Micro-Geomechanics Across Multiple Strain Scales

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Unresolved scale-dependent phenomena:

1. **Spontaneous localization**
2. **Dependence of stiffness on gradients of strain**
3. **Effect of confining pressure and inter-particle friction on mechanical behavior**
4. **Material softening**

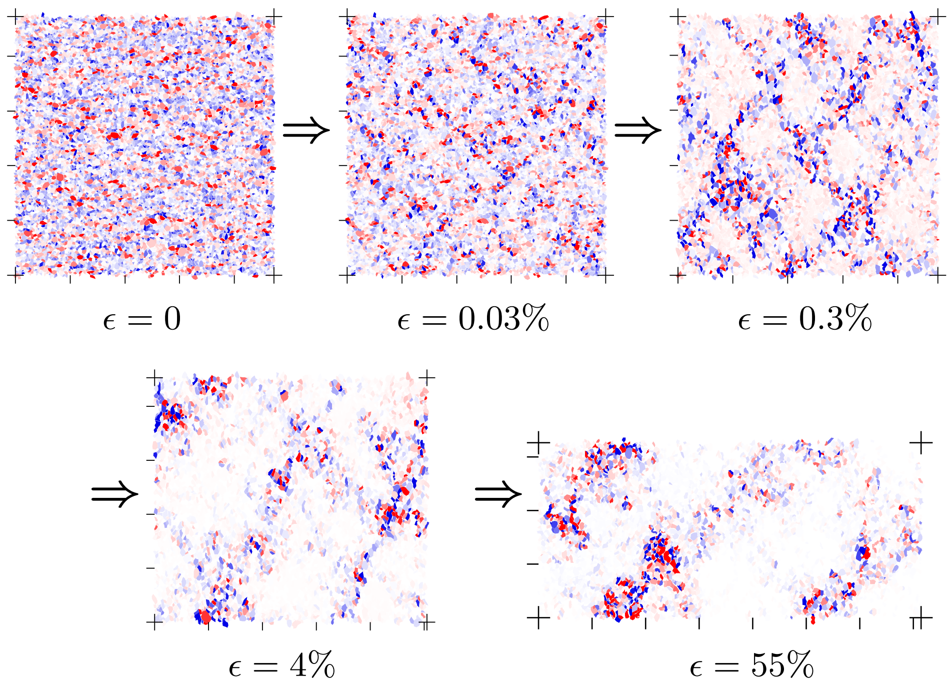
Characteristics common to these phenomena:

- **Predominant at moderate to large strains**
- **Accompanied by rapid particle rotations and rolling**
- **Particle motions deviate greatly from uniform deformation**
- **Sensitive to particle shapes**

Example: Localization and patterning at multiple strain scales.

Biaxial compression of 4000 ovals.

Cluster dilations.



Granular stiffness is geometric as well as mechanical

Example: Behavior of particle pairs at the peak and steady states

$$\overline{\boldsymbol{\sigma}} = \frac{1}{V} \sum_{\text{pairs}} \mathbf{l} \otimes \mathbf{f}$$

$$\frac{d\overline{\boldsymbol{\sigma}}}{d\boldsymbol{\epsilon}} = 0 \quad \Rightarrow \textbf{ stationary stress}$$

Stress increment

$$d\,\overline{\boldsymbol{\sigma}} = \underbrace{-\frac{dV}{V}\overline{\boldsymbol{\sigma}}}_{\boxed{\mathbf{1}}} \quad + \underbrace{\frac{1}{V} \sum_{\text{pairs}} \mathbf{l} \otimes d\mathbf{f}}_{\boxed{\mathbf{2}}} \quad + \underbrace{\frac{1}{V} \sum_{\text{pairs}} d\mathbf{l} \otimes \mathbf{f}}_{\boxed{\mathbf{3}}}$$

Results: Changes in the contact forces produce hardening. Changes in the contact directions produce softening.

Conclusions:

- **Granular stiffness likely depends on the particle shapes at their contacts.**
- **Granular stiffness has both mechanical and geometric origins.**

Possible framework for granular stiffness at the scales of particle pairs/clusters/assemblies:

$$\left[\ \mathbf{H}\ \right]\left[\frac{d\mathbf{u}}{d\boldsymbol{\theta}}\right] = \left[\frac{d\mathbf{f}}{d\mathbf{m}}\right]$$

Stiffness [H] has both mechanical and geometric parts:

$$\left[\ \mathbf{H}\ \right] = \left[\mathbf{H}^{\mathbf{mechanical}}\right] + \left[\mathbf{H}^{\mathbf{geometric}}\right]$$

Stability and softening are associated with second-order work:

$$\Delta_2 W = \left[\ d\mathbf{u}/d\boldsymbol{\theta}\ \right]^{\mathbf{T}}\left[\ \mathbf{H}\ \right]\left[\frac{d\mathbf{u}}{d\boldsymbol{\theta}}\right]$$

Example: Softening before and after shear band formation