DIFFERENT ROLLING MEASURES FOR GRANULAR ASSEMBLIES

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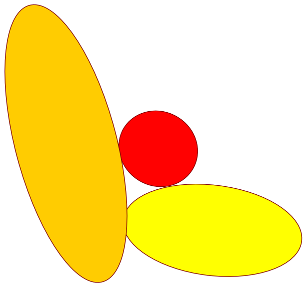
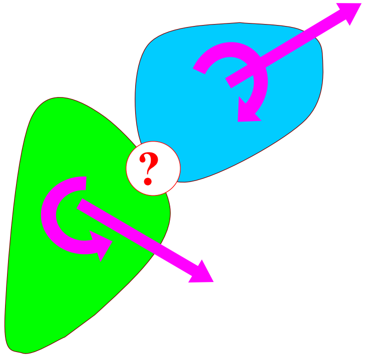


ROLLING



particle translations & rotations:

$\Rightarrow$ contact deformation & sliding

$\Rightarrow$ rigid-body-like displacements

$\Rightarrow$ rolling

in general case:

ALL OF THEM, AT THE SAME TIME

This presentation:

a) What to mean by 'rolling'? (3 different proposals)

b) 'Rolling curl': assigned to the particles

} how they behave in numerical simulations

Basic assumptions:

perfectly rigid particles

contacts: infinitesimally small; deformable ($\Leftarrow$ like in most DEM models)

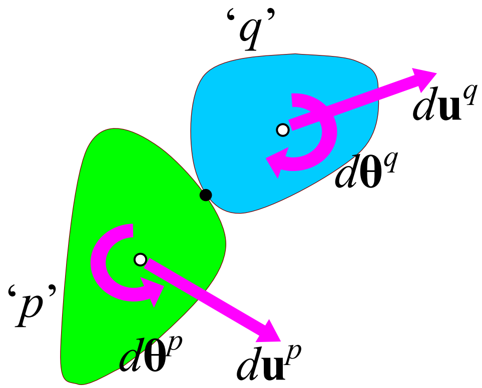
incremental approach

purely kinematical analysis

Rolling measure # 1.:

[analysis of relative rotations]

"The motion when the particles have a relative rotation about a common tangential axis"



$$d\boldsymbol{\theta}^{rel} = d\boldsymbol{\theta}^{q} - d\boldsymbol{\theta}^{p}$$

about the contact normal:

$$d\boldsymbol{\theta}^{rel,\,twist} = (d\boldsymbol{\theta}^{rel} \cdot \mathbf{n})\mathbf{n}$$

about a tangential axis

$$\boxed{d\boldsymbol{\theta}^{roll,1} = d\boldsymbol{\theta}^{rel} - (d\boldsymbol{\theta}^{rel} \cdot \mathbf{n})\mathbf{n}}$$

OBJECTIVE!

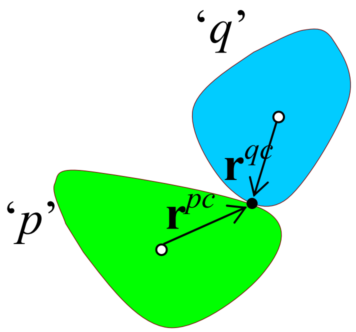
((Objectivity:

observers having different locations & different velocities:
experience the same rolling in the contact))

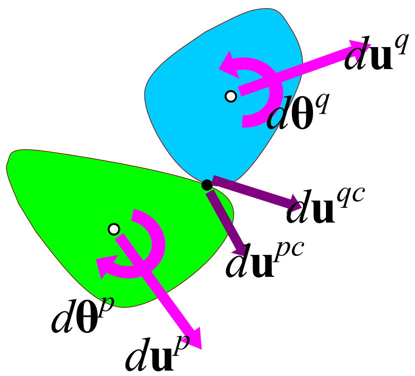
Rolling measure # 2.: [analysis of the average translation of the contact]

"The motion that changes the distance of the contact point from the branch vector"

Contact point:



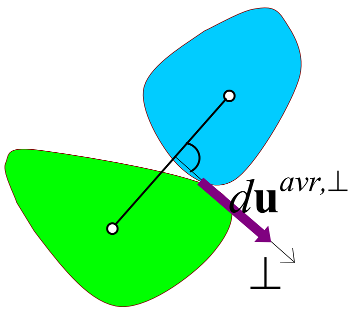
Displacements:



