FABRIC AND DEFORMATION IN GRANULAR MATERIALS

Matthew R. Kuhn, Member, ASCE
School of Engineering, Univ. of Portland,
5000 N. Willamette Blvd., Portland, OR 97203. e-mail: kuhn@egr.up.edu

Abstract

The paper presents results of numerical experiments on a large assembly of smooth circular disks. The assembly was subjected to quasi-static biaxial loading at small to moderate strains by using the Discrete Element Method. Unlike the usual particle-based and coordination number approaches, the fabric was locally measured in relation to the shapes and orientations of voids. Deformation was measured at the smallest possible scale of individual particle clusters. Throughout loading the voids became more elongated in the direction of the major principal compressive stress, even as the average volumetric behavior changed from compressive to dilatant. A direct correlation was observed between local void shape and dilation, which accounts for the transition from compressive to dilatant behavior. The predominant pattern of nonhomogeneous deformation was in the form of thin obliquely trending bands of void cells within which slip deformations were most intense. The fabric in these regions can be characterized by large elongated voids, whose direction of elongation was slightly oblique to the direction of the major principal compressive stress.

Introduction

Deformation occurs nonuniformly in granular materials, particularly at the microscale of particle groups. We investigated the relationship between local deformation and local fabric by using the Discrete Element Method to track and measure the movements of individual particles within a large assembly of smooth circular disks. The same assembly was used in previous work, in which the author reported that a particular pattern of slip deformation, termed *microbands*, was the dominant feature of local, nonuniform deformation at low and moderate strains (Kuhn 1998; Kuhn 1999). In this paper, we present an analysis of local fabric and its relation to two forms of local deformation: volume change and slip deformation. We begin with a brief description of the assembly and its loading.

The Particle Assembly

The assembly contains 4008 smooth circular disks that were initially compacted into a dense and isotropic configuration. Table 1 summarizes essential aspects of the

Table 1. Assembly characteristics

Number of particles	4008
Particle sizes	Multiple
Particle size range	$0.45D_{50}^{*}$ to $1.40D_{50}$
Initial void ratio, e_{init}	0.179
Assembly size	$54D_{50} \times 54D_{50} \times 54D_{50}$

$*$ D_{50} represents the median particle diameter

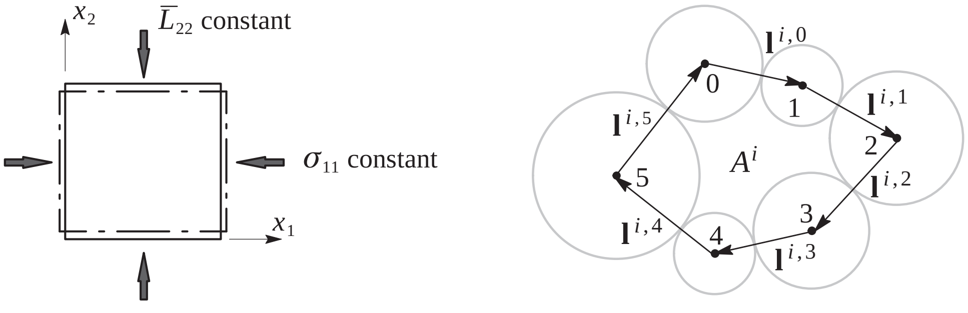
assembly, which is illustrated in a previous work (refer to Fig. 1a in Kuhn 1998). The assembly was slowly loaded in biaxial compression by reducing its height at a constant rate $\overline{\mathbf{L}}_{22}$, while maintaining a constant horizontal stress σ_{11} (Fig. 1a).

