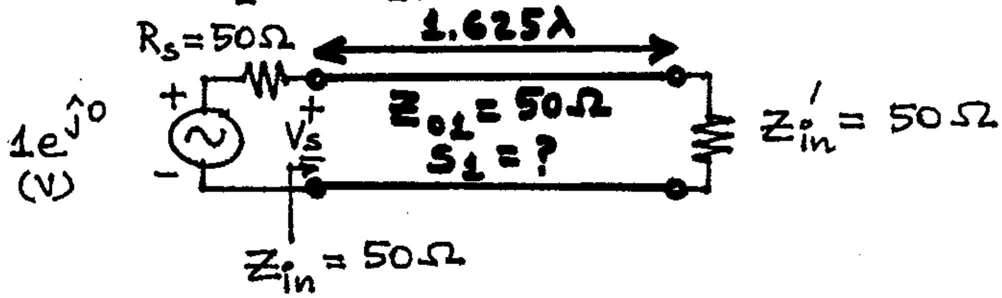
#6



(a) $$\Gamma_L = \frac{R_L - Z_{02}}{R_L + Z_{02}} = \frac{200-100}{200+100} = \frac{1}{3}$$

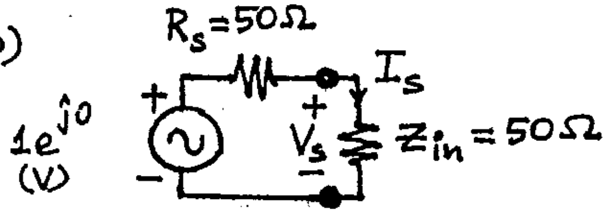
$$\therefore\ S_2 = \frac{1+|\Gamma_2(z)|}{1-|\Gamma_2(z)|} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+1/3}{1-1/3} = \frac{4/3}{2/3} = \boxed{2}$$

$$Z_{in}' = \frac{Z_{02}^2}{R_L} = \frac{(100)^2}{200} = 50\ \Omega$$



$$\therefore\ \Gamma_1(z) = 0 \rightarrow S_1 = \frac{1+|\Gamma_1(z)|}{1-|\Gamma_1(z)|} = \boxed{1}$$

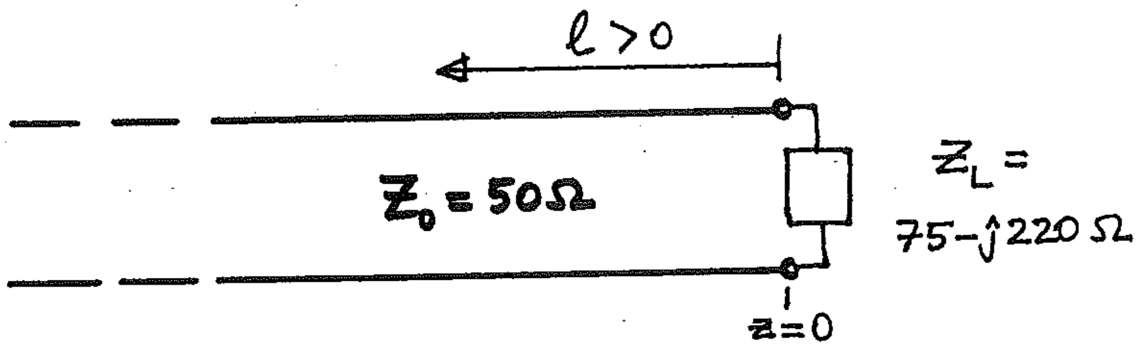
(b)



$$V_s = \frac{Z_{in}}{R_s + Z_{in}}(1e^{j0}) = 0.5\text{V},\quad I_s = V_s/Z_{in} = 10\,\text{mA}$$

$$P_{Z_{in}} = P_{R_L} = \frac{1}{2}|V_s|\,|I_s|\cos(\underbrace{\angle V_s - \angle I_s}_{0}) = \frac{1}{2}(0.5)(10\text{m}) = \boxed{2.5\,\text{mW}}$$

#7



(a)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 - j220 - 50}{75 - j220 + 50}$$

$$= \frac{25 - j220}{125 - j220} = \frac{5 - j44}{25 - j44} \cong \boxed{0.875 e^{-j23.1^\circ}}$$

$$S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \cong \frac{1 + 0.875}{1 - 0.875} \cong \boxed{15.0}$$

(b) $\%$ reflected power $= |\Gamma_L|^2 \times 100 \cong (0.875)^2 \times 100 \cong \boxed{76.6\%}$

(c) $\text{Im}\{Z_{in}\} = X_{in} = 0$

From Inan's Z_{in} handout!

$$X_L Z_0 + T(Z_0^2 - R_L^2 - X_L^2) - Z_0 X_L T^2 = 0$$

$$(-220)(50) + T(50^2 - 75^2 - (-220)^2) - (50)(-220)T^2 = 0$$

$$11{,}000T^2 - 51{,}525T - 11{,}000 = 0$$

$$440T^2 - 2,061T - 440 = 0$$

$$T_1, T_2 = \frac{2,061 \pm \sqrt{(2,061)^2 + 4(440)^2}}{2 \times 440}$$

$$\cong 4.89, \ -2.05 \times 10^{-1}$$

Since $T = \tan(\beta l) = \tan\left(2\pi \frac{l}{\lambda}\right)$, then

From Inan's Z_{in} handout !

