4/18/2011

University ☺f P☺rtland
Sch☺☺l ☺f Engineering

EE 301-Electromagnetic Fields-3 cr. hrs.
Spring 2011

Midterm Exam # 2
Sinusoidal Steady-State Waves on Transmission Lines

(Prepared by Professor A. S. Inan)

(Monday, April 18, 2011)
(Closed Book Exam; Formula Sheets Allowed)
(Total Time: 55 mins.)

Did you know that yesterday in the 21st century was the 221st anniversary of Benjamin Franklin's death? ☺ (He died on April 17, 1790, at age 84.)

Name: SOLUTIONS ☺

Signature: Solutions ☺

"Honesty is the best policy."
Aesop (~ 620B.C. –?)

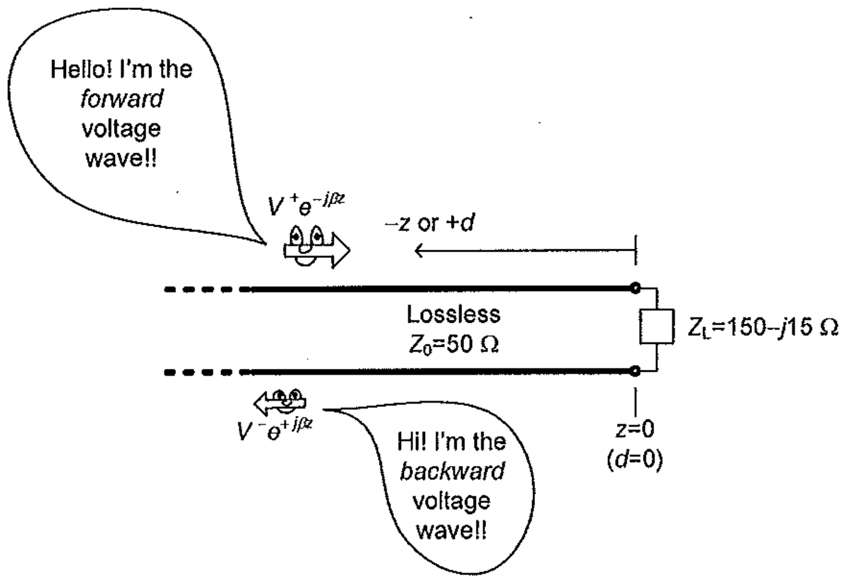
"An honest mind possesses a kingdom."
Lucius Annaeus Seneca (4B.C.–65A.D.)

"Honest people are the true winners of the universe."
Anonymous

This table will be used by Inan for grading!

Problem #	Points gained
#1	
#2	
#3	
Total	

(1) (*15 mins., <u>Total:</u> 40 points*) **A lossless transmission line terminated with a complex impedance.** A $50\,\Omega$ transmission line is terminated with an capacitive load impedance given by $Z_L = 150 - j15\,\Omega$, as shown.



(a) (*10 points*) Calculate the load reflection coefficient, Γ_L. (Provide your answer in polar form.) Show your work!

I'm the load reflection coefficient!

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{150 - \hat{j}15 - 50}{150 - \hat{j}15 + 50} = \frac{100 - \hat{j}15}{200 - \hat{j}15}$$

$$= \frac{20 - \hat{j}3}{40 - \hat{j}3} \times \frac{40 + \hat{j}3}{40 + \hat{j}3} = \frac{809 - \hat{j}60}{(40)^2 + 3^2}$$

$$\approx \boxed{0.5042\, e^{-\hat{j}4.2416^\circ}}$$

(b) (*10 points*) What is the value of the standing wave ratio, S, on the line?

I'm the magnitude of Γ_L!

$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \approx \frac{1+0.5042}{1-0.5042} \approx \boxed{3.034}$$

(c) (*10 points*) Find the percentage time-average incident power that is absorbed by the load.

$$\begin{aligned}\text{\% of incident wave time-average power absorbed by the load} &= 100\left(1-|\Gamma_L|^2\right)\\ &\approx 100\left(1-(0.5042)^2\right)\\ &\approx \boxed{74.58\,\%}\end{aligned}$$

I would have been closer to the load if the load was inductive!

I'm closer to the load compared to the location of V_{max}!

(d) (*10 points*) Find the V_{max} and V_{min} positions nearest to the load. Provide your answers as electrical lengths.

I'm the angle of the load reflection coefficient!