Average translation of the contact point:

$$d\mathbf{u}^{avr} = \tfrac{1}{2}\left[d\mathbf{u}^{pc} + d\mathbf{u}^{qc}\right] =$$

$$= \tfrac{1}{2}\left[(d\mathbf{u}^{p} + d\boldsymbol{\theta}^{p}\times\mathbf{r}^{p}) + (d\mathbf{u}^{q} + d\boldsymbol{\theta}^{q}\times\mathbf{r}^{q})\right]$$



$d\mathbf{u}^{avr,\perp}$ → rigid-body-like

$d\mathbf{u}^{avr,\perp}$ → rolling:

$$\mathrm{d}\mathbf{u}^{roll,2} = \tfrac{1}{2}\Big[(\mathrm{d}\boldsymbol{\theta}^{p}\times\boldsymbol{\lambda})(\mathbf{r}^{p}\cdot\boldsymbol{\lambda}) + (\mathrm{d}\boldsymbol{\theta}^{q}\times\boldsymbol{\lambda})(\mathbf{r}^{q}\cdot\boldsymbol{\lambda}) - \frac{((\mathbf{r}^{p}+\mathbf{r}^{q})\cdot\boldsymbol{\lambda})}{((\mathbf{r}^{p}-\mathbf{r}^{q})\cdot\boldsymbol{\lambda})}(\mathrm{d}\mathbf{u}^{q}-\mathrm{d}\mathbf{u}^{p})\Big]$$

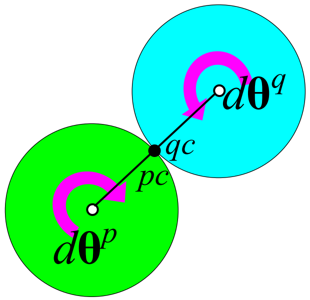
OBJECTIVE !

Rolling measure # 2.: [analysis of the average translation of the contact]

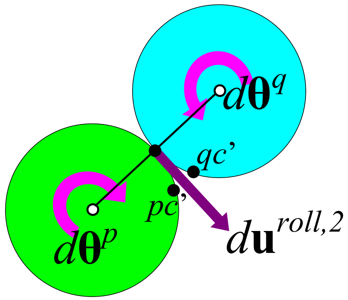
Example 1:

Two equal circles that do not translate: $R^p = R^q := R, \quad d\boldsymbol{\theta}^p = -d\boldsymbol{\theta}^q, \quad \left|d\boldsymbol{\theta}^p\right| = \left|d\boldsymbol{\theta}^p\right| := d\theta$:

Before displacements:



After displacements:



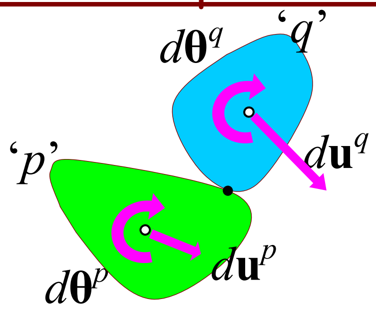
Rolling:

$$\left|d\mathbf{u}^{roll,2}\right| = \left|\tfrac{1}{2}\left[(d\boldsymbol{\theta}^p \times \mathbf{r}^p) + (d\boldsymbol{\theta}^q \times \mathbf{r}^q)\right]\right| = R\, d\theta$$

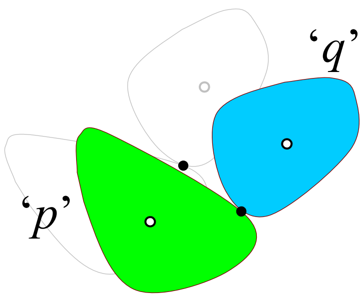
Example 2:

Rigid-body motion of a pair of arbitrary particles: $d\boldsymbol{\theta}^q = d\boldsymbol{\theta}^p, \quad d\mathbf{u}^q = d\mathbf{u}^p + d\boldsymbol{\theta}^p \times \mathbf{r}^p$

Before displacements:



After displacements:



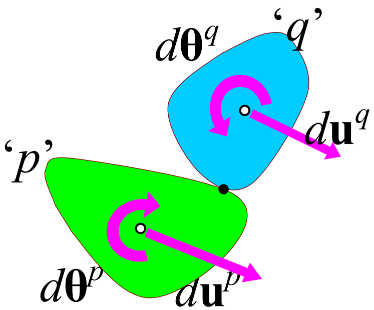
Rolling:

$$\left|d\mathbf{u}^{roll,2}\right| = 0$$

Rolling measure # 3.: [analysis of the shift of the contact point]

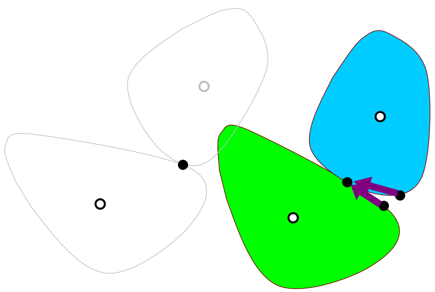
"The motion that changes the location of the contact point on the particle surface"

Before displacements:



(Local surface geometry $\mathbf{K}^p$, $\mathbf{K}^q$, $\mathbf{n}$)

After displacements:



Average shift of contact point:

$$d\mathbf{u}^{roll,3} = -(\mathbf{K}^p + \mathbf{K}^q)^{-1}\left[(d\boldsymbol{\theta}^q - d\boldsymbol{\theta}^p)\times\mathbf{n} + \right.$$
$$\left. + \frac{1}{2}(\mathbf{K}^p - \mathbf{K}^q) d\bar{\mathbf{u}}^{def} \right]$$

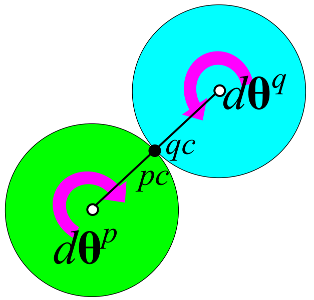
OBJECTIVE !

Rolling measure # 3.: [analysis of the shift of the contact point]

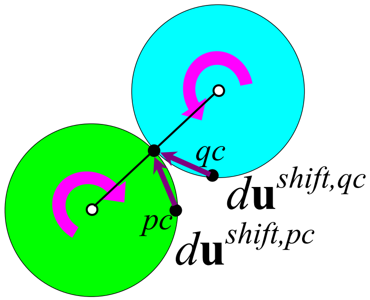
Example 1:

Two equal circles that do not translate: $R^p = R^q := R, \quad d\boldsymbol{\theta}^p = -d\boldsymbol{\theta}^q, \quad \left|d\boldsymbol{\theta}^p\right| = \left|d\boldsymbol{\theta}^p\right| := d\theta$:

Before displacements:



After displacements:



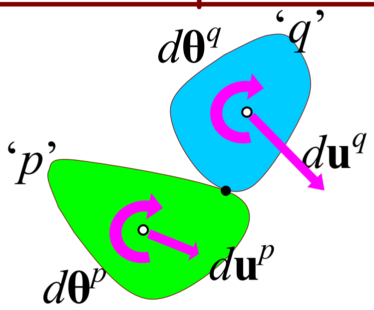
Rolling:

$$\left|d\mathbf{u}^{roll,3}\right| = \left|\tfrac{1}{2}\left[d\mathbf{u}^{shift,pc} + d\mathbf{u}^{shift,qc}\right]\right| = R\, d\theta$$

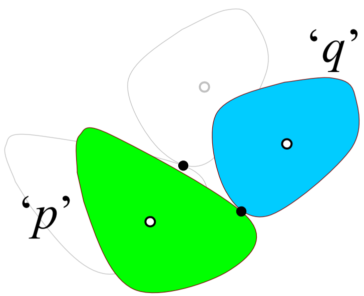
Example 2:

Rigid-body motion of a pair of arbitrary particles: $d\boldsymbol{\theta}^q = d\boldsymbol{\theta}^p, \quad d\mathbf{u}^q = d\mathbf{u}^p + d\boldsymbol{\theta}^p \times \mathbf{r}^p$

Before displacements:



After displacements:

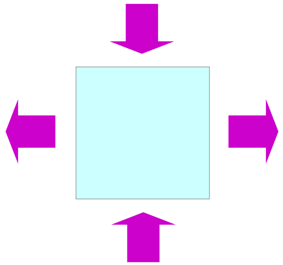


Rolling:

$$\left|d\mathbf{u}^{roll,3}\right| = 0$$

Rolling measures: Numerical simulation results

The simulations: Biaxial/Triaxial tests



Circles/spheres, ovals/ovoids
2D: 10816 3D: 4096
contacts: linear
periodic boundaries

