4/21/2006

University of Portland
School of Engineering

EE 301-Electromagnetic Fields-3 cr. hrs.
Spring 2006

Midterm Exam # 2
Sinusoidal Steady-State Waves on Transmission Lines
(Prepared by Professor A. S. Inan)

(Friday, April 21, 2006)
(Closed Book Exam; 2 Formula Sheets Allowed)
(Total Time: 55 mins.)

Name: SOLUTIONS!

Signature: [illegible]

"Honesty is the best policy."
Aesop (~ 620B.C. –?)

"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

"Honest people are the true winners of the universe."
Anonymous

"Honesty is not for sale."
A. Inan

Inan said I can use the Smith Chart if I choose to do so!

This is a good sign, it shows that Inan is leaning more towards democracy... I heard that in the past that there were times when he banned the use of smith Charts!

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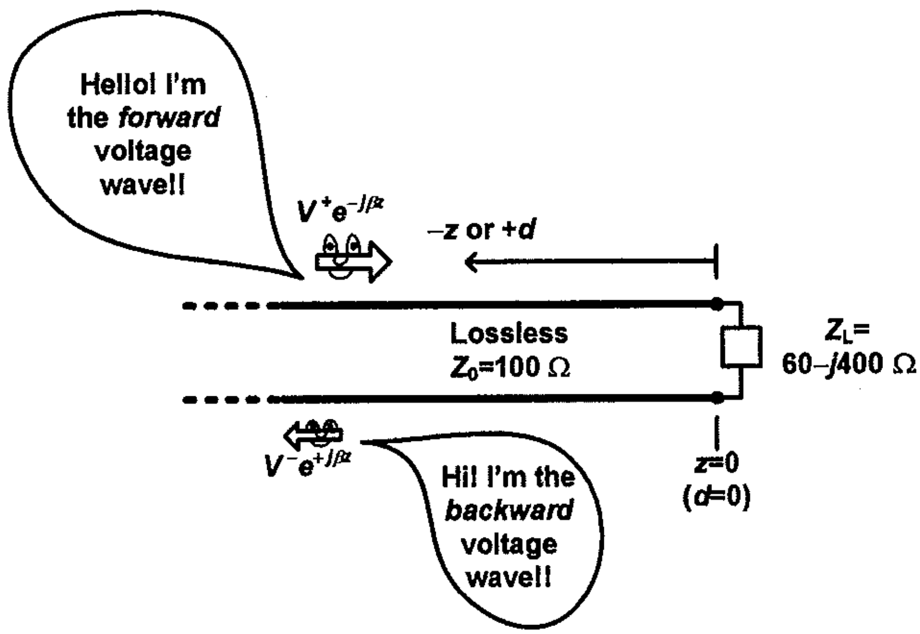
This table will be used by Inan for grading!

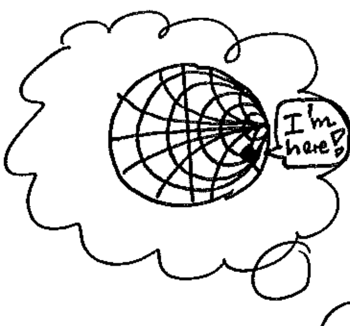
Problem #	Points gained
#1	
#2	
#3	
Total	

(1) (*15 mins., <u>Total:</u> 40 points*) **A lossless transmission line terminated with a complex impedance.** A $100\,\Omega$ transmission line is terminated with an inductive load impedance given by $Z_L = 60 - j400\,\Omega$, as shown.



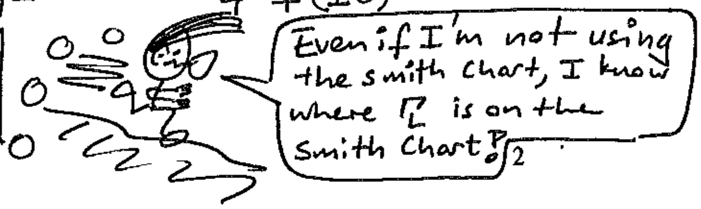
(a) (*10 points*) Find the load reflection coefficient, Γ_L. (Provide your answer in polar form.) Show your work!

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 - j400 - 100}{60 - j400 + 100} = \frac{-40 - j400}{160 - j400}$$



$$= \frac{-1 - j10}{4 - j10} \times \frac{4 + j10}{4 + j10} = \frac{96 - j50}{4^2 + (10)^2}$$

$$\cong \boxed{0.933\, e^{-j27.5^\circ}}$$



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(b) (*10 points*) What is the value of the standing wave ratio, S, on the line? (Show your work!)