I'm in radians!

$$\phi_L + 2\beta z_{min} = -\pi$$

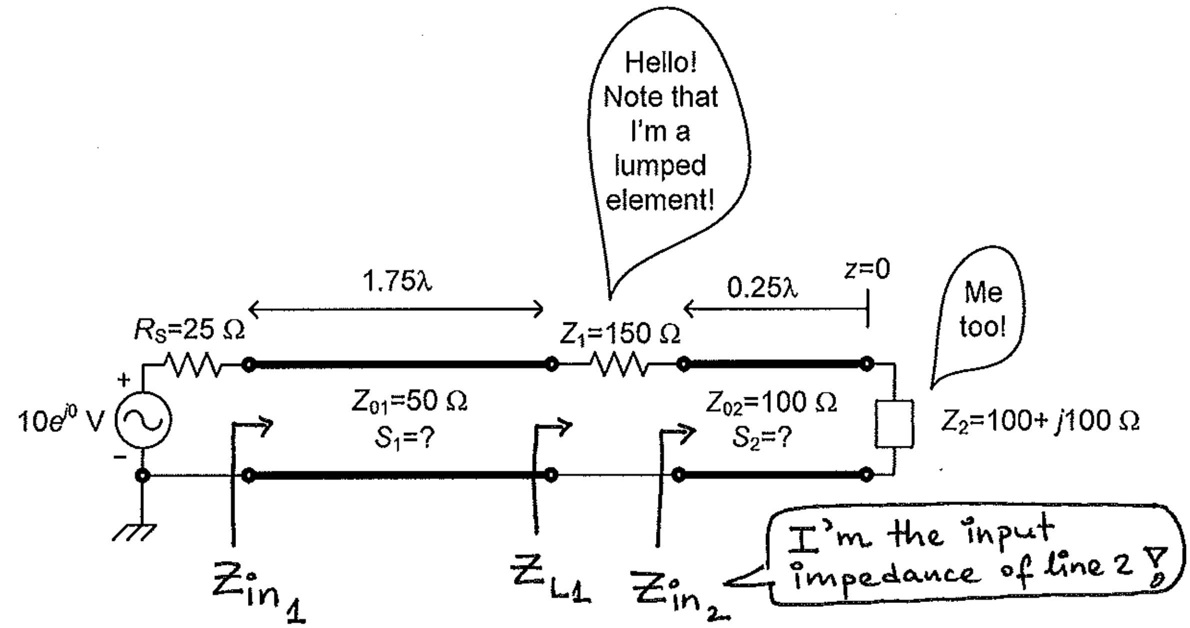
$$\rightarrow \quad -4.2416^\circ\left(\frac{\pi}{180^\circ}\right) + 4\pi\left(\frac{z_{min}}{\lambda}\right) = -\pi$$

$$\rightarrow \quad \frac{z_{min}}{\lambda} \approx \boxed{-0.2441}$$

$$\rightarrow \quad \frac{z_{max}}{\lambda} = \frac{z_{min}}{\lambda} - 0.25 \approx \boxed{-0.4941}$$

We are apart by 0.25λ!

(2) (*15 mins., Total: 30 points*) **Two cascaded transmission lines.** Consider the transmission line circuit as shown.



(a) (*15 points*) Find the standing wave ratio on each line. Show your work!

I'm the load reflection coefficient for line 2

$$\Gamma_{L2} = \frac{Z_2 - Z_{02}}{Z_2 + Z_{02}} = \frac{100+j100-100}{100+j100+100} = \frac{j100}{200+j100}$$

$$= \frac{j}{2+j}\cdot\frac{2-j}{2-j} = \frac{1-j2}{5} \approx \frac{1}{\sqrt{5}}e^{-j63.435^\circ}$$

$$Z_{in_2} = \frac{Z_{02}^2}{Z_2} = \frac{(100)^2}{100+j100} = \frac{100}{1+j}\cdot\frac{1-j}{1-j} = 50-j50\ \Omega$$

$$Z_{L1} = Z_1 + Z_{in_2} = 150+50-j50 = 200-j50\,\Omega$$

I'm the load reflection coefficient for line 1

$$\Gamma_{L1} = \frac{Z_{L1}-Z_{01}}{Z_{L1}+Z_{01}} = \frac{200-j50-50}{200-j50+50} = \frac{150-j50}{250-j50}$$

$$= \frac{3-j}{5-j}\cdot\frac{5+j}{5+j} = \frac{16-j2}{26} = \frac{8-j}{13} \approx 0.6202e^{-j7.125^\circ}$$

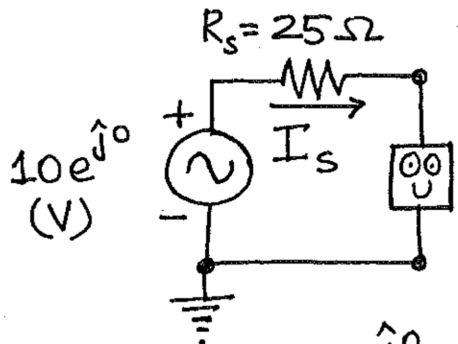
$$S_2 = \frac{1+|\Gamma_{L2}|}{1-|\Gamma_{L2}|} = \frac{1+1/\sqrt{5}}{1-1/\sqrt{5}} \approx \boxed{2.618},\quad S_1 = \frac{1+|\Gamma_{L1}|}{1-|\Gamma_{L1}|} \approx \frac{1+0.6202}{1-0.6202} \approx \boxed{4.2659}$$

(b) (*15 points*) Find the time-average power delivered to the load impedance Z_2.

$$Z_{in_1} = \frac{Z_{01}^2}{Z_{L1}} \quad \text{since } 1.75\lambda \text{ is } 1.5\lambda + 0.25\lambda.$$

$$Z_{in_1} = \frac{50^2}{200 - \hat{j}50} = \frac{50}{4-\hat{j}} \cdot \frac{4+\hat{j}}{4+\hat{j}}$$

$$= \frac{50(4+\hat{j})}{17} \approx 11.765 + \hat{j}\,2.9412\ \Omega$$



I'm the input impedance of line 1!

