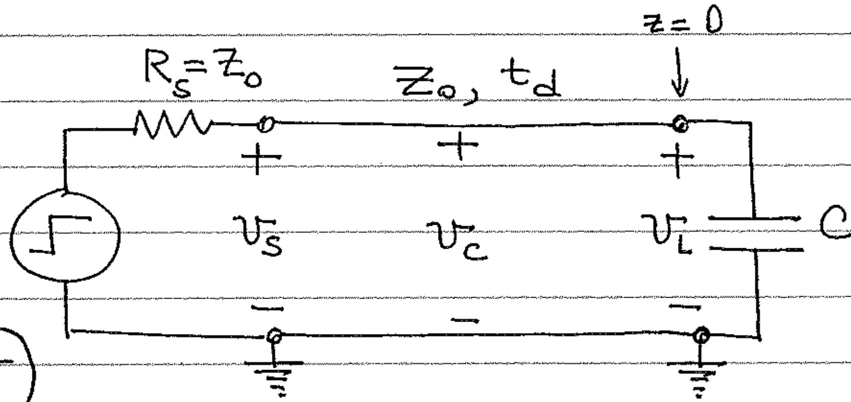
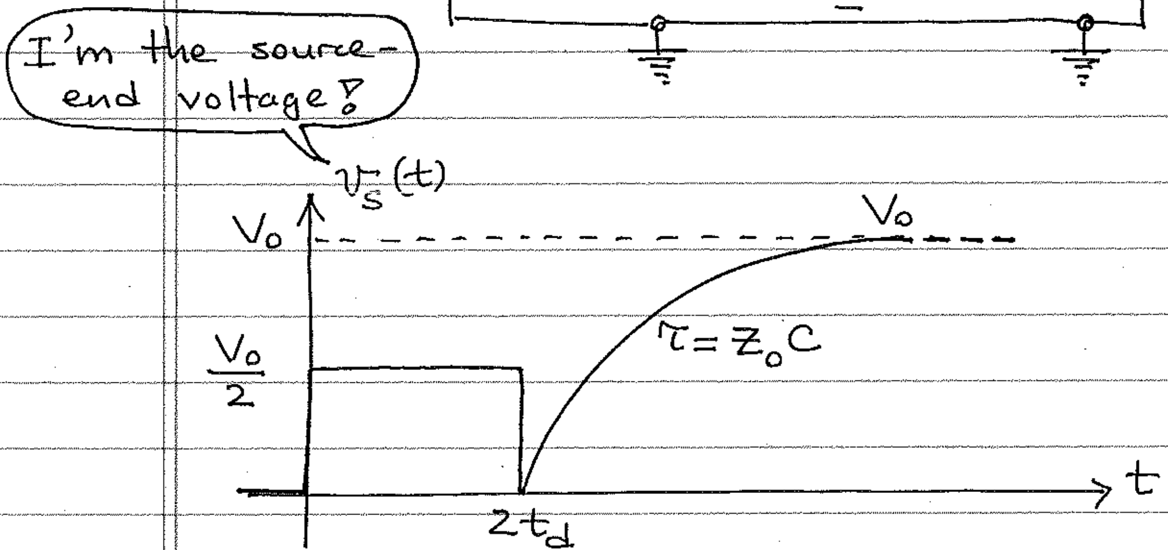
EE 301
SPRING 2012
A. INAN