I hope Inan won't change his mind in the middle of the exam!

$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \cong \frac{1+0.933}{1-0.933} \cong \boxed{28.9}$$

And it's a big relief to know that I can use it if I want to!

(c) (*10 points*) Calculate the percentage time-average incident power that reflects back from the load.

% power incident that reflects back $= |\Gamma_L|^2 \times 100$

$$\cong (0.933)^2 \times 100$$

$$\cong \boxed{87.1\,\%}$$

Only ~13% of the incident power is delivered to the load!

(d) (*10 points*) Find the V_{max} and V_{min} positions nearest to the load. Provide your answers as electrical lengths.

I've to convert into radians!

$$\phi_L + 2\beta z_{min} = -\pi$$

I'm in radians!

$$\rightarrow -\frac{27.5^\circ \times \cancel{\pi}}{180^\circ} + 4\cancel{\pi}\left(\frac{z_{min}}{\lambda}\right) = -\cancel{\pi}$$

$$\rightarrow \frac{z_{min}}{\lambda} \cong \boxed{-0.212} \rightarrow \frac{z_{max}}{\lambda} = \frac{z_{min}}{\lambda} - 0.25 \cong \boxed{-0.462}$$

And it makes sense that V_{min} is closer to the load since the load is capacitive!

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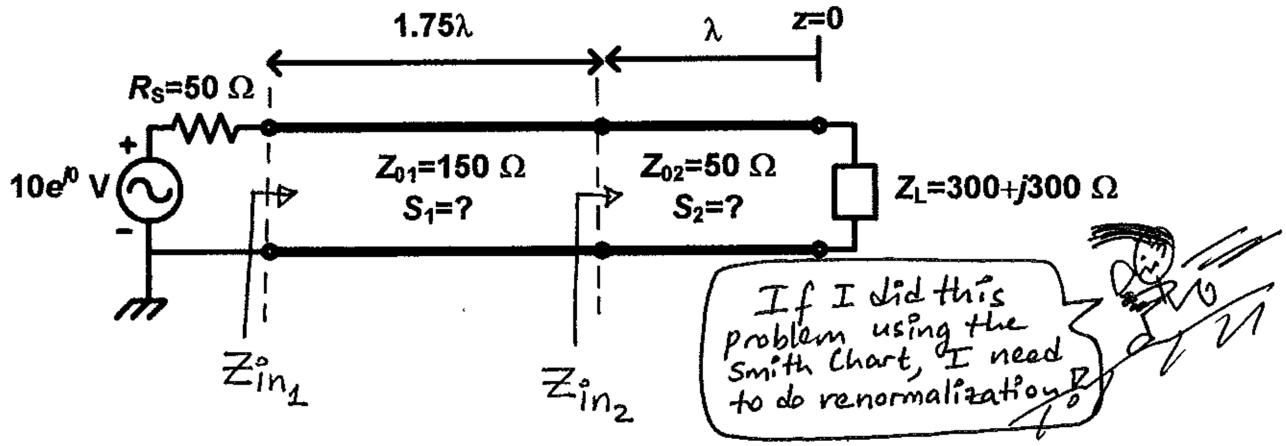
(2) (*15 mins., Total: 30 points*) **Two cascaded transmission lines.** Consider the transmission line circuit as shown. Assume lossless lines.



(a) (*15 points*) Find the standing wave ratio on each line. Show your work!

$$\Gamma_L = \frac{Z_L - Z_{02}}{Z_L + Z_{02}} = \frac{300 + j300 - 50}{300 + j300 + 50} = \frac{250 + j300}{350 + j300}$$

$$= \frac{5+j6}{7+j6} \times \frac{7-j6}{7-j6} = \frac{71 + j12}{(7)^2 + (6)^2} \cong 0.847 e^{j9.59^\circ}$$

$$\therefore S_2 = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \cong \frac{1+0.847}{1-0.847} \cong \boxed{12.1}$$

Note that $Z_{in_2} = Z_L = 300 + j300\ \Omega$ since the second line's length is λ.

