A Mathematical Model of Suicide Risk among US Veterans

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Abstract: Suicide is a major public health issue, especially among our service members. In the VA's National Suicide Data Report, spanning from 2005-2015, they report an average of 16.8 suicides per day for veterans and 3.8 for active-duty service members. The majority of existing work in suicidology provides qualitative research, often in the form of correlational relationships among factors involved in suicidal ideation; however, such studies lack the mathematical sophistication to be useful as a predictive measure, so that at-risk individuals might receive the help they need before it is too late. In an effort to create more of a predictive tool, we developed an ordinary differential equations model based on a 2013 study conducted by Leemput et al.[1], where they implemented the theory of critical slowing to flag individuals who were near a dangerous tipping point in their emotional landscape. We identified seven major factors involved in suicidal behavior and, through our modeling effort, analyzed the complex interactions among each of these factors. Here we share results from global and local sensitivity analyses and discuss ways in which this research may be used to further improve the care we provide to our veterans.

Prominent Interactions in Suicidal Behavior

When looking at suicide risk and the factors that play a role in increasing one's risk for ideation or attempts, there is a complex web of interactions at play. Though we encountered over 39 potential factors from the literature, for our purposes, we elected to model interactions among the seven characteristics that were most prominent in the literature: *desire for death, acquired capability, social isolation, social connection, drug and alcohol use, anger level*, and *happiness level (see Figure 1)*.

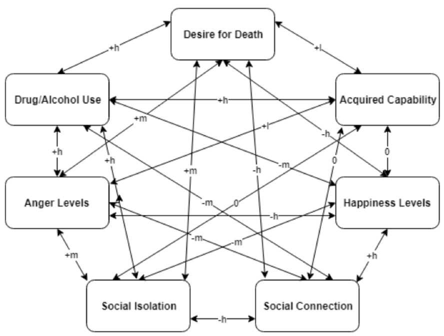

Figure 1. The simplified version of interactions in suicidal behavior used for our model. We assume all characteristics interact with all others, to a varying degree: h-high level of interaction; m-medium level; l-low level. A plus represents positive interaction (same valence) while a minus indicates a negative interaction (opposite valence).

Desire for death[2] is an individual's increasingly pessimistic outlook on future events, leading to compromised belongingness. ***Acquired capability***[2] is the degradation of self-preservation, or one's ability "to inflict lethal harm." ***Drug and alcohol*** includes any excess alcohol consumption or inappropriate use of drugs, legal or illegal. ***Social isolation***[3] is the counter to ***social connection***, which involves the level of integration into social circles and affects sense of belonging. ***Happiness*** is defined as a level of contentment, while ***Anger***[4] is defined as a level of displeasure.

Mathematical Model

We employ a Lotka-Volterra relationship among the key characteristics, with added stochastic elements to account for random fluctuations in human behavior and environmental factors. Our model, shown below, was simulated using an Euler-Maruyama scheme in MATLAB.

$$\frac{dx_i}{dt} = (r_i + \epsilon_r) + \sum_{j}^{7} C_{ij} x_j x_i + \mu \qquad \begin{cases} x_1 = \text{desire for death levels} \\ x_2 = \text{acquired capability levels} \\ x_3 = \text{drug/alcohol use levels} \\ x_4 = \text{anger levels} \\ x_5 = \text{social isolation levels} \\ x_6 = \text{happiness levels} \\ x_7 = \text{social connection levels} \end{cases}$$

Here r_i dictates the maximum rate of change of variable i (indicating "stress level"); C_{ij} governs the level of interaction between variables i and j; μ represents a small increase of the strength of a variable (μ=1); and ϵ_r is a noise term. The valence of each interaction is determined by the respective valences of the variables interacting; if an interaction is with two variables of the same valence they will work cooperatively, whereas interactions between opposite valences will work competitively.

Equilibria and Global Sensitivity Analysis

$*$	*best*	*average*	*worst*
x_1	1	3	9
x_2	1	3	9
x_3	1	3	9
x_4	1	5	9
x_5	1	5	9
x_6	9	5	1
x_7	9	5	1

Figure 2. Equilibrium levels for each variable, for best, average, and worst case scenarios.

We determined values for each of the interaction rates C_{ij}, in order to arrive at three different sets of equilibria for three different emotional landscapes: *best case*, *average case*, and *worst case*. Figure 2 shows the equilibria used for all three of our scenarios. The equilibria for the respective scenarios were supported by literature and data. For these scenarios, we only conducted analysis on a deterministic version of the model.