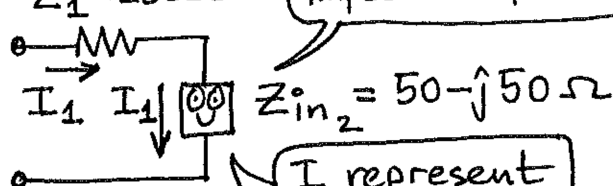
$$I_s = \frac{10e^{\hat{j}0}}{36.765 + \hat{j}2.9412} \approx 0.27113\, e^{-\hat{j}4.5739^\circ}\ \text{(A)}$$

I'm $P_{Z_1} + P_{Z_2}$!

$$P_{Z_{in_1}} = \frac{1}{2}|I_s|^2 R_{in_1} \approx \frac{1}{2}(0.27113)^2(11.765)$$

$$\approx \boxed{0.43243\ \text{W}}$$

Using the junction equivalent lumped circuit as shown:



I'm the input impedance of line 2!

I represent load Z_2!

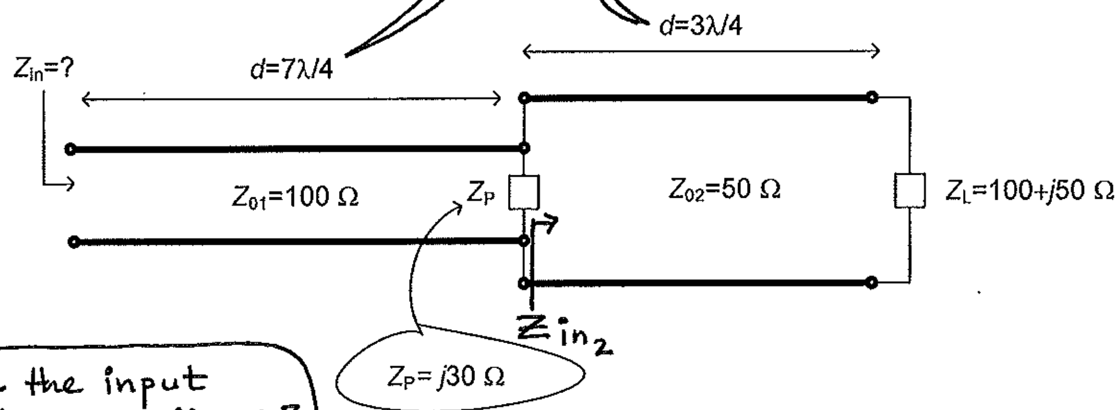
Since Z_1 and Z_{in_2} share the same current

and $P_{Z_1} = \frac{1}{2}|I_1|^2 R_1 = 75|I_1|^2$, $P_{Z_{in_2}} = \frac{1}{2}|I_1|^2 R_{in_2} = 25|I_1|^2$

and $P_{Z_{in_1}} = P_{Z_1} + P_{Z_{in_2}}$, $P_{Z_{in_2}} = P_{Z_2} = \frac{1}{4} P_{Z_{in_1}} \approx \boxed{0.1081\ \text{W}}$

(3) (*15 mins., 30 points*) **Input impedance.** Consider the transmission line circuit as shown where Z_P impedance represents a parallel lumped element. Find the input impedance Z_{in}. Show your work step by step.

We cause the same input impedance as the 0.25λ long line!



I'm the input impedance of line 2!

$$Z_{in_2} = \frac{Z_{02}^2}{Z_L} = \frac{50^2}{100+j50} = \frac{50}{2+j}\cdot\frac{2-j}{2-j}$$

$$= \frac{50(2-j)}{5} = 20 - j10\,\Omega$$

$$\underbrace{Z_P \,//\, Z_{in_2}}_{Z_{L_{eq}}} = \frac{j3\cancel{0}(20-j10)}{\underbrace{j30+20-j10}_{2\cancel{0}+j2\cancel{0}}} = \frac{j3(10-j5)}{1+j}\cdot\frac{1-j}{1-j}$$

I'm the equivalent load impedance for line 1!

$$= \frac{45+j15}{2}\,\Omega$$

$$Z_{in} = \frac{Z_{01}^2}{Z_{L_{eq}}} = \frac{100^2}{\left(\frac{45+j15}{2}\right)} = \frac{4000}{\underbrace{9+j3}_{3(3+j)}}\cdot\frac{3-j}{3-j}$$

$$= \frac{400}{3}(3-j) = \boxed{400 - j\,\frac{400}{3}\,\Omega}$$