$$\boxed{l_1, l_2 \cong 0.218\lambda, \ 0.468\lambda}$$

Using $Re\{Z_{in}\} = R_{in} = \dfrac{R_L Z_0^2 (1+T^2)}{(Z_0 - X_L T)^2 + R_L^2 T^2}$

we can find Z_{in} at the above two positions as follows:

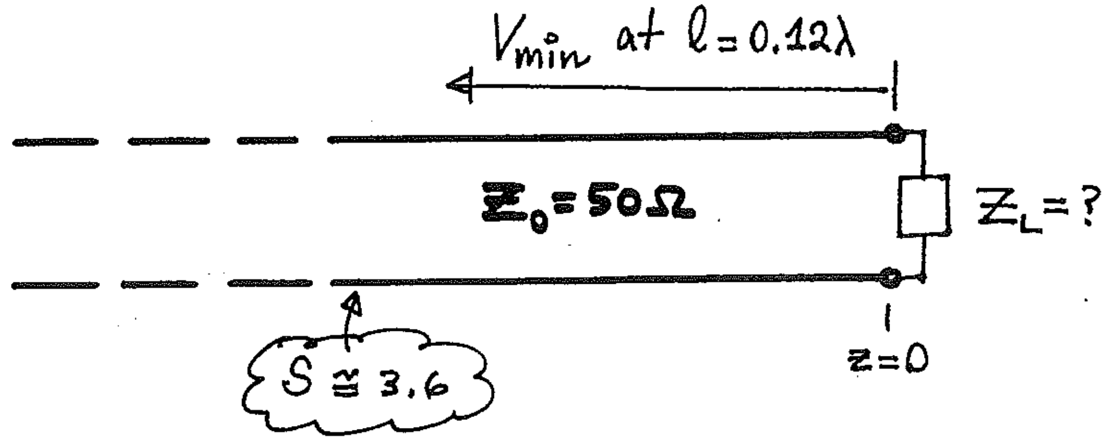
At $l_1 \cong 0.218\lambda$, we have

$$Z_{in_1} \cong \frac{75(50)^2(1+(4.89)^2)}{(50-(-220)(4.89))^2 + (75)^2(4.89)^2} \cong \boxed{3.33\,\Omega}$$

At $l_2 \cong 0.468\lambda$, we have

$$Z_{in_2} \cong \frac{75(50)^2(1+(-0.205)^2)}{(50-(-220)(-0.205))^2 + (75)^2(-0.205)^2} \cong \boxed{750.3\,\Omega}$$

#8



From Inan² text, p. 139, we have

$$Z_L = Z_0 \frac{1 - \hat{j} S \tan(\beta l_{min})}{S - \hat{j} \tan(\beta l_{min})}$$

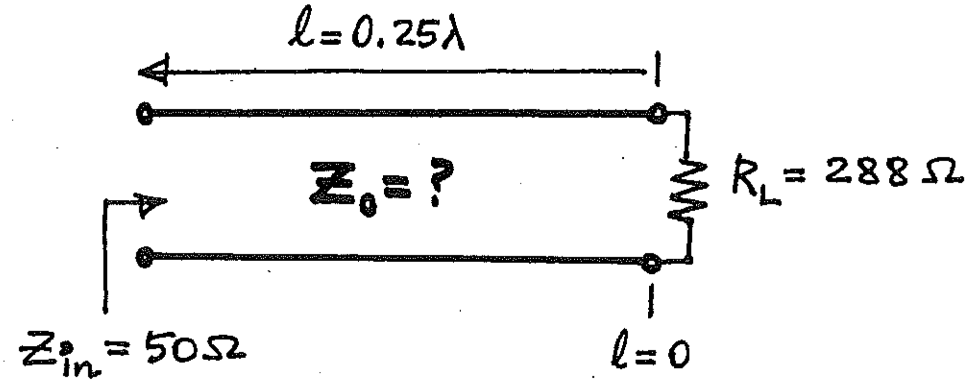
$$\cong 50 \frac{1 - \hat{j}\, 3.6 \tan(2\pi(0.12))}{3.6 - \hat{j} \tan(2\pi(0.12))}$$

$$\cong 50 \frac{1 - \hat{j}\, 3.38}{3.6 - \hat{j}\, 0.939} \cdot \frac{3.6 + \hat{j}\, 0.939}{3.6 + \hat{j}\, 0.939}$$

$$\cong 50 \cdot \frac{3.6 + 3.38(0.939) - \hat{j}\left[(3.38)(3.6) - 0.939\right]}{(3.6)^2 + (0.939)^2}$$

$$\cong \boxed{24.5 - \hat{j}\, 40.6\ \Omega}$$

#9



$$Z_{in} = \frac{Z_o^2}{R_L} \rightarrow Z_o = \sqrt{(50)(288)}$$

$$= \boxed{120\,\Omega}$$