$$\Gamma_{in_2} = \frac{Z_{in_2} - Z_{01}}{Z_{in_2} + Z_{01}} = \frac{300 + j300 - 150}{300 + j300 + 150} = \frac{1+j2}{3+j2} \times \frac{3-j2}{3-j2}$$

$$= \frac{7 + j4}{13} \cong 0.620 e^{j29.7^\circ}$$

$$\therefore S_1 = \frac{1+|\Gamma_{in_2}|}{1-|\Gamma_{in_2}|} \cong \frac{1+0.62}{1-0.62} \cong \boxed{4.27}$$

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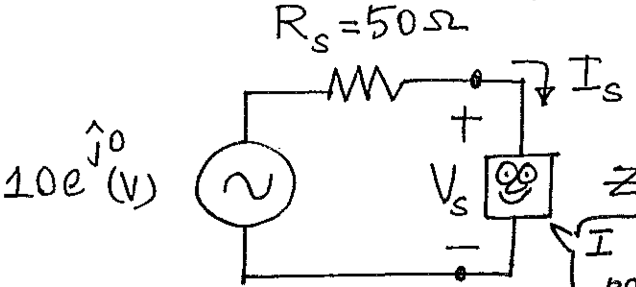
(b) (*15 points*) Find the time-average power delivered to the load impedance.

Since the length of the first line is 1.75λ, then, Z_{in_1} can be found as

$$Z_{in_1} = \frac{Z_{01}^2}{Z_{in_2}} = \frac{(150)^2}{300+\hat{j}300} = \frac{75}{1+\hat{j}} \times \frac{1-\hat{j}}{1-\hat{j}}$$

$$= \frac{75}{2}(1-\hat{j})\ \Omega$$



I'm R_{in}!

And I'm X_{in}!

$$Z_{in_1} = \left(\frac{75}{2} - \hat{j}\frac{75}{2}\right)\ \Omega \cong 53.0e^{-\hat{j}45^\circ}\ \Omega$$

I absorb the same power as Z_L!

I'm Z_L's representative!

$$I_s = \frac{10e^{\hat{j}0}}{87.5-\hat{j}37.5} \cong 0.105\,e^{\hat{j}23.2^\circ}\ \text{(A)}$$

∴ Power delivered to Z_{in_1} can be calculated as

$$P_{Z_{in_1}} = \frac{1}{2}|I_s|^2|Z_{in_1}|\cos(\angle Z_{in_1})$$

$$\cong \frac{1}{2}(0.105)^2(53)\cos(-45^\circ) \cong 0.207\ \text{W}$$

∴ Since both transmission lines are lossless, then,

$$P_L = P_{Z_{in_1}} \cong \boxed{0.207\ \text{W}}$$

This power is dissipated by the R_{in} part of Z_{in}!

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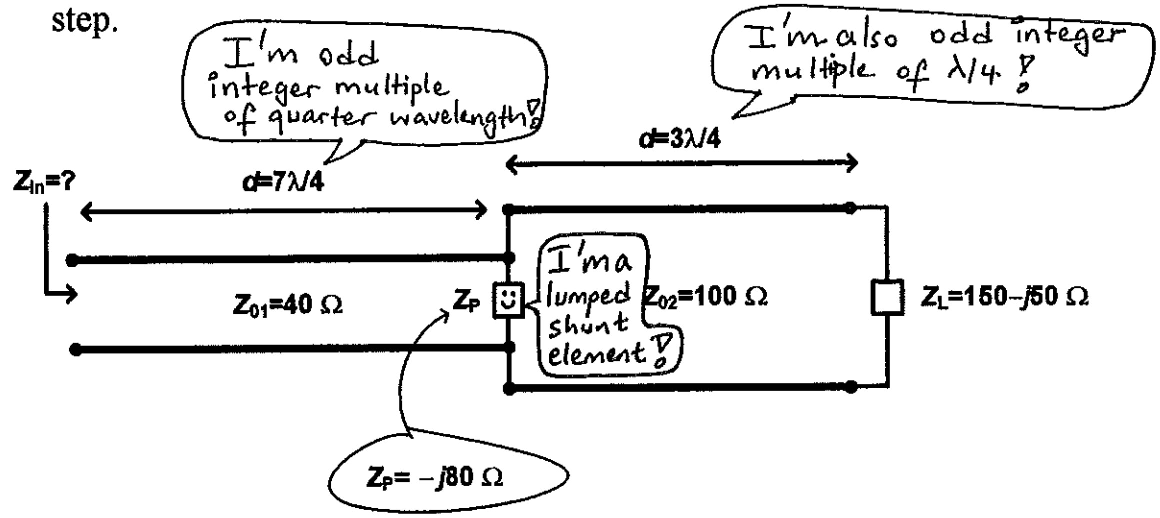
(3) (*15 mins., Total: 30 points*) **Input impedance.** Consider the transmission line circuit as shown where Z_P impedance represents a parallel lumped element.

