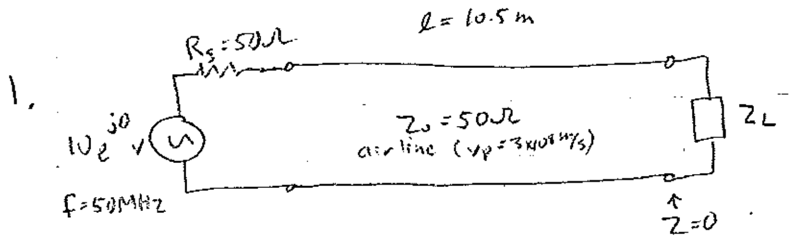
EE 301
Spring 2011
HW 5

1.



i $Z_L = 70 + j10\Omega$
ii $Z_L = 15 - j35\Omega$

$$\lambda = \frac{v_p}{f} = \frac{3\times10^8\,m/s}{50\,MHz} \qquad \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{6m} = \frac{\pi}{3}\,\frac{rad}{m}$$

$$\lambda = 6\,m$$

a) Calculate Γ_L

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

i

$$= \frac{70 + j10 - 50}{70 + j10 + 50}$$

$$\Gamma_L = \frac{20 + j10}{120 + j10}$$

a: $\boxed{\text{i, } \Gamma_L = \frac{5}{29} + j\frac{2}{29}}$

$$= .01857e^{0.38053j}$$

ii

$$\Gamma_L = \frac{15 - j35 - 50}{15 - j35 + 50}$$

$$\Gamma_L = \frac{-35 - j35}{65 - j35}$$

$\boxed{\text{ii } \Gamma_L = -\frac{21}{109} - j\frac{70}{109}}$

$$= 0.67048e^{-1.862j}$$

b) Calculate S

$$S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

i

$$S = \frac{1 + 0.1857}{1 - 0.1857}$$

b: $\boxed{\text{i, } S = 1.456}$

ii

$$S = \frac{1 + 0.67048}{1 - 0.67048}$$

$\boxed{\text{ii } S = 5.069}$

c) Find Locations of V_{max} and V_{min} on the line

i, $z_{max\,first} = -\frac{\Phi_L}{2\beta} = -\frac{0.3805\,rad}{2\,\pi/3\,rad}$

$$z_{max\,first} = -0.182\,m$$

$$z_{max} = z_{max\,first} - n\frac{\lambda}{2} = z_{max\,first} - n\cdot 3m$$

$\boxed{\text{i, } z_{max} = -0.182m,\ -3.182m,\ -6.182m,\ -9.182m}$

$$z_{min\,first} = \frac{-\pi - \Phi}{2\beta} = -\frac{\pi - 0.3805}{2\pi/3}$$

$$z_{min\,first} = -1.682m$$

$$z_{min} = z_{min\,first} - n\frac{\lambda}{2}$$

$\boxed{\text{i. } z_{min} = -1.682m,\ -4.682m,\ -7.682m}$

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c) cont.

ii $Z_{min\,first} = \dfrac{-\pi - \theta_L}{2\beta}$

$= -\dfrac{\pi - (-1.862)}{2\pi/3}$

$Z_{min\,first} = -0.61\ m$

$\boxed{ii.\ Z_{min} = -0.61m,\ -3.61m,\ -6.61m,\ -9.61m}$

$Z_{max\,first} = \dfrac{-2\pi - \theta_L}{2\beta}$

$= \dfrac{-2\pi - (-1.862)}{2\pi/3}$

$Z_{max\,first} = -2.11\ m$

$\boxed{ii.\ Z_{max} = -2.11m,\ -5.11m,\ -8.11m}$

d) $Z_{in} = Z_0 \dfrac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$

i $Z_{min\,in} = 50 \dfrac{70 + j10 + j50\tan(\frac{\pi}{3}(.182))}{50 + j(70+j10)\tan(\frac{\pi}{3}(.182))}$

$\boxed{i\ Z_{min\,in} = 72.8\,\Omega}$

$Z_{max\,in} = 50 \dfrac{70 + j10 + j50\tan(\frac{\pi}{3}(1.682))}{50 + j(70+10j)\tan(\frac{\pi}{3}(1.682))}$

$\boxed{i\ Z_{max\,in} = 34.34\,\Omega}$

ii $Z_{min\,in} = 50 \dfrac{15 - j35 + j50\tan(\frac{\pi}{3}(0.61))}{50 + j(15-j35)\tan(\pi/3(0.61))}$

