4/23/2007

University of Portland
School of Engineering

<u>EE 301-Electromagnetic Fields-3 cr. hrs.</u>
<u>Spring 2007</u>

<u>Midterm Exam # 2</u>
<u>*Sinusoidal Steady-State Waves on Transmission Lines*</u>

(Prepared by Professor A. S. Inan)

(Monday, April 16, 2007)
(Closed Book Exam; 2 Formula Sheets Allowed)
(Total Time: 55 mins.)

Name: SOLUTIONS ☺

Signature: Solutions ☺

"Honesty is the best policy."
Aesop (~ 620B.C. –?)

"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

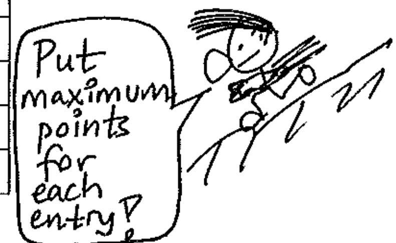
"Honest people are the true winners of the universe."
Anonymous

"Honesty is not for sale."
A. Inan

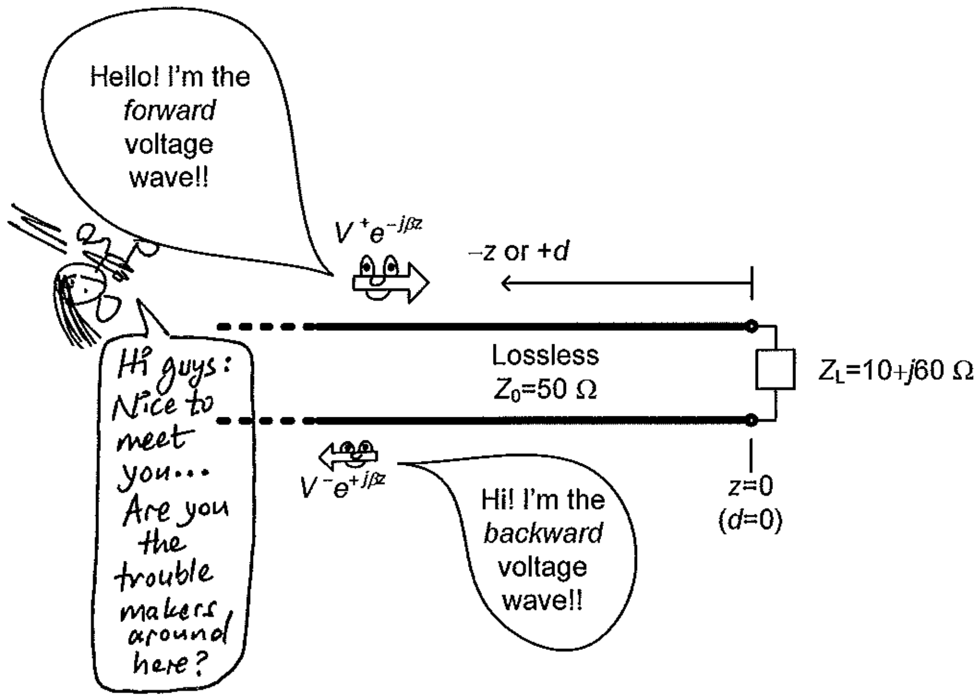
4/23/2007

This table will be used by Inan for grading!

Problem #	Points gained
#1	
#2	
#3	
Total	



(1) (*15 mins., <u>Total:</u> 40 points*) **A lossless transmission line terminated with a complex impedance.** A $50\,\Omega$ transmission line is terminated with an inductive load impedance given by $Z_L = 10 + j60\,\Omega$, as shown.



(a) (*10 points*) Find the load reflection coefficient, Γ_L. (Provide your answer in polar form.) Show your work!

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{10 + \hat{j}60 - 50}{10 + \hat{j}60 + 50} = \frac{-40 + \hat{j}60}{60 + \hat{j}60}$$

$$= \frac{-2 + \hat{j}3}{3 + \hat{j}3} \times \frac{3 - \hat{j}3}{3 - \hat{j}3} = \frac{(-6+9) + \hat{j}(6+9)}{9+9}$$

$$= \frac{1 + \hat{j}5}{6} \cong \boxed{0.85e^{\hat{j}78.7^\circ}}$$

4/23/2007

(b) (*10 points*) What is the value of the standing wave ratio, *S*, on the line? (Show your work!)