The simulation results:

- Correlations between Type 1 / Type 2 / Type 3 measures: > 95%

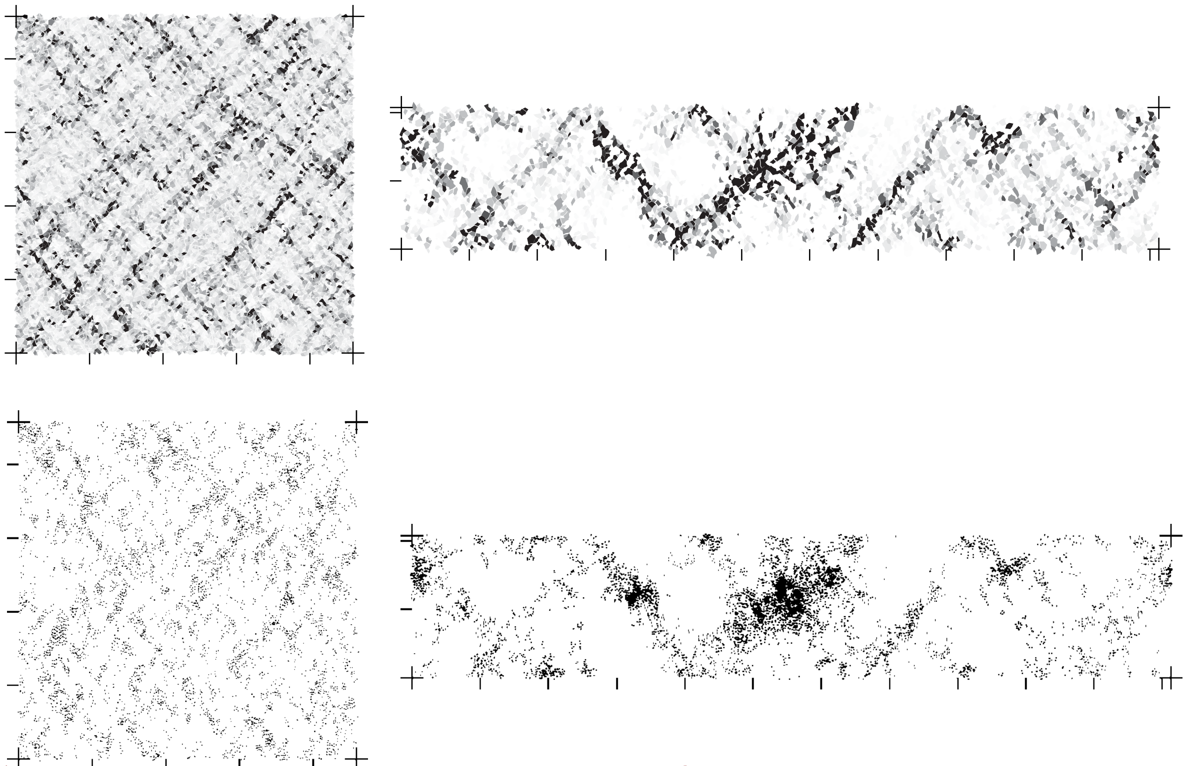
Rolling measures: Numerical simulation results

The simulation results:

- Pattern # 1:

Dilatation of voids:

Contact rolling:



hardening stage

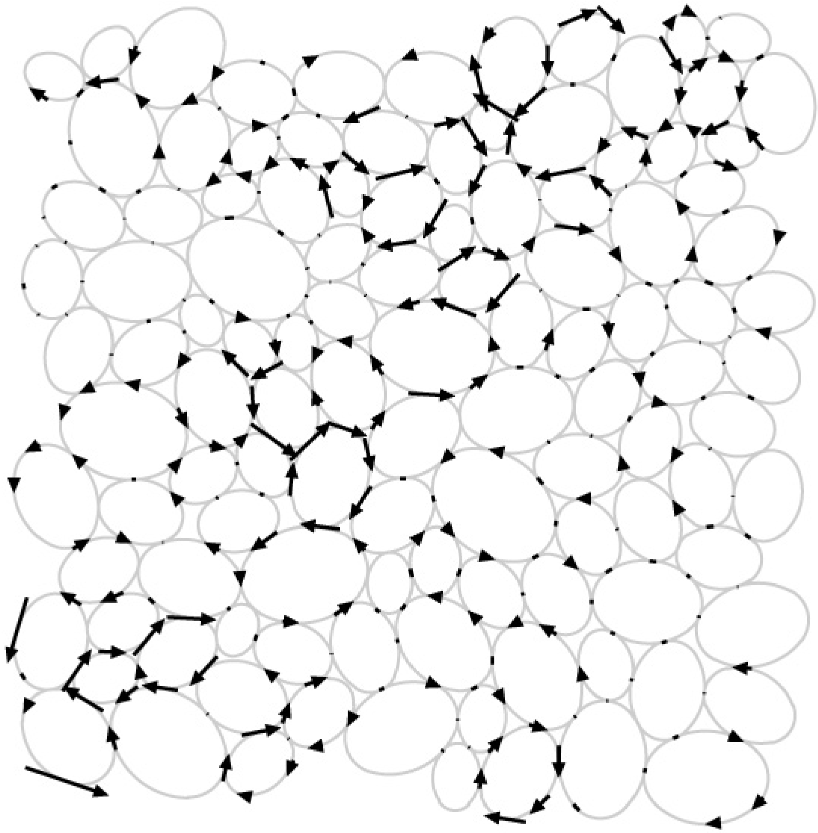
softening stage

- deforming strips with rolling
- quiet regions between them

Rolling measures: Numerical simulation results

The simulation results:

- Pattern # 2:



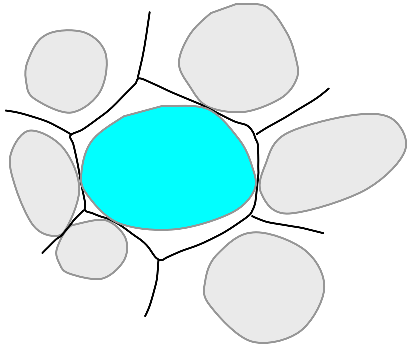
- Rolling vectors around an individual particle: typically, either all of them clockwise, or all of them counter-clockwise
- GEAR-LIKE PATTERN
- observed:
  - at all particle shapes
  - at all strain levels
  - before shear bands and also within shear bands

(see later in 3D)

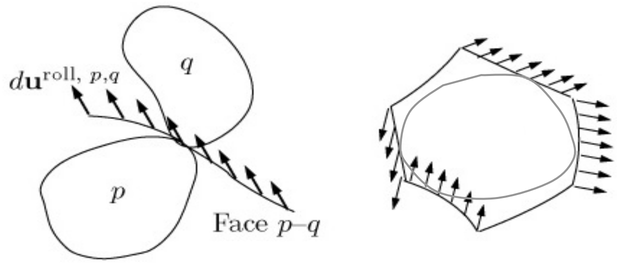
Rolling curl

~~contact~~ ⇒ particle

Equivalent continuum:



Define a vector field:



The rolling curl:

$$d\bar{\boldsymbol{\rho}}^{p} = \frac{1}{V^{p}} \iint_{(S^{p})} \mathbf{n}(\mathbf{x}^{S}) \times d\mathbf{u}^{roll}(\mathbf{x}^{S})\, dS$$

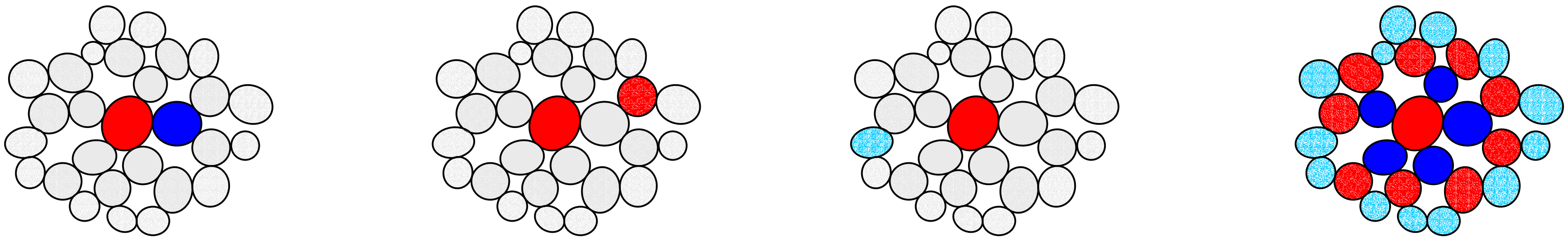
Physical meaning:

≈ that part of the particle rotation which leads to rolling

Rolling curl: Simulation results

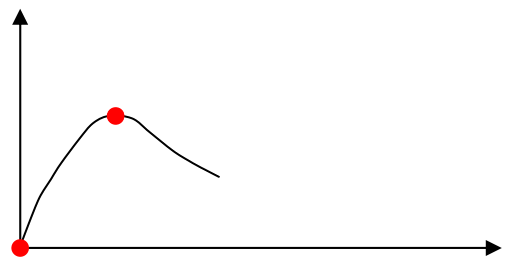
Correlations between particle curls

Discrete distance between two grains:



$d = 1$ $d = 2$ $d = 3$ particles at distance 1, 2, 3

Correlations between rolling curls
of particles at distance 1, 2, 3, …
were analyzed in biaxial/triaxial tests
circles/spheres, ovals/ovoids



Rolling curl: Simulation results

Correlations between particle curls

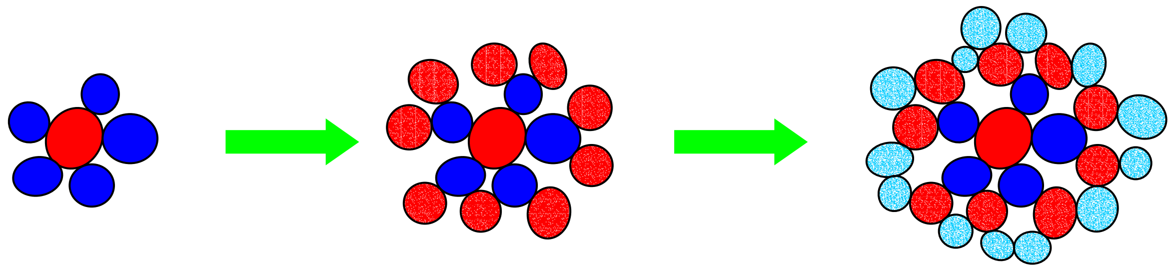
Zero strain level:

Distance *d*	Circles (2D)	Ovals (2D)	Spheres (3D)	Ovoids (3D)
0	1.00	1.00	1.00	1.00
1	-0.55	-0.29	-0.37	-0.21
2	0.18	0.02	0.08	0.02
3	-0.04	0.01	-0.01	0.00
4	0.01	0.00	0.00	0.00
5	0	0	0	0

Peak stress level:

Distance *d*	Circles (2D)	Ovals (2D)	Spheres (3D)	Ovoids (3D)
0	1.00	1.00	1.00	1.00
1	-0.63	-0.51	-0.42	-0.33
2	0.33	0.20	0.14	0.09
3	-0.13	-0.06	-0.03	-0.01
4	0.05	0.02	0.00	0.00
5	-0.01	-0.00	0	0
6	0.01	0.00	0	0

Physical meaning of the results:



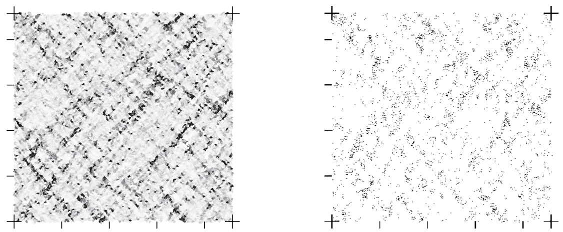
FUTURE RESEARCH

HOW TO USE IT IN A CONSTITUTIVE THEORY ?

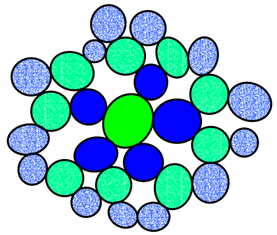
Total deformation of an assembly:

⇐ Elastic energy (elastic particle deformations)
⇐ Dissipated energy (contact sliding etc.)
⇐ ROLLING

SUMMARY



- 3 different objective measures for contact rolling
  - ⇒ simulation results: very large correlations; characteristic spatial patterns



- Rolling curl: assigned to the particles
  - ⇒ simulation results: gear-like pattern
- Idea of future research

THANKS !