$\boxed{ii\ Z_{min\,in} = 9.86\,\Omega}$

$Z_{max\,in} = 50 \dfrac{15 - j35 + j50\tan(\frac{\pi}{3}(2.11))}{50 + j(15-j35)\tan(\pi/3(2.11))}$

$\boxed{ii\ Z_{max\,in} = 253.5\,\Omega}$

e) Zin at the source end

$Z_{in} = Z_0 \cdot \dfrac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$

i $Z_{ins} = 50 \dfrac{70 + j10 + j50\tan(\frac{\pi}{3}\cdot 10.5)}{50 + j(70+j10)\tan(\pi/3 \cdot 10.5)}$

$\boxed{i\ Z_{ins} = 35 - 5j\ \Omega}$

ii $Z_{ins} = 50 \dfrac{15 - j35 + j50\tan(\frac{\pi}{3}(10.5))}{50 + j(15-j35)\tan(\frac{\pi}{3}(10.5))}$

$\boxed{ii\ Z_{ins} = 25.86 + 60.34j\ \Omega}$

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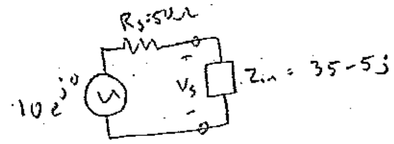
f) Find V_s, V^+ and V_L

i.



$$V_s = 10e^{j0}\,\frac{35-5j}{35-5j+50}$$
$$= 10e^{j0}\cdot\left(\frac{12}{29} - \frac{1}{29}j\right)$$

$$\boxed{i\;\; V_s = 4.138 - 0.345j\ V}$$

$$V_s = V^+e^{j\beta\ell} + V^-e^{-j\beta\ell}$$
$$= V^+e^{j\beta\ell}\left(1 + \frac{V^-}{V^+}e^{-2j\beta\ell}\right)$$
$$V_s = V^+e^{j\beta\ell}\left(1 + \Gamma_L e^{-2j\beta\ell}\right)$$
$$V^+ = \frac{V_s}{e^{j\beta\ell}(1+\Gamma_L e^{-2\beta\ell})}$$
$$V^+ = \frac{4.138 - 0.345j}{e^{j\frac{\pi}{3}\cdot 10.5}\left(1+\left(\frac{5}{29}+j\frac{2}{29}\right)e^{-2\frac{\pi}{3}\cdot 10.5}\right)}$$

$$\boxed{i.\;\; V^+ = 0.345 + 4.138j\ V}$$

$$V_L = V^+ + V^-$$
$$= V^+ + \Gamma_L V^+ = (1+\Gamma_L)V^+$$

$$V_L = (1+\Gamma_L)V^+$$
$$= \left(1+\left(\frac{5}{29}+j\frac{2}{29}\right)\right)(0.345+4.138j)$$

$$\boxed{i.\;\; V_L = 0.119 + 4.875j\ V}$$

ii

$$V_s = 10e^{j0}\left(\frac{25.86+60.34j}{50+25.86+60.34j}\right)$$

$$\boxed{ii\;\; V_s = 5.963 + 3.211j\ V}$$

$$V^+ = \frac{V_s}{e^{j\beta\ell}(1+\Gamma_L e^{-2\beta\ell})}$$
$$V^+ = \frac{5.963+3.211j}{e^{j\pi/3\cdot 10.5}\left(1+\left(-\frac{21}{109}-j\frac{70}{109}\right)e^{-2\frac{\pi}{3}\cdot 10.5}\right)}$$

$$\boxed{ii\;\; V^+ = -3.211 + 5.963j\ V}$$

$$V_L = (1+\Gamma_L)V^+$$
$$= \left(1+\left(-\frac{21}{109}-j\frac{70}{109}\right)\right)(-3.211+5.963j)$$

$$\boxed{ii\;\; V_L = 1.237 + 6.876j\ V}$$

g) Find P^+, P^-, P_{RS}, P_L, P_{source}

$$P^+ = \frac{1}{2}|V^+||I^+|$$
$$= \frac{1}{2}\frac{|V^+|^2}{Z_0}$$
i $$= \frac{1}{2}\frac{|0.345+4.138j|^2}{50}$$

$$\boxed{\text{i} \quad P^+ = 0.1724 \text{ watts}}$$

ii $$P^+ = \frac{1}{2}\frac{|-3.211+5.963j|^2}{50}$$

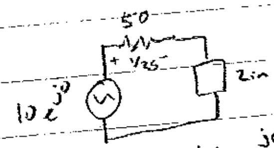
$$\boxed{\text{ii} \quad P^+ = 0.4587 \text{ watts}}$$

