and

$$\Gamma = -\frac{Y_n^{TM} - 1}{Y_n^{TM} + 1}$$

$$T = \frac{2}{Y_n^{TM} + 1}$$

$$Y_n^{TM} = \frac{\epsilon_t / k_{tz}}{\epsilon_i / k_{iz}}$$

where $Y_n^{TM} = 1/Z_n^{TM}$.

At *Brewster's angle*, the reflection coefficient is zero. For media in which $\epsilon_i \neq \epsilon_t$ but $\mu_i = \mu_t$, the Brewster's angle exists only for *TM* modes and is equal to

$$\theta_B = \tan^{-1}\sqrt{\epsilon_t/\epsilon_i}$$

For media in which $\mu_i \neq \mu_t$ but $\epsilon_i = \epsilon_t$, Brewster's angle exists only for *TE* modes.

Current sheets may also be located at boundaries where they may radiate, scatter, or absorb power. For surfaces along which the phase variation of the current is sinusoidal [i.e., $\overline{J} = \overline{J}_{sA}\, e^{-jk_{Ax}x}\delta(z)$], it is particularly simple to find the $\overline{E}$ and $\overline{H}$ fields on either side of the surface because they have the form of plane waves. Boundary conditions are then used to determine the magnitudes and directions of each of the fields.

4.7 PROBLEMS

4.1.1 An exotic material has the properties $\mu = \infty$, $\sigma = 0$, and $\epsilon = 0$. What boundary conditions does such a surface impose on $\overline{E}$, $\overline{H}$, $\overline{B}$, and $\overline{D}$ in an adjacent free space region at nonzero frequencies?

4.1.2 Repeat Problem 4.1.1 for $\epsilon = \infty$, $\mu = 0$, and $\sigma = 0$.

4.2.1 Ambient light in a room can produce annoying reflections from CRT displays. Prepare a simple plot showing the percent power reflected at normal incidence from a flat plate of permittivity ϵ for $\epsilon_0 < \epsilon < 10\epsilon_0$. Note that ϵ/ϵ_0 is approximately 1.0006, 2.5, 3.75, 6.00, and 9.5 for air, lucite, fused quartz, Pyrex 1710, and Corning 8879, respectively.

4.2.2 Narcissus looked at his reflection in a pool of water with $\mu = \mu_0$, $\epsilon = 1.8\epsilon_0$. What fraction of the light flux density ($\lambda = 0.5 \times 10^{-6}$ m) incident upon the water was reflected at normal incidence? What standing wave ratio would be measured at this boundary?

4.2.3 A uniform plane wave of intensity 1 W/m^2 in free space is normally incident on a planar perfect conductor located at $z = 0$. The total electric field is

$$\overline{E} = (\hat{x} + j\hat{y})\, E_0\, e^{-j\pi z} + \overline{E}_r$$

(a) What is the frequency f of this wave?
(b) Evaluate E_0 quantitatively.
(c) What is the reflected term $\overline{E}_r$ in the expression for the total electric field? Give your answer in the same format as that of the incident wave.
(d) What is the polarization of the reflected wave?
(e) At $z = -\lambda/2$, what is the time-dependent magnetic field $\overline{H}(x, y, z, t)$?

4.2.4 What is the highest frequency at which 99.9 percent of the energy of an incident plane wave would be reflected from a flat copper slab (thick compared to a skin depth) having conductivity $\sigma = 5 \times 10^7$ S/m? What is the frequency for 99 percent reflection?

4.2.5 Complex optical systems require low-loss mirrors because the signals may be reflected many times. Consider a polished silver mirror with $\sigma = 5 \times 10^7$ S/m.

(a) What is the skin depth δ in the silver for infrared light of 10 μm wavelength?
(b) What is the wavelength λ inside the silver?
(c) What fraction of the power in this infrared beam would be reflected at normal incidence?

4.3.1 A uniform laser beam in free space is normally incident on a glass surface that reflects nine percent of the power and transmits the rest.

(a) What is the permittivity ϵ of the glass?
(b) If the beam is incident at 60°, what is the angle of transmission θ_t?

4.3.2 A 10-W $\hat{y}$-polarized uniform plane wave in free space is incident upon a planar interface at $\theta_i = 60°$; the plane of incidence is the x–z plane. The medium on the other side of the interface ($z > 0$) is characterized by $\epsilon = \epsilon_0/2$ and $\mu = \mu_0$.

(a) What is the direction of the transmitted electric field?
(b) Give the magnitude of the transmitted electric field and its phase relative to the incident field.

4.3.3 A plane wave is incident at 45° upon a perfectly conducting surface in the y–z plane. The plane wave consists of both *TE* and *TM* components as follows:

$$\overline{E}^{TE} = \hat{y}\, E_0\, e^{jk(x-z)/\sqrt{2}}$$

$$\overline{E}^{TM} = (\hat{x} + \hat{z})\, \frac{jE_0}{\sqrt{2}}\, e^{jk(x-z)/\sqrt{2}}$$

(a) Write the *TE* and *TM* components of the electric fields of the reflected wave in the same form used above in describing the incident wave, and show that the tangential electric fields satisfy the boundary conditions.
(b) What are the polarizations of the incident and the reflected waves?

4.3.4 A 1-MHz uniform plane wave is incident on a planar plasma boundary at 60° with $\mu = \mu_0$.

(a) If no wave is reflected, what is the dielectric constant of the transmitting medium?
(b) If no wave is transmitted, what is the minimum plasma frequency of the transmitting medium?

4.3.5 Light can be trapped inside optical fibers if it is reflected internally at angles θ greater than the critical angle θ_c. If the wavelength is small compared to the fiber radius, the reflection is approximately planar. The field strength in the evanescent region decays