Local fabric and deformation were studied by constructing the assembly's particle graph (Satake 1992). This graph partitions the entire two-dimensional region A into small subregions A^i. Each subregion is a void (or *void cell*) that is surrounded by neighboring particles and bounded by the m^i branch vectors $\mathbf{l}^{i,j}$ of these contacting particles (Fig. 1b). The initial particle graph contained 3950 void cells and is shown in Fig. 2a. We used the void-based loop tensor $\mathbf{F}^i$ of Konishi and Naruse (1988) to characterize the local fabric of ith subregion A^i,

$$\mathbf{F}^i = \frac{1}{2}\sum_{j=1}^{m^i} \mathbf{l}^{i,j} \otimes \mathbf{l}^{i,j}. \tag{1}$$

Tensor $\mathbf{F}^i$ depends on the size, shape, and orientation of the void cell. Of particular interest are the following fabric measures, which are derived from $\mathbf{F}^i$:

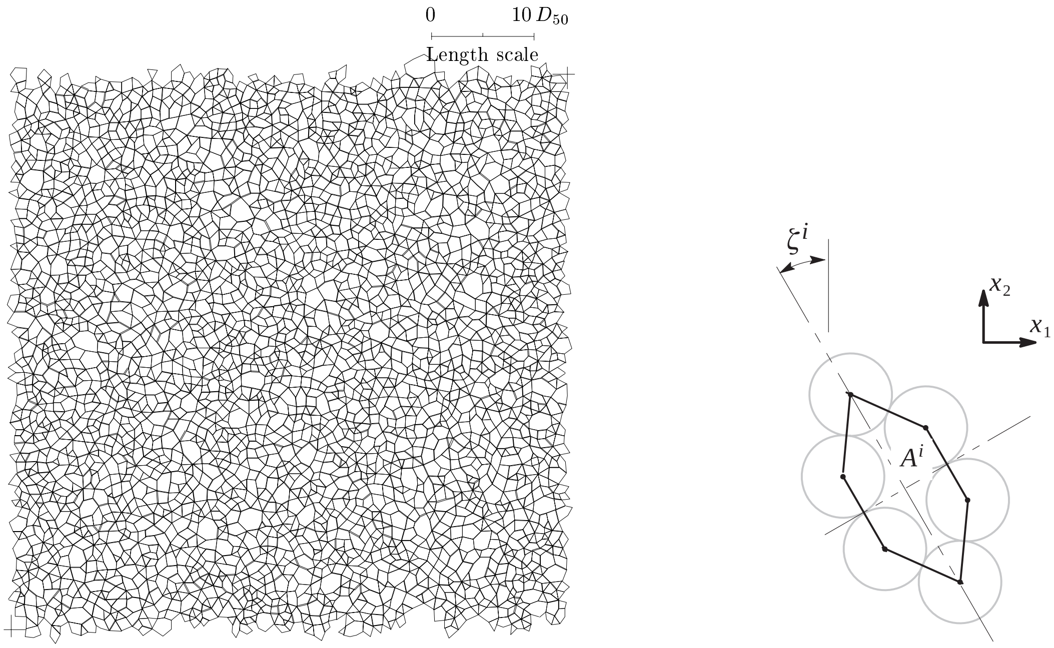
1. The elongation ratio F^i_{22}/F^i_{11} is a measure of the average vertical elongation of the ith void cell. The cell's *height-to-width ratio* is given roughly by $\alpha^i = \sqrt{F^i_{22}/F^i_{11}}$.
2. The orientation angle ζ^i of the tensor's major principal axis represents the orientation of the ith void cell elongation with respect to vertical (Fig. 2b, refer also to Konishi and Naruse 1988).



(a) Biaxial loading conditions

(b) Void cell with branch vectors

Fig. 1. Biaxial loading conditions, void cell, and branch vectors



(a) Particle graph for the initial arrangement of 4008 particles (the 3950 void cells are appear as small polygonal subregions.)

(b) Void cell orientation ζ^i

Fig. 2. Particle graph for 4008 particles; Void cell orientation

Fabric Evolution

Figs. 3a and b show the changes in stress and void ratio during biaxial compression of the disc assembly. The average height-to-width ratio $\overline{\alpha}$ of all void cells is shown in Fig. 3c. The voids are seen to grow continually more anisotropic, with the voids becoming more vertically elongated as the assembly is compressed. This *void* anisotropy corresponds to the contact anisotropy that has been measured by Oda et al. (1980) and others. During biaxial compression, particles that were originally touching become disengaged, and this loss of contacts occurs primarily among horizontal neighbors. The loss of such contacts leads to vertical elongation of the voids.

