University ☺f P☺rtland
Sch☺☺l ☺f Engineering

EE 301-Electromagnetic Fields-3 cr. hrs.
Spring 2014

Midterm Exam # 2
Sinusoidal Steady-State Waves on Transmission Lines

(Prepared by Professor A. S. Inan)

(Monday, April 4, 2014)
(Closed Book Exam; 3 Formula Sheets Allowed)
(Total Time: 55 mins.)

Name: SOLUTIONS ☺

Signature: SOLUTIONS ☺

"*Honesty is the best policy.*"
Aesop (~ 620B.C. –?)

"*An honest mind possesses a kingdom.*"
Lucius Annaeus Seneca (4B.C.–65A.D.)

"*Honest people are the true winners of the universe.*"
Anonymous

This table will be used by Inan for grading!

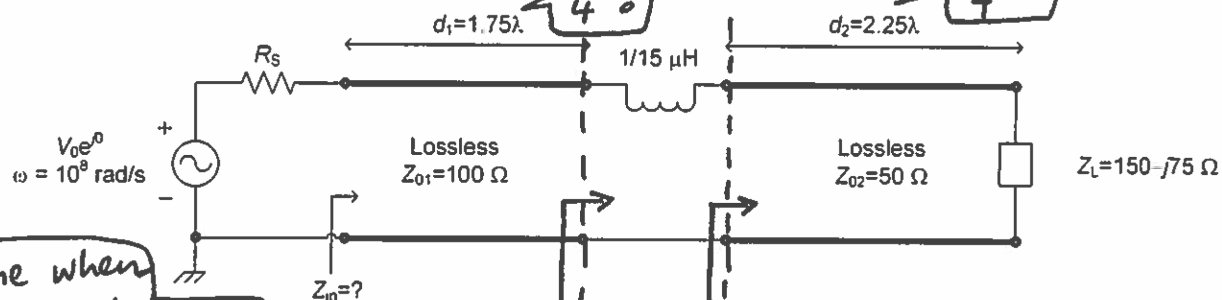
Problem #	Points gained
#1	
#2	
#3	
#4	
Total	

We are both odd integer multiples of quarter wavelength!

(1)(10 mins., 20 points) **Input impedance of a transmission-line circuit.** For the double transmission-line circuit shown, find the input impedance Z_{in} at $\omega = 10^8$ rad-s^{-1}.

$\frac{7\lambda}{4}$!

$\frac{9\lambda}{4}$!



Apply me when the line length equals odd integer multiple of quarter wavelength!

Z_{L1} $\quad$ Z_{in2}

$$Z_{in_2} = \frac{Z_{02}^2}{Z_L} = \frac{(50)^2}{150-\hat{j}75} = \frac{100}{6-\hat{j}3} = \frac{100}{3(2-\hat{j})} \times \frac{2+\hat{j}}{2+\hat{j}}$$

I'm the impedance of the inductor!

$$= \frac{20(2+\hat{j})}{3}\,\Omega \simeq 13.3+\hat{j}6.67\,\Omega$$

$$Z_{L1} = Z_{ind} + Z_{in2} = \hat{j}(10^8)\left(\frac{10^{-6}}{15}\right) + \frac{20(2+\hat{j})}{3} = \frac{40(1+\hat{j})}{3}\,\Omega$$

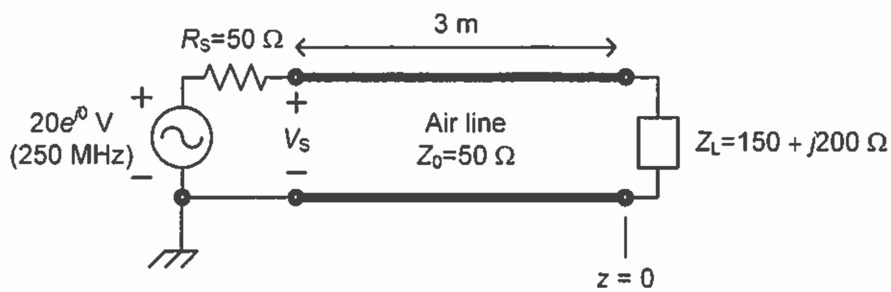
$$\simeq 13.3\,(1+\hat{j})\,\Omega$$

$$Z_{in} = \frac{Z_{01}^2}{Z_{L1}} = \frac{3(100)^2}{40(1+\hat{j})} = \frac{750}{1+\hat{j}} \times \frac{1-\hat{j}}{1-\hat{j}} = \frac{750}{2}(1-\hat{j})$$

$$= \boxed{375\,(1-\hat{j})\,\Omega}$$

Just like $9\lambda/4$, $7\lambda/4$ is also an odd integer multiple of quarter wavelength!