(a) (*20 points*) Find the input impedance Z_{in}. Show your work step by step.



$$Z_{in_2} = \frac{Z_{02}^2}{Z_L} = \frac{(100)^2}{150-\hat{j}50} = \frac{200}{3-\hat{j}} \times \frac{3+\hat{j}}{3+\hat{j}}$$

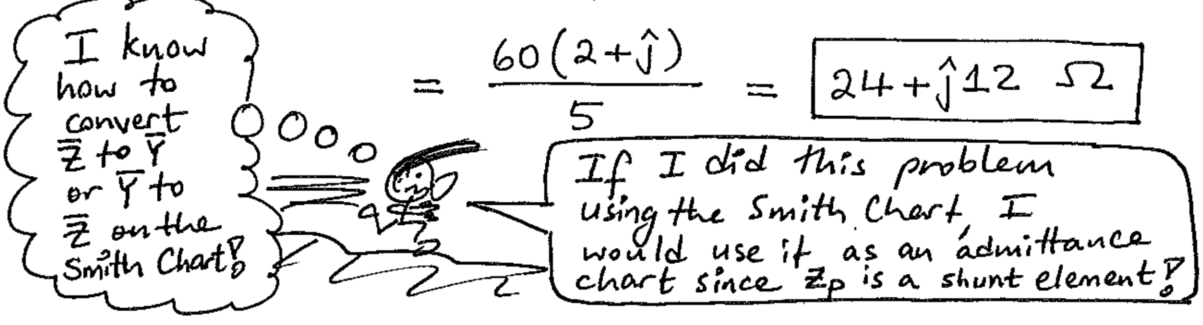
$$= 20(3+\hat{j}) = 60+\hat{j}20\ \Omega.$$

$$\underbrace{Z_P \,//\, Z_{in_2}}_{Z_{eq}} = \frac{(-\hat{j}80)(60+\hat{j}20)}{\underbrace{-\hat{j}80+60+\hat{j}20}_{60-\hat{j}60}} = \frac{(-\hat{j}80)(3+\hat{j})}{3(1-\hat{j})} \times \frac{1+\hat{j}}{1+\hat{j}}$$

$$= -\frac{\hat{j}40}{3}(2+\hat{j}4) = \frac{160-\hat{j}80}{3}\ \Omega$$

$$\therefore\ Z_{in_1} = \frac{Z_{01}^2}{Z_{eq}} = \frac{(40)^2 \times 3}{160-\hat{j}80} = \frac{60}{2-\hat{j}} \times \frac{2+\hat{j}}{2+\hat{j}}$$

$$= \frac{60(2+\hat{j})}{5} = \boxed{24+\hat{j}12\ \Omega}$$



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(b) (*10 points*) Assume that the physical length of each transmission line stays the same, however, the wavelength reduces by a factor of 2. What will be the new value of the input impedance Z_{in}? Why? (Assume the values of Z_L, Z_P, Z_{01} and Z_{02} to stay the same.)

Then, $d_1 = \frac{7\lambda_2}{2}$ & $d_2 = \frac{3\lambda_2}{2}$ where $\lambda_2 = \frac{\lambda_1}{2}$.

$$\therefore \quad Z_{in_2} = Z_L = 150 - \hat{j}50\ \Omega.$$

$$\therefore \quad Z_{eq} = Z_P \parallel Z_{in_2} = \frac{(-\hat{j}8\cancel{0})(150-\hat{j}50)}{15\cancel{0}-\hat{j}13\cancel{0}} \times \frac{15+\hat{j}13}{15+\hat{j}13}$$

$$= \frac{(-\hat{j}8)(50)(3-\hat{j})(15+\hat{j}13)}{(15)^2+(13)^2}$$

$$= \frac{(-\hat{j}400)(58+\hat{j}24)}{\underbrace{(15)^2+(13)^2}_{394}} = \frac{4800-\hat{j}11{,}600}{197}$$

$$\cong \boxed{24.4 - \hat{j}58.9\ \Omega}$$