DIFFERENT ROLLING MEASURES FOR GRANULAR ASSEMBLIES

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ROLLING

particle translations & rotations:
⇒ contact deformation & sliding
⇒ rigid-body-like displacements
⇒ rolling

in general case:             ALL OF THEM, AT THE SAME TIME

This presentation:

a) What to mean by 'rolling'? (3 different proposals)
b) 'Rolling curl': assigned to the particles

Basic assumptions:

perfectly rigid particles
contacts: infinitesimally small; deformable  (⇐ like in most DEM models)
incremental approach
purely kinematical analysis

how they behave in numerical simulations
Rolling measure # 1: [analysis of relative rotations]

“The motion when the particles have a relative rotation about a common tangential axis”

\[
\begin{align*}
\theta_{rel, \text{twist}} &= (\theta_{rel} \cdot n)n \\
\theta_{roll, 1} &= \theta_{rel} - (\theta_{rel} \cdot n)n
\end{align*}
\]

(( Objectivity: observers having different locations & different velocities: experience the same rolling in the contact ))
Rolling measure # 2.: [analysis of the average translation of the contact]

“The motion that changes the distance of the contact point from the branch vector”

Contact point:

Displacements:

Average translation of the contact point:

\[
\mathbf{d}_u^{\text{avr}} = \frac{1}{2} \left[ \mathbf{d}_u^{pc} + \mathbf{d}_u^{qc} \right] \\
= \frac{1}{2} \left[ (\mathbf{d}_u^p + \mathbf{d}_\theta^p \times \mathbf{r}^p) + (\mathbf{d}_u^q + \mathbf{d}_\theta^q \times \mathbf{r}^q) \right]
\]

\[
\mathbf{d}_u^{\text{roll},2} = \frac{1}{2} \left[ (\mathbf{d}_\theta^p \times \lambda)(\mathbf{r}^p \cdot \lambda) + (\mathbf{d}_\theta^q \times \lambda)(\mathbf{r}^q \cdot \lambda) - \frac{((\mathbf{r}^p + \mathbf{r}^q) \cdot \lambda)}{((\mathbf{r}^p - \mathbf{r}^q) \cdot \lambda)} (\mathbf{d}_u^q - \mathbf{d}_u^p) \right]
\]
Rolling measure # 2.: [analysis of the average translation of the contact]

Example 1:
Two equal circles that do not translate: \( R^p = R^q := R, \ d\theta^p = -d\theta^q, \ \left| d\theta^p \right| = \left| d\theta^p \right| := d\theta \):

Before displacements: \( d\theta^p \)

After displacements: \( d\theta^q \)

Rolling: \( \left| d\mathbf{u}^{\text{roll},2} \right| = \frac{1}{2} \left[ (d\theta^p \times \mathbf{r}^p) + (d\theta^q \times \mathbf{r}^q) \right] = R \ d\theta \)

Example 2:
Rigid-body motion of a pair of arbitrary particles: \( d\theta^q = d\theta^p, \ d\mathbf{u}^q = d\mathbf{u}^p + d\theta^p \times \mathbf{r}^p \):

Before displacements: \( d\theta^p \)

After displacements: \( d\theta^q \)

Rolling: \( \left| d\mathbf{u}^{\text{roll},2} \right| = 0 \)
Rolling measure # 3.: [analysis of the shift of the contact point]

“The motion that changes the location of the contact point on the particle surface”

Before displacements:          After displacements:          Average shift of contact point:

\[
d\mathbf{u}_{\text{roll},3} = -(K^p + K^q)^{-1} \left[ (d\mathbf{\theta}^q - d\mathbf{\theta}^p) \times \mathbf{n} + \frac{1}{2} (K^p - K^q) d\mathbf{u}^{\text{def}} \right]
\]

( Local surface geometry \( K^p, K^q, \mathbf{n} \) )
**Rolling measure #3:** [analysis of the shift of the contact point]