(2)(15 mins., <u>Total:</u> 40 points) **A lossless transmission line terminated with a complex impedance.** A $50\,\Omega$ air transmission line is terminated with an inductive load impedance given by $Z_L = 150 + j200\,\Omega$ and excited by a sinusoidal voltage source as shown.



(a) (10 points) Calculate the load reflection coefficient Γ_L. (Provide your answer in polar form.) Show your work!

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 + \hat{j}200 - 50}{150 + \hat{j}200 + 50} = \frac{100(1+\hat{j}2)}{200(1+\hat{j})} \times \frac{1-\hat{j}}{1-\hat{j}}$$

$$= \frac{3+\hat{j}}{4} = \frac{\sqrt{10}}{4} e^{\hat{j}\tan^{-1}(1/3)} \simeq \boxed{0.791\, e^{\hat{j}18.4^\circ}}$$

We use the magnitude of Γ_L!

(b) (5 points) What is the value of the standing wave ratio S on the line?

$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+\frac{\sqrt{10}}{4}}{1-\frac{\sqrt{10}}{4}} = \frac{4+\sqrt{10}}{4-\sqrt{10}}$$

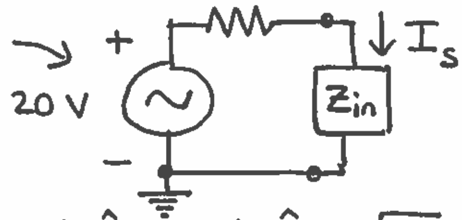
$$\simeq \boxed{8.55}$$

(c) (15 points) Calculate the time-average power delivered to the load.

I'm the electrical length of the transmission line

$$\lambda_{air} = \frac{c}{f} \simeq \frac{3\times10^{8}\ m\text{-}s^{-1}}{250\times10^{6}\ Hz} = 1.2\,m \rightarrow \bar{l} = \frac{l}{\lambda} = \frac{3m}{1.2m} = 2.5$$

$$Z_{in} = Z_L = 150 + \hat{j}200\,\Omega \rightarrow$$



$$I_s = \frac{20e^{\hat{j}0}}{50+150+\hat{j}200} = \frac{1}{10(1+\hat{j})} \times \frac{1-\hat{j}}{1-\hat{j}} = \frac{1-\hat{j}}{20} = \frac{\sqrt{2}}{20}e^{-\hat{j}45^\circ}\ A$$

$$P_L = P_{Z_{in}} = \frac{1}{2}|I_s|^2 R_{in} = \frac{1}{2}\left(\frac{\sqrt{2}}{20}\right)^2(150) = \boxed{0.375\ W}$$

Since the line is lossless...

(d) (10 points) Find the first two V_{max} and the first two V_{min} positions nearest to the position of the load on this transmission line. Provide your answers in units of distance.

Since the load is inductive, Vmax is closer to the load.

$$\phi_L + 2\beta z_{max_1} = 0 \rightarrow z_{max_1} = -\frac{\phi_L}{2\beta} \simeq \boxed{-0.0307\,m} = -3.07\,cm$$

$\phi_L \sim 0.322$ rad

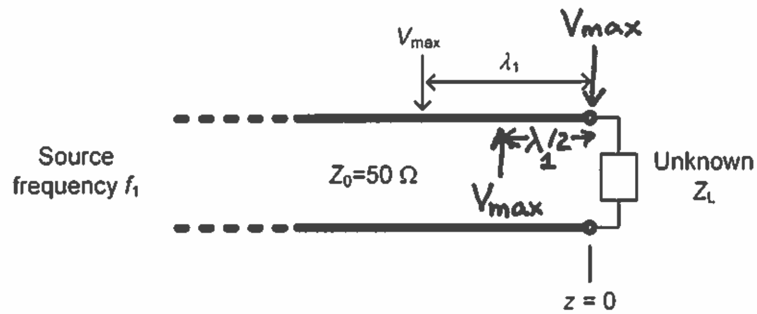
$\beta = \frac{2\pi}{1.2}$

$$z_{max_2} = z_{max_1} - \frac{\lambda_{air}}{2} \simeq -0.0307 - 0.6 = \boxed{-0.6307\,m}$$