$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \cong \frac{1+0.85}{1-0.85} \cong \boxed{12.3}$$

This is baby stuff! I had more difficult stuff in high school!

(c) (*10 points*) Calculate the percentage time-average incident power that reflects back from the load.

I'm the time-average reflected power!

I'm the time-average incident power!

$$\frac{P^-}{P^+} \times 100 = \frac{\cancel{\frac{1}{2}}\frac{|V^-|^2}{\cancel{Z_0}}}{\cancel{\frac{1}{2}}\frac{|V^+|^2}{\cancel{Z_0}}} \times 100 = |\Gamma_L|^2 \times 100$$

$$\cong (0.85)^2 \times 100 \cong \boxed{72.2\,\%}$$

So only 27.8% of the incident power is delivered to the load!

(d) (*10 points*) Find the V_{max} and V_{min} positions nearest to the load. Provide your answers as electrical lengths.

I'm the angle of Γ_L and I'm in radians!

$$\phi_L + 2\beta z_{max} = 0$$

$$\rightarrow 78.7^\circ \left(\frac{\pi}{180^\circ}\right) + 2\left(\frac{2\pi}{\lambda}\right) z_{max} = 0$$

$$\rightarrow \therefore z_{max} \cong -0.109\lambda \quad \text{or} \quad \bar{z}_{max} = \frac{z_{max}}{\lambda} \cong \boxed{-0.109}$$

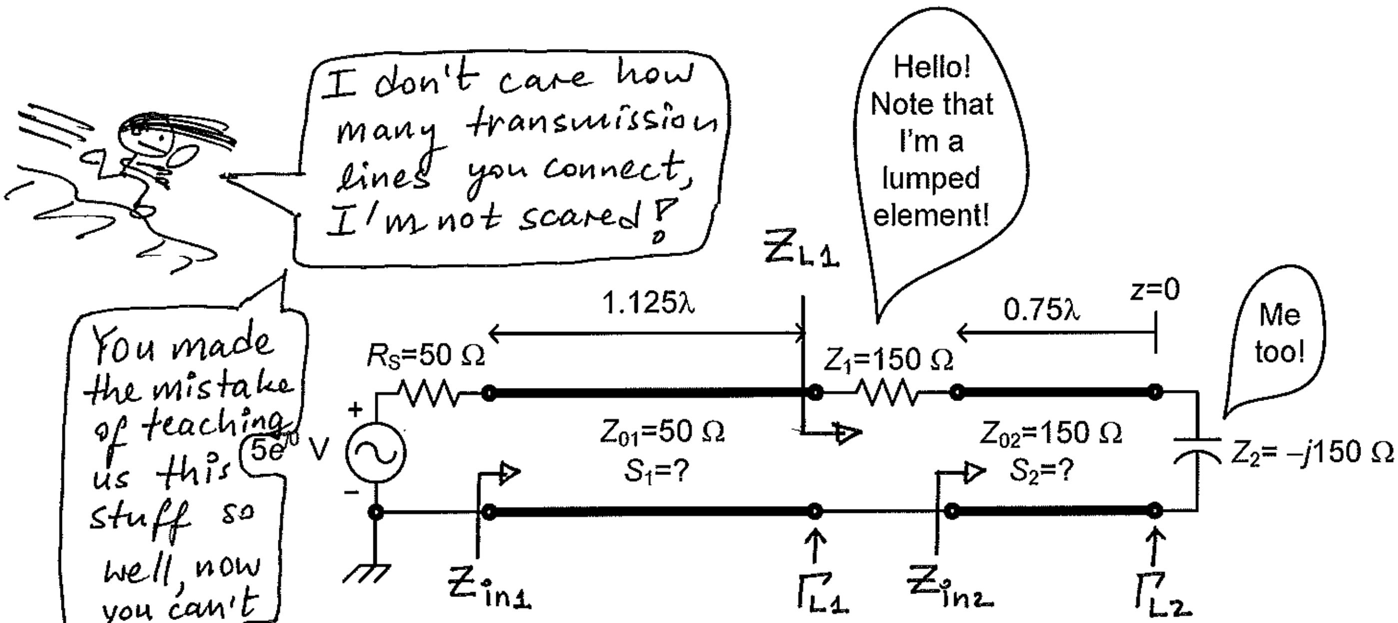
$$\therefore \bar{z}_{min} = \frac{z_{min}}{\lambda} \cong -0.109 - 0.25 = \boxed{-0.359}$$

Max-min points are always apart by quarter wavelength if they are neighbors!

Come out you coward Inan problem!