SOLUTIONS TO HOMEWORK #3

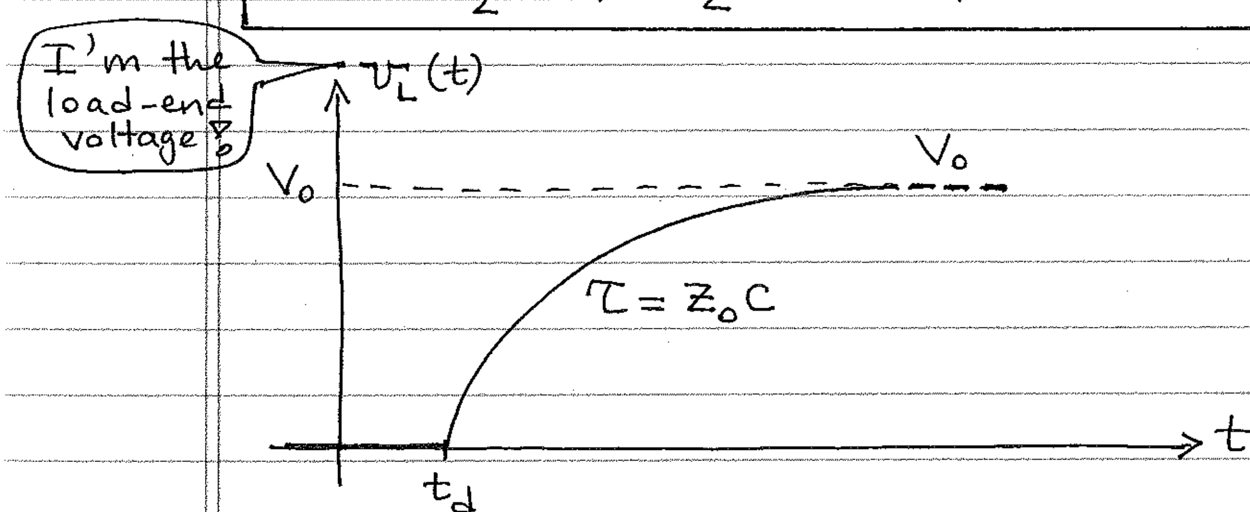
(#1)



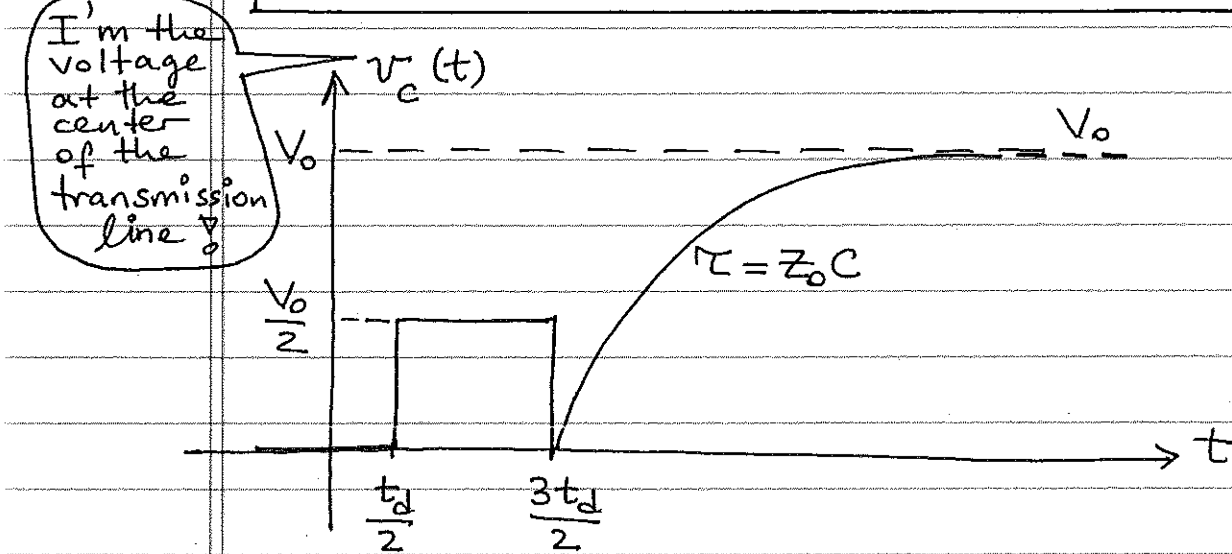


$$v_S(t) = \frac{V_0}{2}\left[u(t) - u(t-2t_d)\right] + V_0\left(1 - e^{-(t-2t_d)/(Z_0 C)}\right) u(t-2t_d)$$

$$\text{or} = \frac{V_0}{2} u(t) + \frac{V_0}{2} u(t-2t_d) - V_0 e^{-(t-2t_d)/(Z_0 C)} u(t-2t_d)$$



