EE 301
SPRING 2012
A. INAN

SOLUTIONS TO HOMEWORK #5

(1) (a)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{70+\hat{j}10-50}{70+\hat{j}10+50} = \frac{20+\hat{j}10}{120+\hat{j}10}$$

$$= \frac{2+\hat{j}}{12+\hat{j}} \times \frac{12-\hat{j}}{12-\hat{j}} = \frac{25+\hat{j}10}{(12)^2+1^2} = \frac{5+\hat{j}2}{29}$$

$$= \frac{\sqrt{5^2+2^2}}{29} e^{\hat{j}\tan^{-1}(2/5)} = \frac{1}{\sqrt{29}} e^{\hat{j}\tan^{-1}(0.4)}$$

$$\cong \boxed{0.1857 e^{\hat{j}21.8^\circ}}$$

(b)

$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+1/\sqrt{29}}{1-1/\sqrt{29}} = \frac{\sqrt{29}+1}{\sqrt{29}-1}$$

$$\cong \boxed{1.456}$$

(c)

$$\phi_L + 2\beta z_{max_1} = 0$$

$$\lambda_{air} = \frac{c}{f} = \frac{3\times10^8 \text{ m/s}}{50\times10^6 \text{ Hz}} = 6 \text{ m}$$

$$(21.8^\circ)\frac{\cancel{\pi}}{180^\circ} + \frac{4\cancel{\pi}}{6} z_{max_1} = 0 \rightarrow z_{max_1} \cong \boxed{-0.5708 \text{ (m)}}$$

$$z_{min_1} = z_{max_1} - \frac{\lambda_{air}}{4} = -0.5708 - 1.5 = \boxed{-2.0708 \text{ (m)}}$$

$$z_{max_2} = z_{max_1} - \frac{\lambda_{air}}{2} = \boxed{-3.5708 \text{ (m)}}$$

$$z_{min_2} = z_{min_1} - \frac{\lambda_{air}}{2} = \boxed{-5.0708 \text{ (m)}}$$

$$z_{max_3} = z_{max_2} - \frac{\lambda_{air}}{2} = \boxed{-6.5708\,(m)}$$

$$z_{min_3} = z_{min_2} - \frac{\lambda_{air}}{2} = \boxed{-8.0708\,(m)}$$

$$z_{max_4} = z_{max_3} - \frac{\lambda_{air}}{2} = \boxed{-9.5708\,(m)}$$

z_{max} (m)	z_{min} (m)
−0.5708	−2.0708
−3.5708	−5.0708
−6.5708	−8.0708
−9.5708	

So all together, there are four V_{max} and three V_{min} positions on the transmission line.

(d) The input impedances at both V_{max} and V_{min} positions are real (purely resistive). The input impedance at each V_{max} position is given by

$$Z_{in}\Big|_{V_{max}} = S Z_0 \cong (1.456)(50) \cong \boxed{72.804\,(\Omega)}$$

The input impedance at each V_{min} position is given by

$$Z_{in}\Big|_{V_{min}} = \frac{Z_0}{S} \cong \frac{50}{1.456} \cong \boxed{34.34\,(\Omega)}$$

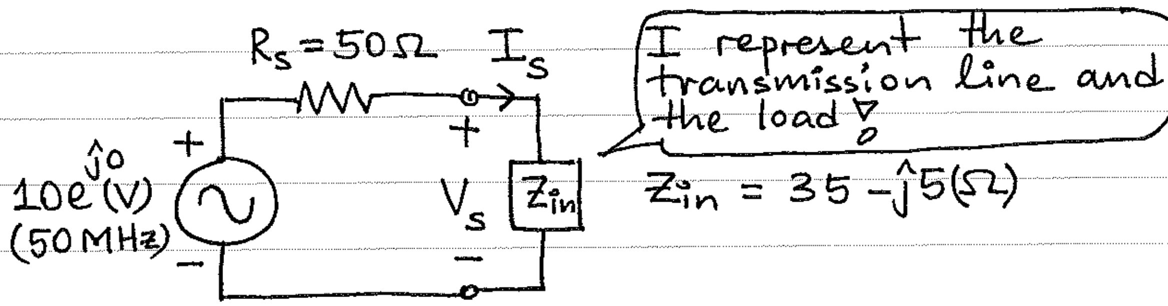
(e) Since the electrical length of the transmission line is given by

$$\frac{\ell}{\lambda_{air}} = \frac{10.5\,m}{6\,m} = 1.75$$

the input impedance of this line can be calculated as

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{50^2}{70+\hat{j}10} \times \frac{70-\hat{j}10}{70-\hat{j}10}$$