Dilation and Fabric

At the start of biaxial compression, the assembly's volume is reduced, but as deformation advances, the assembly becomes strongly dilatant (Fig. 3b). Dilation is most pronounced among vertically elongated void cells. This trend is illustrated in Fig. 4, which shows the local, void cell dilation rate $\phi^{\mathrm{vol},i}$ and its relation to the height-to-width ratio α^i of the same void cells. This figure was produced at a vertical strain ε_{22} of 0.60%. Each of the ten bars represents one-tenth of the 2370 void cells, which have been sorted into ranges of their ratios α^i. The figure establishes a moderate to strong correlation between a void cell's vertical elongation and its propensity to dilate. This relationship between fabric and dilation has been previously speculated on the basis of theoretical analyses (e.g., Chang et al. 1995).

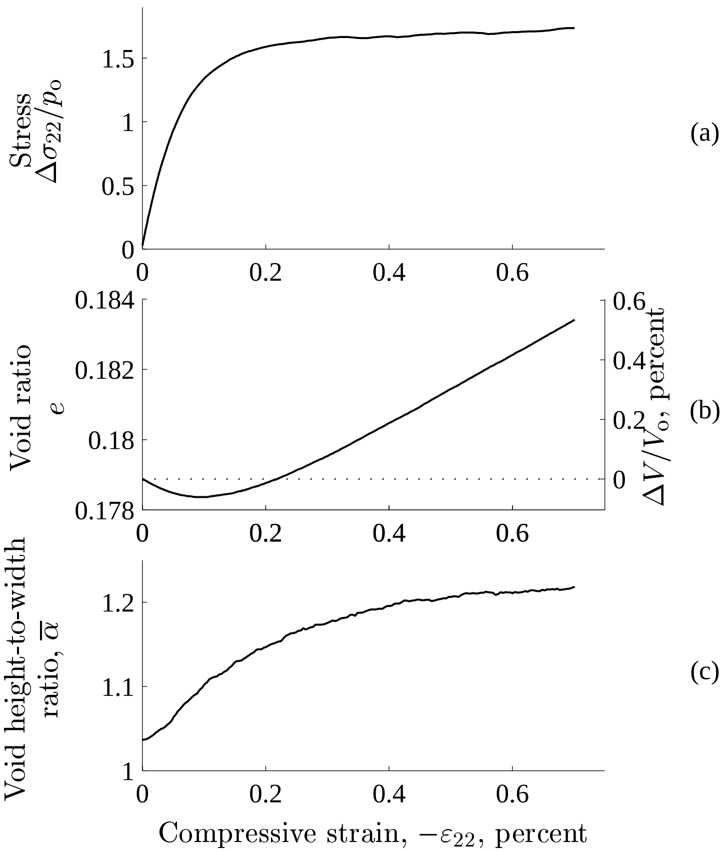


Fig. 3. Stress, volume change, and fabric during biaxial loading

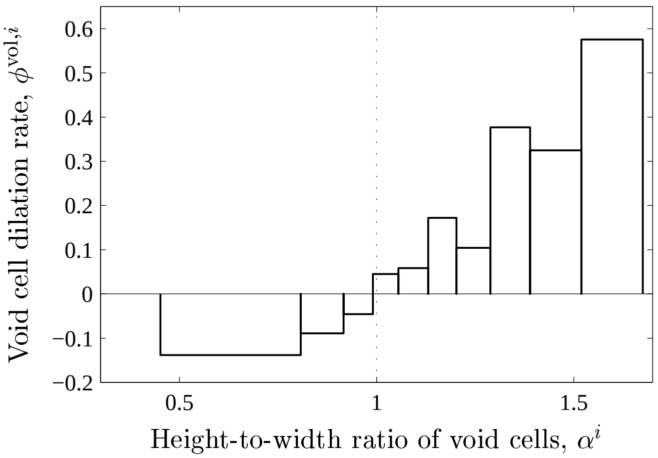
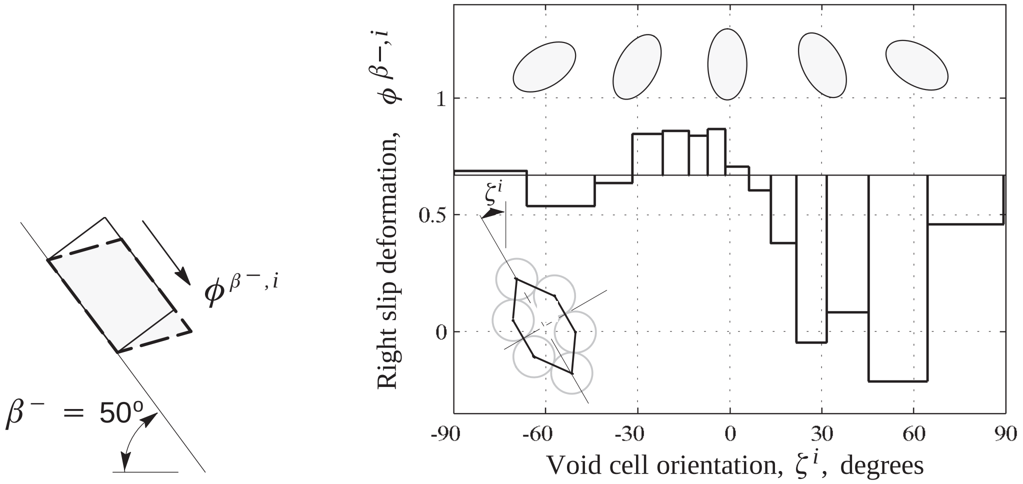