$$v_L(t) = V_0\left(1 - e^{-(t-t_d)/(Z_0C)}\right) u(t-t_d)$$



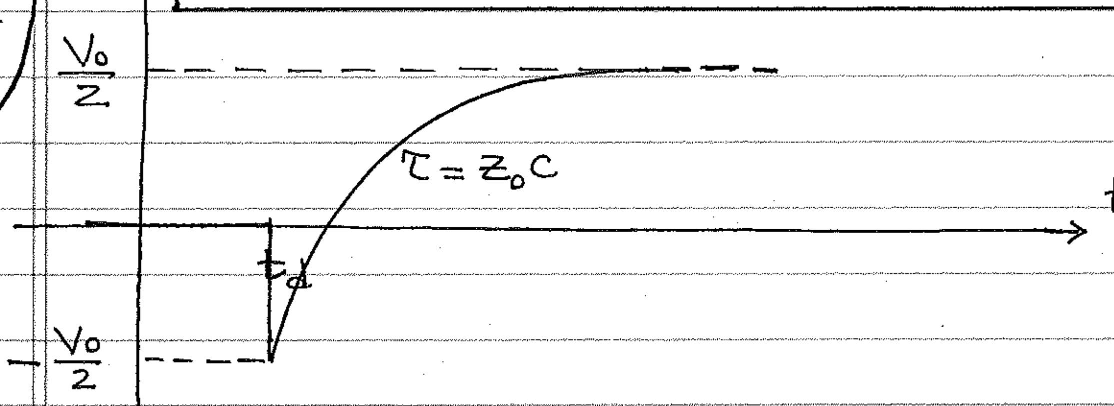
$$v_c(t) = \frac{V_0}{2}\left[u\left(t-\frac{t_d}{2}\right) - u\left(t-\frac{3t_d}{2}\right)\right] + V_0\left(1 - e^{-(t-3t_d/2)/(Z_0C)}\right) u\left(t-\frac{3t_d}{2}\right)$$

$$\text{or} = \frac{V_0}{2}u\left(t-\frac{t_d}{2}\right) + \frac{V_0}{2}u\left(t-\frac{3t_d}{2}\right) - V_0 e^{-(t-3t_d/2)/(Z_0C)}\, u(t-3t_d/2)$$

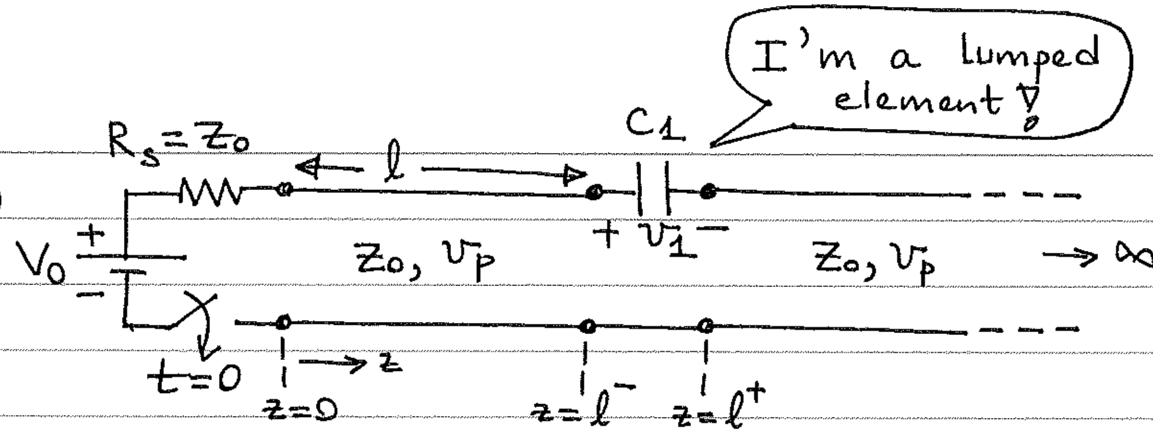
I'm the reflected voltage wave observed at the load end of the line!

$v_1^-(z=0,t)$ ← Load end

$$v_1^-(z=0,t) = \left[-\frac{V_0}{2}e^{-(t-t_d)/(Z_0C)} + \frac{V_0}{2}\left(1-e^{-(t-t_d)/(Z_0C)}\right)\right]u(t-t_d)$$

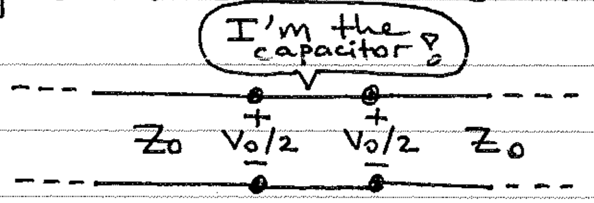


(#2)



(a) At $t = \ell/v_p$, $\Gamma_L(t=\ell/v_p) = 0$ since the capacitor initially behaves like a short circuit.