For each of the three scenarios from Figure 2, we conducted a global sensitivity analysis on the seven rate of change variables r_i and the 28 unique interaction parameters C_{ij}. Extended Fourier Sensitivity Analysis (eFAST) was used to gain knowledge of which specific interactions are the most important to the overall outcome of each variable. Developed by Saltelli et al[5], eFAST is a variance-based method capable of partitioning output variance to the appropriate parameter in a model. Due to the innate complexity of suicidal behavior, eFAST is a natural choice compared to less sophisticated methods. In all three scenarios, interactions between each variable and itself were the strongest (see, for example, C_{11} in Table 1, which displays a subset of our SA indices).

Table 1. First 7 Sensitivity Indices for Average Case Scenario at t = 50. Each parameter was varied by +/- 25% of their nominal value

Output Variables		C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}
	x_1	0.5527	0.0002	0.0255	0.0302	0.0311	0.0047	0.0046
	x_2	0.0172	0.0002	0.0031	0.0037	0.0039	0.0000	0.0000
	x_3	0.0479	0.0000	0.0400	0.0042	0.0043	0.0001	0.0001
	x_4	0.0444	0.0000	0.0033	0.0350	0.0047	0.0001	0.0001
	x_5	0.0433	0.0000	0.0035	0.0049	0.0362	0.0001	0.0001
	x_6	0.0091	0.0000	0.0012	0.0013	0.0013	0.0014	0.0000
	x_7	0.0096	0.0000	0.0011	0.0013	0.0013	0.0000	0.0014

Local Sensitivity Analysis

Test 1: Varying initial conditions. Initial conditions were perturbed systematically over the range of possible values to test for changes in equilibria and overall system dynamics. For each of the three scenarios from Figure 2, the system was not sensitive to changes in initial conditions (data not shown).

Test 2: Varying interaction terms. Interaction terms (C_{ij}) in key, high-level interactions were perturbed across a wide range of possible values, for each of the three scenarios. System dynamics were not affected in any unexpected manner:

- As interaction parameters between variables of the same valence increased, so did the equilibria for both variables.
- As interaction parameters between variable of opposite valence increased, both equilibria decreased.
- No bifurcations were observed.

Local Sensitivity Analysis con'd

Test 3: Varying stress level. A local sensitivity analysis was performed on the r_i values, representing the effects of stress. The r values for the positive variables were held constant at 1 while we systematically perturbed the r values of the negative variables, since we were concerned primarily with how stress exacerbates suicidal behavior. Results are shown in Figures 3-5, for best, average, and worst case, respectively.

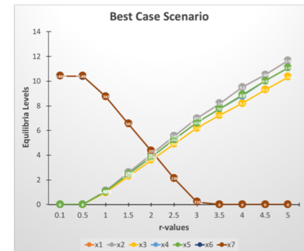

Figure 3. Bifurcation diagram for best case scenario.

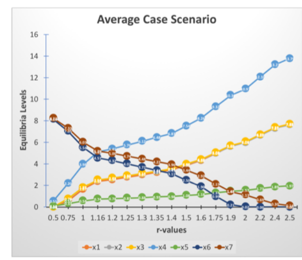

Figure 4. Bifurcation diagram for average case scenario.

Figure 5. Bifurcation diagram for worst case scenario.

Conclusions and Further Work

Here we develop and conduct sensitivity analysis on a mathematical model that captures the dynamic interplay between seven of the most important factors in suicidal behavior. Our global sensitivity analysis suggests that under three different scenarios of mental landscape (Figure 2), the most influential interactions under constant stress levels are the ones involving self promotion of an individual characteristic. Further, when stress is allowed to vary among the characteristics with negative valence, we observe bifurcations in equilibria. These findings suggest the existence of a tipping point in the emotional landscape, at which point an individual dramatically shifts from a healthy state to an unhealthy one.

Our next steps are to calibrate our model to patient data and investigate the presence of these tipping points in the stochastic version of the model, where random fluctuations may further contribute to shifts in mental landscape. Once it is further validated by data, this model will have the potential to assist in preventing suicidal behavior, as health care providers will be better able to detect points at which an individual is approaching a dangerous tipping point.

References

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