$$= \frac{(50)^2(70-\hat{j}10)}{(70)^2+(10)^2} = \boxed{35-\hat{j}5\,(\Omega)}$$



$Z_{in} = 35 - \hat{j}5\,(\Omega)$

(f) Using VDP:

$$V_s = \frac{Z_{in}}{R_s + Z_{in}}\left(10e^{\hat{j}0}\right) = \frac{35-\hat{j}5}{85-\hat{j}5}(10)$$

$$= \frac{70-\hat{j}10}{17-\hat{j}} \times \frac{17+\hat{j}}{17+\hat{j}} = \frac{1200-\hat{j}100}{(17)^2+1^2}$$

$$= \frac{10}{29}(12-\hat{j}) \cong \boxed{4.152\,e^{-\hat{j}4.764^\circ}\,(V)}$$

Using $V^+ = \dfrac{V_s}{e^{\hat{j}\beta \ell}\left(1+\Gamma_L e^{-\hat{j}2\beta\ell}\right)}$

we have

$$V^+ = \frac{4.152\, e^{-\hat{j}4.764^\circ}}{\underbrace{e^{\hat{j}\frac{7\pi}{2}}}_{\hat{j}}\left(1+0.1857\, e^{\hat{j}21.8^\circ}\underbrace{e^{-\hat{j}7\pi}}_{-1}\right)}$$

$$\cong \frac{4.152\, e^{-\hat{j}4.764^\circ}}{\hat{j}\left(\underbrace{1-0.1724}_{0.8276} - \hat{j}0.06897\right)}$$

$$= \frac{4.152\, e^{-\hat{j}4.764^\circ}}{0.06897 + \hat{j}0.8276}$$

$$\cong \frac{4.152\, e^{-\hat{j}4.764^\circ}}{0.83045\, e^{\hat{j}85.24^\circ}}$$

$$\cong \boxed{5.00\, e^{-\hat{j}90^\circ} \text{ (V)}}$$

$$V^- = \Gamma_L V^+ \cong \left(0.1857\, e^{\hat{j}21.8^\circ}\right)\left(5\, e^{-\hat{j}90^\circ}\right)$$

$$\cong \boxed{0.9285\, e^{-\hat{j}68.2^\circ} \text{ (V)}}$$

$$V_L = V^+ + V^- \cong 5e^{-\hat{j}90°} + 0.9285e^{-\hat{j}68.2°}$$

$$\cong -\hat{j}5 + 0.3448 - \hat{j}0.8621$$

$$\cong \boxed{5.872e^{-\hat{j}86.63°} \text{ (V)}}$$

(g) $V_{max} = |V^+|(1+|\Gamma_L|) \cong 5(1+0.1857)$

$$\cong \boxed{5.9285 \text{ (V)}}$$

$V_{min} = |V^+|(1-|\Gamma_L|) \cong 5(1-0.1857)$

$$\cong \boxed{4.0715 \text{ (V)}}$$

(h) $P^+ = \dfrac{|V^+|^2}{2Z_0} \cong \dfrac{5^2}{2\times 50} = \boxed{0.25 \text{ (W)}}$

$$P^- = \frac{|V^-|^2}{2Z_0} = |\Gamma_L|^2 P^+ \cong (0.1857)^2(0.25)$$

$$\cong \boxed{8.621 \text{ (mW)}}$$

From the equivalent circuit, we have

$$I_s = \frac{10e^{\hat{j}0}}{R_s + Z_{in}} = \frac{10}{\underbrace{50+35-\hat{j}5}_{85-\hat{j}5}} = \frac{2}{17-\hat{j}}$$

$$= \frac{2(17+\hat{j})}{(17)^2+1^2} = \frac{17+\hat{j}}{145} \cong 0.1174e^{\hat{j}3.3665°} \text{ (A)}$$

$$P_{R_s} = \frac{1}{2}|I_s|^2 R_s \cong \frac{1}{2}(0.1174)^2(50)$$

$$\cong \boxed{0.3448\ (W)}$$

$$P_L = P_{Z_{in}} = \frac{1}{2}|I_s|^2 R_{in} \cong \frac{1}{2}(0.1174)^2(35)$$

$$\cong \boxed{0.2414\ (W)}$$

$$P_{source} = P_{R_s} + P_{Z_{in}} \cong 0.3448 + 0.2414$$

$$= \boxed{0.5862\ (W)}$$

% of incident wave power that reflects back to the source can be found as

$$|\Gamma_L|^2 \times 100 \cong (0.1857)^2 \times 100$$

$$\cong \boxed{3.448\ \%}$$