**Example 1:**
Two equal circles that do not translate: \( R^p = R^q := R, \quad d\theta^p = -d\theta^q, \quad |d\theta^p| = |d\theta^q| := d\theta; \)

Before displacements:

After displacements:

**Rolling:**

\[
|d\mathbf{u}_{\text{roll},3}| = \frac{1}{2} \left[ d\mathbf{u}^{\text{shift,pc}} + d\mathbf{u}^{\text{shift,qc}} \right] = R \, d\theta
\]

**Example 2:**
Rigid-body motion of a pair of arbitrary particles: \( d\theta^q = d\theta^p, \quad d\mathbf{u}^q = d\mathbf{u}^p + d\theta^p \times \mathbf{r}^p \)

Before displacements:

After displacements:

**Rolling:**

\[
|d\mathbf{u}_{\text{roll},3}| = 0
\]
Rolling measures: Numerical simulation results

The simulations: Biaxial/Triaxial tests

Circles/spheres, ovals/ovoids
2D: 10816  3D: 4096
contacts: linear
periodic boundaries

The simulation results:

• Correlations between Type 1 / Type 2 / Type 3 measures: > 95%
Rolling measures: Numerical simulation results

The simulation results:

• Pattern # 1:

  Dilatation of voids:

  Contact rolling:

  - deforming strips with rolling
  - quiet regions between them

Rolling measures: Numerical simulation results

The simulation results:

• Pattern # 1:

  Dilatation of voids:

  Contact rolling:

  - deforming strips with rolling
  - quiet regions between them
Rolling measures: Numerical simulation results

The simulation results:

- Pattern #2:
  - Rolling vectors around an individual particle: typically, either all of them clockwise, or all of them counter-clockwise
  - GEAR-LIKE PATTERN
  - observed:
    - at all particle shapes
    - at all strain levels
    - before shear bands and also within shear bands

(see later in 3D)
Rolling curl

contact ⇒ particle

Equivalent continuum:  Define a vector field:  The rolling curl:

\[ d\bar{p}^p = \frac{1}{V_p} \int_{S_p} n(x^S) \times d\mathbf{u}^{\text{roll}}(x^S) \, dS \]

Physical meaning:
\[ \approx \text{that part of the particle rotation which leads to rolling} \]
Rolling curl: Simulation results

Correlations between particle curls

Discrete distance between two grains:

\[ d = 1 \]
\[ d = 2 \]
\[ d = 3 \]

particles at distance 1, 2, 3

Correlations between rolling curls of particles at distance 1, 2, 3, ... were analyzed in biaxial/triaxial tests circles/spheres, ovals/ovoids
**Rolling curl: Simulation results**

**Correlations between particle curls**

### Zero strain level:

<table>
<thead>
<tr>
<th>Distance $d$</th>
<th>Circles (2D)</th>
<th>Ovals (2D)</th>
<th>Spheres (3D)</th>
<th>Ovoids (3D)</th>
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</table>

### Peak stress level:

<table>
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<th>Ovals (2D)</th>
<th>Spheres (3D)</th>
<th>Ovoids (3D)</th>
</tr>
</thead>
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</tr>
</tbody>
</table>

**Physical meaning of the results:**

[Diagram showing particle curls]
FUTURE RESEARCH

HOW TO USE IT IN A CONSTITUTIVE THEORY?

\[
\begin{align*}
\text{Total deformation of an assembly:} & \quad \Leftarrow \quad \text{Elastic energy (elastic particle deformations)} \\
& \quad \Leftarrow \quad \text{Dissipated energy (contact sliding etc.)} \\
& \quad \Leftarrow \quad \text{ROLLING}
\end{align*}
\]
SUMMARY

- 3 different objective measures for contact rolling
  ⇒ simulation results: very large correlations
  characteristic spatial patterns

- Rolling curl: assigned to the particles
  ⇒ simulation results: gear-like pattern

- Idea of future research

THANKS!