4/23/2007

(2) (*15 mins., Total: 30 points*) **Two cascaded transmission lines.** Consider the transmission line circuit as shown. Assume lossless lines.



(a) (*15 points*) Find the standing wave ratio on each line. Show your work!

$$\Gamma_{L2} = \frac{Z_2 - Z_{02}}{Z_2 + Z_{02}} = \frac{-j150 - 150}{-j150 + 150}$$

$$= \frac{-j-1}{-j+1} = \frac{\cancel{\sqrt{2}}\, e^{j5\pi/4}}{\cancel{\sqrt{2}}\, e^{-j\pi/4}} = 1\, e^{j3\pi/2} = 1 e^{-j\pi/2}$$

$$\therefore\ S_2 = \frac{1+|\Gamma_{L2}|}{1-|\Gamma_{L2}|} = \frac{1+1}{1-1} = \boxed{\infty\ !}$$

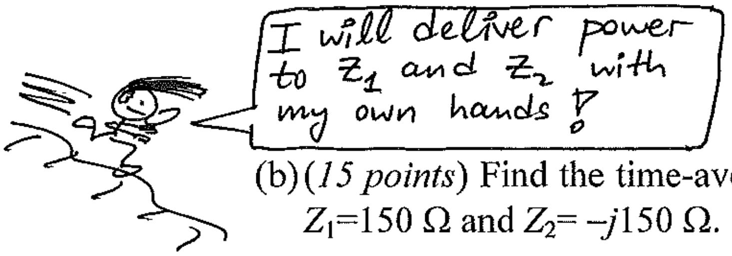
$$Z_{in2} = \frac{Z_{02}^2}{Z_2} = \frac{(150)^2}{-j150} = j150\ \Omega$$

$$Z_{L1} = Z_1 + Z_{in2} = 150 + j150\ \Omega$$

$$\Gamma_{L1} = \frac{Z_{L1} - Z_{01}}{Z_{L1} + Z_{01}} = \frac{150 + j150 - 50}{150 + j150 + 50} = \frac{2+j3}{4+j3} \times \frac{4-j3}{4-j3}$$

$$= \frac{17 + j6}{25} \cong \underbrace{0.721}_{|\Gamma_{L1}|} e^{j19.4^\circ} \quad \therefore\ S_1 = \frac{1+|\Gamma_{L1}|}{1-|\Gamma_{L1}|} \cong \boxed{6.17}$$

4/23/2007

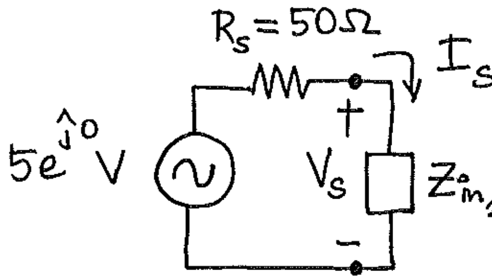


(b) (*15 points*) Find the time-average power delivered to each impedance Z_1=150 Ω and Z_2= $-j$150 Ω.

$$Z_{in1} = 50\,\frac{150 + \hat{j}150 + \hat{j}50\,\cancel{\tan(\pi/4)}^{\,\to 1}}{50 + \hat{j}(150+\hat{j}150)\,\cancel{\tan(\pi/4)}^{\,\to 1}}$$

$$= 50\,\frac{150 + \hat{j}\,200}{-100 + \hat{j}150} = 50\,\frac{3+\hat{j}4}{-2+\hat{j}3} \times \frac{-2-\hat{j}3}{-2-\hat{j}3}$$

$$= 50\,\frac{(-6+12)+\hat{j}(-8-9)}{(-2)^2 + 3^2} = \frac{50}{13}(6-\hat{j}17)\ \Omega$$

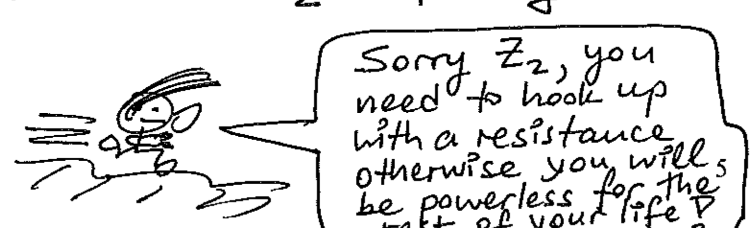


$$Z_{in_1} = \frac{300 - \hat{j}\,850}{13}\ \Omega$$

$$I_s = \frac{5e^{\hat{j}0}}{R_s + Z_{in1}} \cong \frac{5e^{\hat{j}0}}{\underbrace{50 + 23.1}_{73.1} - \hat{j}65.4} \cong 51\,e^{\hat{j}41.8^\circ}\ (\text{mA})$$