$$P^- = |\Gamma_L|^2 P^+$$
i $$P^- = 0.1857^2(0.1724)$$

$$\boxed{\text{i} \quad P^- = 5.945 \text{ mW}}$$

ii $$P^- = 0.67048^2(0.4587)$$

$$\boxed{\text{ii} \quad P^- = 0.206 \text{ W}}$$



$$V_{RS} = \frac{R_s}{R_s + Z_{in}} \cdot 10e^{j0}$$
i $$V_{RS} = \frac{50}{50+35-5j} 10e^{j0}$$
$$V_{RS} = 5.862 + 0.3448j \text{ V}$$
ii $$V_{RS} = \frac{50}{50+25.86+60.34j} 10e^{j0}$$
$$V_{RS} = 4.037 - 3.211j \text{ V}$$

$$P_{RS} = \frac{1}{2}\frac{|V_{RS}|^2}{R_s}$$
i $$P_{RS} = \frac{1}{2}\frac{|5.862+0.3448j|^2}{50}$$

$$\boxed{\text{i} \quad P_{RS} = 0.3448 \text{W}}$$

ii $$P_{RS} = \frac{1}{2}\frac{|4.037-3.211j|^2}{50}$$

$$\boxed{\text{i} \quad P_{RS} = 0.266 \text{W}}$$

$$P_L = P_{in} = \frac{1}{2}\frac{|V_s|^2}{|Z_{in}|}\cos(\theta_{Z_{in}})$$
i $$P_L = \frac{1}{2}\frac{|4.138-0.345j|^2}{|35-5j|}\cos\left(\tan^{-1}\left(\frac{-5}{35}\right)\right)$$

$$\boxed{\text{i} \quad P_L = 0.241 \text{ W}}$$

ii $$P_L = \frac{1}{2}\frac{|5.963+3.211j|^2}{|35.86+60.34j|}\cos\left(\tan^{-1}\left(\frac{60.34}{25.86}\right)\right)$$

$$\boxed{\text{ii} \quad P_L = 0.1376 \text{ W}}$$

$$P_{source} = P_{RS} + P_L$$
i $$P_{source} = 0.3448\text{W} + 0.241\text{W}$$

$$\boxed{\text{i} \quad P_{source} = 0.5858 \text{ W}}$$

ii $$P_{source} = 0.266\text{ W} + 0.1376\text{ W}$$

$$\boxed{\text{ii} \quad P_{source} = 0.4036 \text{W}}$$

h) Find the actual positions on the line where
$Z_{in} = Z_0 + jX_{in}$.

$$Z_{in} = Z_0 \frac{R_L Z_0 (1 + (\tan\beta l)^2)}{(Z_0 - X_L \tan\beta l)^2 + (R_L \tan\beta l)^2} + j \cdots$$

$$Z_0 = Z_0 \frac{R_L Z_0 (1 + (\tan\beta l)^2)}{(Z_0 - X_L \tan\beta l)^2 + (R_L \tan\beta l)^2}$$

$$(Z_0 - X_L \tan\beta l)^2 + (R_L \tan\beta l)^2 = R_L Z_0 (1 + (\tan\beta l)^2)$$

(solved by calculator)

i $(50 - 10\tan(\frac{\pi}{3} l))^2 + (70\tan(\frac{\pi}{3} l))^2 = 70 \cdot 50 (1 + (\tan\frac{\pi}{3} l)^2)$

$0.8425 + n\frac{\lambda}{2}$

$2.5209 + n\frac{\lambda}{2}$

$$\boxed{\text{i} \quad l = 0.8425\text{m},\ 3.8425\text{m},\ 6.8425\text{m},\ 9.8425\text{m},\ 2.5209\text{m},\ 5.5209\text{m},\ 8.5209\text{m},}$$

note, $z = -l$

ii $(50 - -35\tan(\frac{\pi}{3} l))^2 + (15\tan(\frac{\pi}{3} l))^2 = 15 \cdot 50 (1 + (\tan\frac{\pi}{3} l)^2)$

$1.7117\text{m} + n\frac{\lambda}{2}$

$2.50997 + n\frac{\lambda}{2}$

$$\boxed{\text{ii} \quad l = 1.7117\text{m},\ 4.7117\text{m},\ 7.7117\text{m},\ 2.50997\text{m},\ 5.50997\text{m}\ 8.50997\text{m}}$$