$$z_{min_1} = z_{max_1} - \frac{\lambda_{air}}{4} \simeq -0.0307 - 0.3 = \boxed{-0.3307\,m}$$

$$z_{min_2} = z_{min_1} - \frac{\lambda_{air}}{2} \simeq \boxed{-0.9307\,m}$$

(3)(10 mins., 20 points) **Unknown load.** The standing wave ratio on a 50 Ω transmission line excited at f_1 frequency and terminated with an unknown load Z_L is measured to be 5. If a voltage maximum position on the line is located at λ_1 distance away from the load position, determine the value of the load impedance Z_L. (Note that λ_1 is the wavelength at source frequency f_1.)



$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \rightarrow |\Gamma_L| = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

If a voltage maximum position on the line is located at λ_1 distance away from the load, since Vmax positions alternate every $\lambda_1/2$, the load position must also be a voltage maximum position. This means

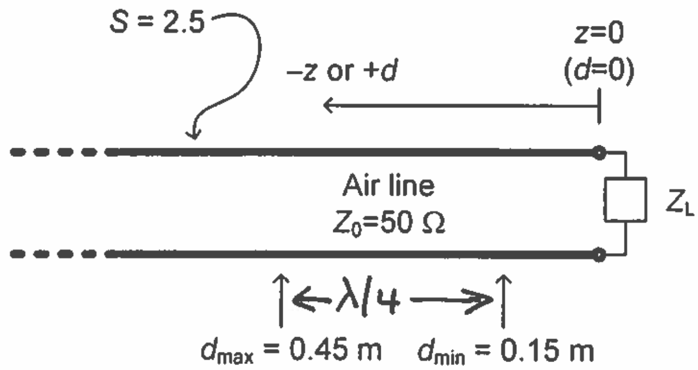
$$\phi_L = 0 \rightarrow \therefore \Gamma_L = \frac{2}{3}e^{j0} = \frac{2}{3}$$

Using $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} \rightarrow \dfrac{2}{3} = \dfrac{Z_L - 50}{Z_L + 50}$

$$\rightarrow 2Z_L + 100 = 3Z_L - 150 \rightarrow Z_L = \boxed{250\,\Omega}$$

As expected, I'm purely resistive and my value is greater than Z_0!

(4)(10 mins., 20 points) **Unknown load.** A 50 Ω air transmission line with a standing wave ratio of $S = 2.5$ has its first voltage minimum and maximum positions on the line nearest to the load at 0.15 m and 0.45 m respectively. Calculate (a) the operating frequency f; and (b) the load impedance Z_L.



(a) $$\frac{\lambda_{air}}{4} = 0.45 - 0.15 = 0.3\,\text{m} \rightarrow \lambda_{air} = 1.2\,\text{m}$$

$$f = \frac{c}{\lambda_{air}} \simeq \frac{3 \times 10^8\ \text{m-s}^{-1}}{1.2\,\text{m}} = \boxed{250\ \text{MHz}}$$

(b) $$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \rightarrow |\Gamma_L| = \frac{S-1}{S+1} = \frac{2.5-1}{2.5+1} = \frac{3}{7}$$

I represent a capacitive load!

$$\phi_L + 2\beta z_{min_1} = -\pi$$

$$\rightarrow \phi_L = -\pi - 2\left(\frac{2\pi}{1.2}\right)(-0.15) = -\pi + \frac{\pi}{2} = -\frac{\pi}{2}$$

$$\therefore \Gamma_L = \frac{3}{7} e^{-\hat{j}\frac{\pi}{2}} = -\hat{j}\frac{3}{7}$$

Using $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \rightarrow Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$$

$$\rightarrow Z_L = 50 \frac{1 - \hat{j}\frac{3}{7}}{1 + \hat{j}\frac{3}{7}} = 50 \frac{7-\hat{j}3}{7+\hat{j}3} \times \frac{7-\hat{j}3}{7-\hat{j}3}$$

$$= 50 \frac{40 - \hat{j}42}{49+9} \simeq \boxed{34.5 - \hat{j}\,36.2\,\Omega}$$

Indeed I'm capacitive!