Some Interesting Numerical Properties "Hidden" Within the Digits of Pi

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Pi Day is celebrated around the world every year on March 14 (3/14, or simply 314) because 314 constitutes the first three digits of number pi. This year, Pi Day is expressed as 3/14/15 (31415) and is particularly unique since 31415 represents the first five digits of pi. This once-in-a-century Pi Day piqued our curiosity and motivated us to revisit the number pi, in search of finding some undiscovered interesting numerical properties "hidden" within its digits.

Historically, pi (or π), the ratio of any circle's circumference to its diameter, have fascinated and inspired mathematicians for four millennia [1-4]. Using basic experimentation, many mathematicians in early civilizations figured out that the length of a rope wound around the circumference of a circle is equal to approximately three times the length of its diameter.

The calculation of the digits of pi was revolutionized by the development of infinite series techniques during the 16th and 17th centuries. Infinite series allowed mathematicians to compute pi with much greater precision than ever before.

Pi is an irrational number, meaning that it cannot be written as the ratio of two integers. Since π is irrational, it has an infinite number of digits, and does not appear to settle into a repeating pattern of digits.

Using powerful computers, mathematicians are now able to compute the value of pi to billions of digits, but still, no one has ever found any evidence that calculating more and more digits of pi will reveal that there is a regular pattern that exists within its digits.

A number consisting of an infinite number of digits is called *normal* when all possible sequences of digits of any given length appear equally often. The conjecture that π is *normal* has not yet been proven or disproven.

In this article, numerous "hidden" number connections are revealed between the digits of pi. The authors discovered most of these number connections by splitting the digits of pi into groups of three consecutive digits. The following table lists the first 45 digits of pi in groups of three digits.

3.14	**159**	**265**	**358**	**979**
323	**846**	**264**	**338**	**327**
950	**288**	**419**	**716**	**939**

The following "hidden" properties were observed:

1. The prime factors of the first three digits of pi, 314, add up to 2 + 157 = 159, the next three digits of pi. (The prime factors of a positive integer are the prime numbers that divide this integer exactly. For example, 314 = 2 x 157, where 2 and 157 are prime numbers.)
2. The reverse of the next three digits of pi, 159, is 951. Interestingly enough, the difference of the prime factors of 951 yields 317 – 3 = 314, the first three digits of pi.

3. The sum of 314 and 951 (which is the reverse of 159) yields 1265, where the rightmost three digits are 265, corresponding to the next three digits (7^{th} to 9^{th}) of pi.
4. The product of 159 and the reverse of 265 (562) yields 89358, where the rightmost three digits (358) are the next three digits (10^{th} to 12^{th}) of pi. Also, interestingly enough, if 89358 is split into numbers 893 and 58, these two numbers add up to 951, which is reverse of 159. In addition, 159 + 265 = 8 x 53, where if numbers 8 and 53 are put side-by-side as 853, the reverse of this number is also 358.
5. If the 5^{th} to 12^{th} digits of pi (59265358) are split as 59, 265, and 358, the sum of 59, the reverse of 265 (562), and 358 equals 979, which is the next three digits (13^{th} to 15^{th}) of pi.
6. Subtracting twice 314 from 951 (the reverse of 159) yields 323, the next three digits (16^{th} to 18^{th}) of pi. Also, the reverse of 323 plus 1 equals 3 times 141, where 3 and 141 side-by-side constitutes the first four digits of pi.
7. 323 plus 1 times 2 yields 648, the reverse of which is 846, which corresponds to the next three digits (19^{th} to 21^{st}) of pi. Also, 141 (the 2^{nd} to 4^{th} digits of pi) times the sum of its digits yields 846.
8. 265 minus 1 yields 264, the next three digits (22^{nd} to 24^{th}) of pi, and since 264 equals 33 times 8, 33 and 8 put side-by-side makes 338, which constitutes the next three digits (25^{th} to 27^{th}) of pi.
9. Numbers 846, 264, and 338 (which side-by-side as 846264338 constitute the 19^{th} to 27^{th} digits of pi) are numerically connected in an interesting way: Reverse of 264 (462) multiplied by reverse of 338 (833) yields 384,846 where the rightmost three digits are 846. In addition, if 384,846 is split in the middle as 384 and 846, 846 minus 384 results in 462, which is the reverse of 264.
10. 979 (which corresponds to the 13^{th} to 15^{th} digits of pi) plus 2 divided by 3 yields 327, the next three digits (28^{th} to 30^{th}) of pi.
11. One less than 951 (the reverse of 159, the 4^{th} to 6^{th} digits of pi) is 950, the next three digits (31^{st} to 33^{rd}) of pi. In addition, 723 (which is the reverse of 327, the 28^{th} to 30^{th} digits of pi) plus the reverse of one less than 723 also equals 950. Also, 592 (the 5^{th} to 7^{th} digits of pi) plus 358 (the 10^{th} to 12^{th} digits of pi) yield 950.
12. The difference of 626 (the 21^{st} to 23^{rd} digits of pi) and 338 (the 25^{th} to 27^{th} digits of pi) is 288, the next three digits (34^{th} to 36^{th}) of pi. Also, 338 plus twice 288 yields 914, the reverse of which is 419, corresponding to the next three digits (37^{th} to 39^{th}) of pi. (Also, note that 914 equals 626 plus 288.)
13. Twice 358 (the 10^{th} to 12^{th} digits of pi) yield 716, the next three digits (40^{th} to 42^{nd}) of pi.
14. Three times half of the difference of the reverse of 419 (the 37^{th} to 39^{th} digits of pi) and 288 (the 34^{th} to 36^{th} digits of pi) yields 939, the next three digits (43^{rd} to 45^{th}) of pi. And, 939 minus 2 results in 937, which constitute the next three digits (46^{th} to 48^{th}) of pi. Also, interestingly enough, 937 is the 159^{th} (the 4^{th} to 6^{th} digits of pi) prime number.

The authors find these "hidden" properties fascinating. And there probably exist many more interesting undiscovered properties hidden within the number π which the authors will continue to investigate. And, who knows, maybe the findings of this article will someday lead mathematicians to make a "breakthrough" to prove or disprove, once and for all, if π is a *normal* irrational number. And, by the way, next year's Pi Day (3/14/16 or 31416) is also interesting since 3.1416 is the value of pi "rounded off" to 5 significant figures.

[1] Petr Beckmann, "A History of Pi," The Golem Press, 1971.
[2] David Blatner, "The Joy of π," Walker, 1997.
[3] Jörg Arndt and Christoph Haenel, "Pi—Unleashed," Springer, 2001.
[4] Alfred S. Posamentier and Ingmar Lehmann, "A Biography of the World's Most Mysterious Number," Prometheus Books, 2004.