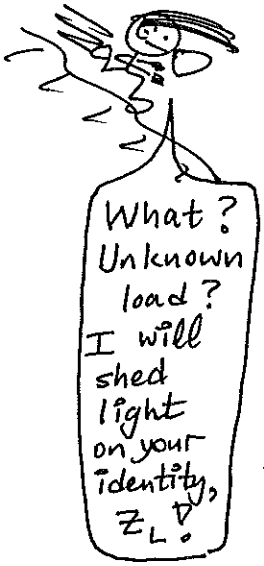
$$\therefore\ P_{Z_{in1}} = \frac{1}{2} R_{in1} |I_s|^2 \cong 30\ \text{mW}$$

$\therefore\ P_{Z_1} \cong \boxed{30\ \text{mW}}$ and $P_{Z_2} = \boxed{0}$ since both lines are lossless and Z_2 is purely reactive impedance.



4/23/2007

(3) (*15 mins.*, *Total: 30 points*) **Unknown load impedance.** Consider a 50 Ω transmission line terminated with an unknown load Z_L. If the standing-wave ratio on the line is measured to be $S \cong 4.2$ and the nearest voltage minimum point with respect to the load is located at 0.21λ, find the following:



(a) (*10 points*) The load impedance Z_L. Show your work step by step.

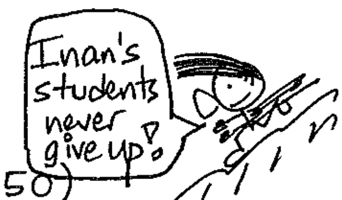
$$|\Gamma_L| = \frac{S-1}{S+1} = \frac{4.2-1}{4.2+1} \cong 0.615$$

$$\phi_L + 2\beta z_{min} = \phi_L + 2\left(\frac{2\pi}{\lambda}\right)(-0.21\lambda) = -\pi$$

$$\rightarrow \phi_L = 0.84\pi - \pi = -0.16\pi \cong -28.8^\circ$$

$$\therefore \Gamma_L \cong 0.615 e^{-\hat{j}0.16\pi} \cong 0.539 - \hat{j}0.296$$

$$\rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{R_L + \hat{j}X_L - 50}{R_L + \hat{j}X_L + 50} \cong 0.539 - \hat{j}0.296$$

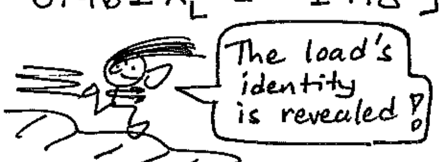


$$\rightarrow R_L + \hat{j}X_L - 50 \cong 0.539(R_L + 50) + 0.296X_L + \hat{j}0.539X_L - \hat{j}0.296(R_L + 50)$$

$$\begin{cases} R_L - 50 = 0.539(R_L + 50) + 0.296X_L & \leftarrow \text{Real parts are equal...} \\ X_L = 0.539X_L - 0.296(R_L + 50) & \leftarrow \text{Imaginary parts are equal...} \end{cases}$$

$$\rightarrow \begin{cases} 0.461R_L - 0.296X_L = 76.96 \\ 0.296R_L + 0.461X_L = -14.8 \end{cases}$$ Solving simultaneously yields

$$\boxed{\begin{aligned} R_L &\cong 103.5\Omega \\ X_L &\cong -98.77\Omega \end{aligned}}$$



4/23/2007

(b) (*10 points*) The nearest voltage maximum position to the load.

$$\bar{z}_{max} = \frac{z_{max}}{\lambda} = \bar{z}_{min} - 0.25$$

$$= -0.21 - 0.25 = -0.46$$

or $z_{max} = \boxed{-0.46\lambda}$

(c) (*10 points*) The input impedance Z_{in} at the nearest voltage minimum and maximum positions found in parts (a) and (b).

Z_{in} at the voltage minimum position is given by

$$Z_{in_{min}} = \frac{Z_0}{S} = \frac{50}{4.2} \cong \boxed{11.9\Omega}$$

Z_{in} at the voltage maximum position is given by

$$Z_{in_{max}} = S Z_0 = (4.2)(50\Omega) = \boxed{210\Omega}$$