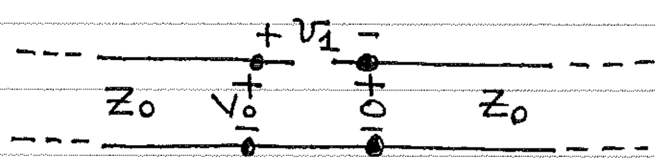
$t = \ell/v_p$



$$v_1(t=\ell/v_p) = 0$$

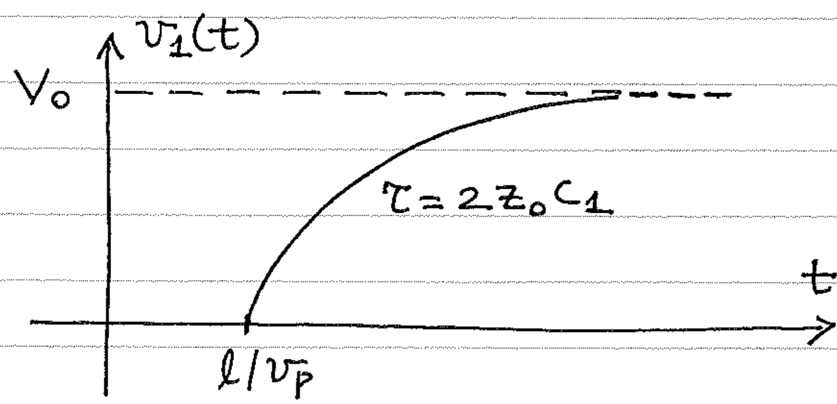
At $t \rightarrow \infty$, $\Gamma_L(t\rightarrow\infty) = +1$ since the capacitor behaves like an open circuit at steady state.

$t \rightarrow \infty$

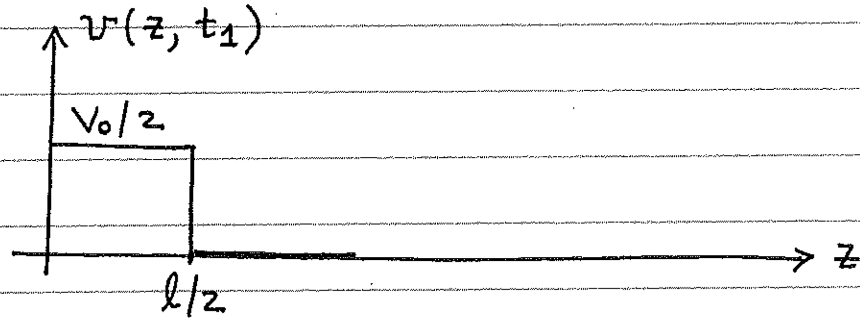


$$v_1(t\rightarrow\infty) = V_0$$

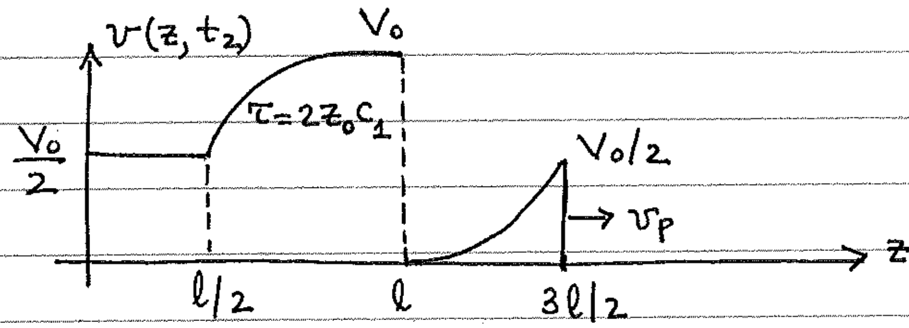
$$\therefore \; v_1(t) = V_0\left(1 - e^{-(t-\ell/v_p)/(2Z_0C_1)}\right)u(t-\ell/v_p)$$