Fig. 4. Void cell elongation and dilation at strain $\varepsilon_{22} = 0.60\%$



(a) Slip deformation

(b) Right slip deformation and void cell orientation at strain $\varepsilon_{22} = 0.60\%$

Fig. 5. Slip deformation and its dependence on void cell orientation

The assembly's change from compressive to dilatant behavior is explained by this link between void elongation and dilation. At the start of loading, volume decreases as a result of an increase in the mean confining stress. As deformation proceeds, the horizontal expansion of the material reduces the number of horizontally oriented contacts, which produces vertically elongated voids. The tendency of elongated voids to dilate leads to overall dilation of the assembly.

Slip Deformation and Fabric

During biaxial compression, the predominant deformation structures are thin obliquely trending bands of void cells within which slip deformation is most intense (Kuhn 1998; Kuhn 1999). These *microbands* appear spontaneously during loading, well before the onset of failure and the initiation of shear bands. Fig. 5a illustrates the sort of "right slip" deformation $\phi^{\beta^-,i}$ that was measured within individual void cells. The relation between such slip deformation and fabric is shown in Fig. 5b at a vertical strain ε_{22} of 0.60%. Each of the fourteen bars represents one-fourteenth of the 2370 void cells, which were sorted according to their orientation angles ζ^i. Right slip deformation is least intense in those void cells that are elongated along the direction of right slip ($\zeta^i \approx 90° - \beta^- = 40°$) and most intense at orientations of between $-5°$ and $-30°$, which is somewhat steeper than an orientation perpendicular to the slip plane ($\zeta^i = -50°$).

References

Chang, C. S., Chao, S. J., and Chang, Y. (1995). "Estimates of elastic moduli for granular material with anisotropic random packing structure." *Int. J. Solids and Structures*, 32(14), 1989–2008.

Konishi, J. and Naruse, F. (1988). "A note on fabric in terms of voids." *Micromechanics of Granular Materials*, M. Satake and J. Jenkins, eds., Elsevier Science

Pub. B.V., Amsterdam, The Netherlands, 39–46.

Kuhn, M. R. (1998). “Slip deformation structures in granular materials.” *Engineering Mechanics: A Force for the 21st Century*, H. Murakami and J. E. Luco, eds., ASCE, Reston, VA, 1736–1739.

Kuhn, M. R. (1999). “Structured deformation in granular materials.” *Mechanics of Materials*, in print.

H. Murakami and J. E. Luco, eds. (1998). *Engineering Mechanics: A Force for the 21st Century.* ASCE, Reston, VA. Proc. of the 12th Engineering Mechanics Conference.

Oda, M., Konishi, J., and Nemat-Nasser, S. (1980). “Some experimentally based fundamental results on the mechanical behaviour of granular materials.” *Géotechnique*, 30(4), 479–495.

Satake, M. (1992). “A discrete-mechanical approach to granular materials.” *Int. J. Engng. Sci.*, 30(10), 1525–1533.