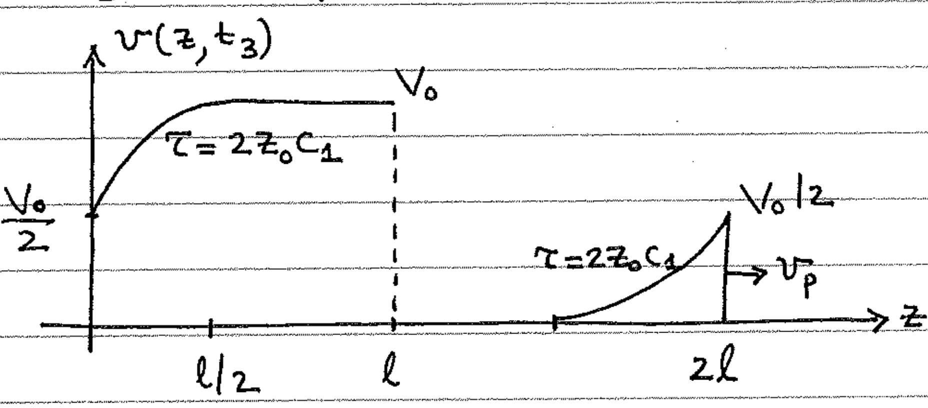
(b) $t_1 = \ell/(2v_p)$



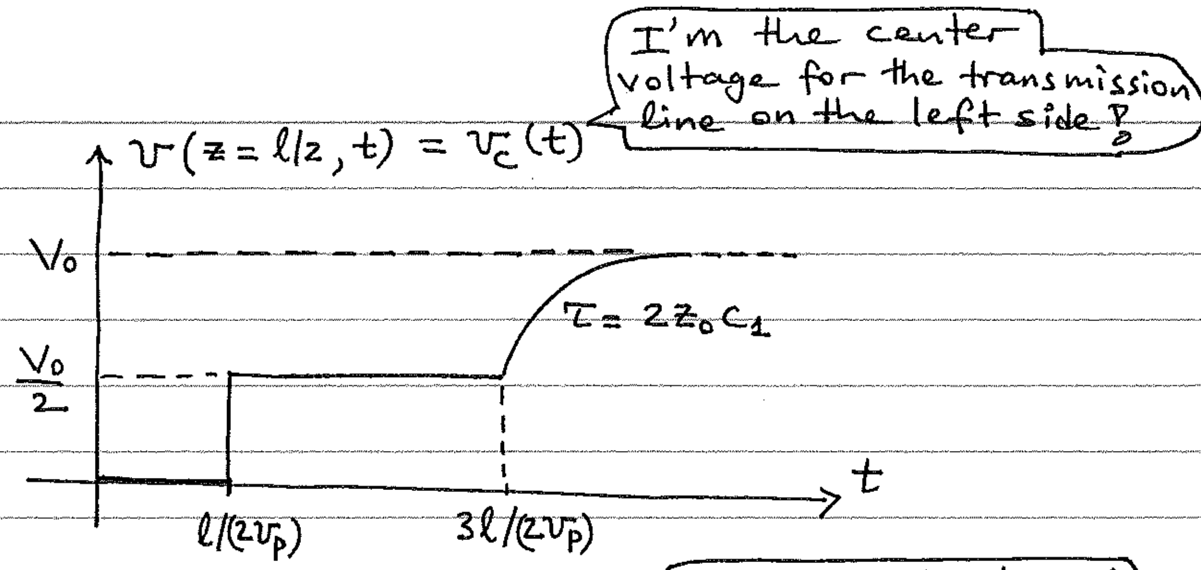
$t_2 = 3\ell/(2v_p)$

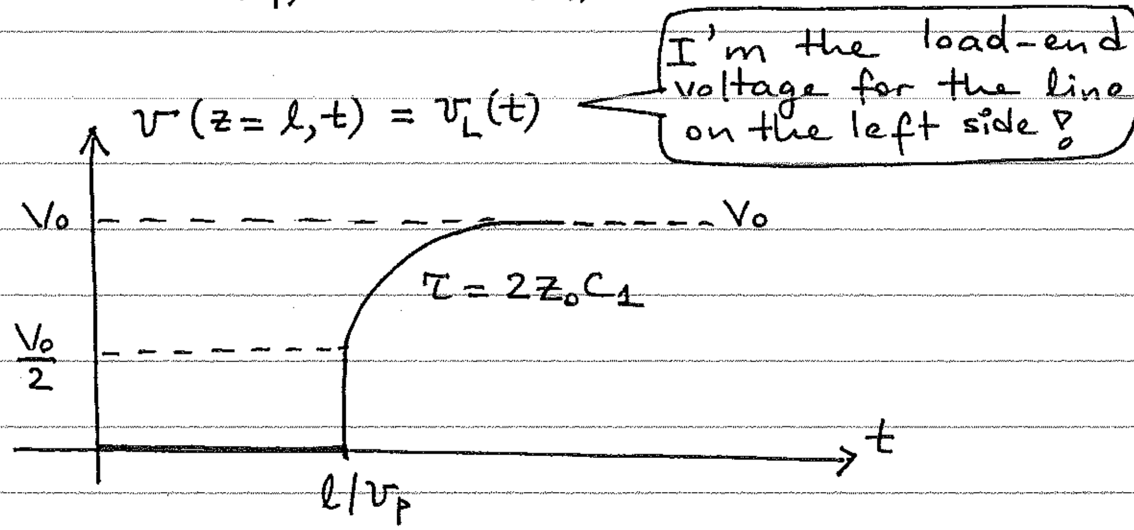


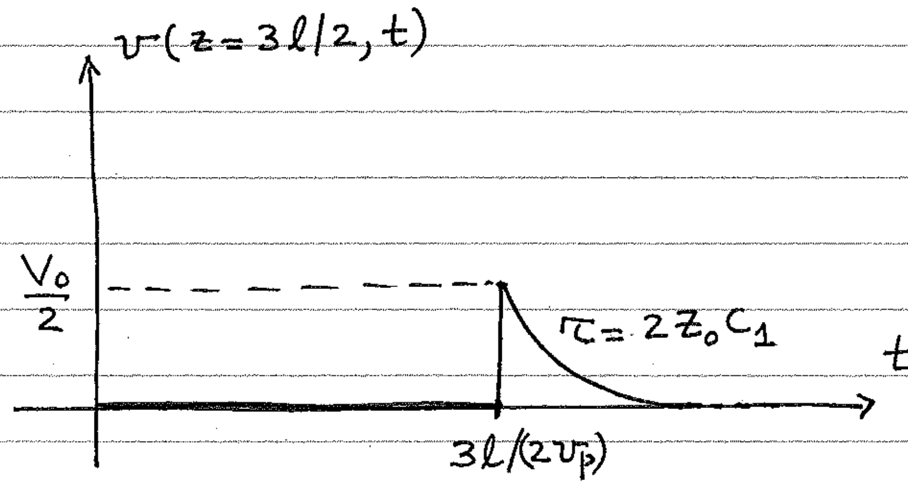
$t_3 = 2\ell/v_p$



(c)


I'm the center voltage for the transmission line on the left side!
$v(z=\ell/2, t) = v_C(t)$
V_0
$\tau = 2Z_0 C_1$
$\frac{V_0}{2}$
t
$\ell/(2v_p)$
$3\ell/(2v_p)$


I'm the load-end voltage for the line on the left side!
$v(z=\ell, t) = v_L(t)$
V_0
V_0
$\tau = 2Z_0 C_1$
$\frac{V_0}{2}$
t
ℓ/v_p


$v(z=3\ell/2, t)$
$\frac{V_0}{2}$
$\tau = 2Z_0 C_1$
t
$3\ell/(2v_p)$

(#3) The complete mathematical expressions for the TDR waveforms shown in Figure 2.34 (Inan & Inan, page 80) are as follows:

$$v_{s1}(t) = u(t) + \frac{R}{R+2Z_0} u(t-2t_d)$$

$$v_{s2}(t) = u(t) - \frac{Z_0}{2R+Z_0} u(t-2t_d)$$

$$v_{s3}(t) = u(t) + e^{-(t-2t_d)2Z_0/L} u(t-2t_d)$$

$$v_{s4}(t) = u(t) - \left(1-e^{-(t-2t_d)Z_0/(2L)}\right) u(t-2t_d)$$

$$v_{s5}(t) = u(t) + \left(1-e^{-(t-2t_d)/(2Z_0C)}\right) u(t-2t_d)$$

$$v_{s6}(t) = u(t) - u(t-2t_d) + \left(1-e^{-2(t-2t_d)/(Z_0C)}\right) u(t-2t_d)$$

$$= u(t) - e^{-2(t-2t_d)/(Z_0C)} u(t-2t_d)$$

(Note that for all of these waveforms, v_1^+ approaching from the left-hand-side is assumed to be 1 V.)