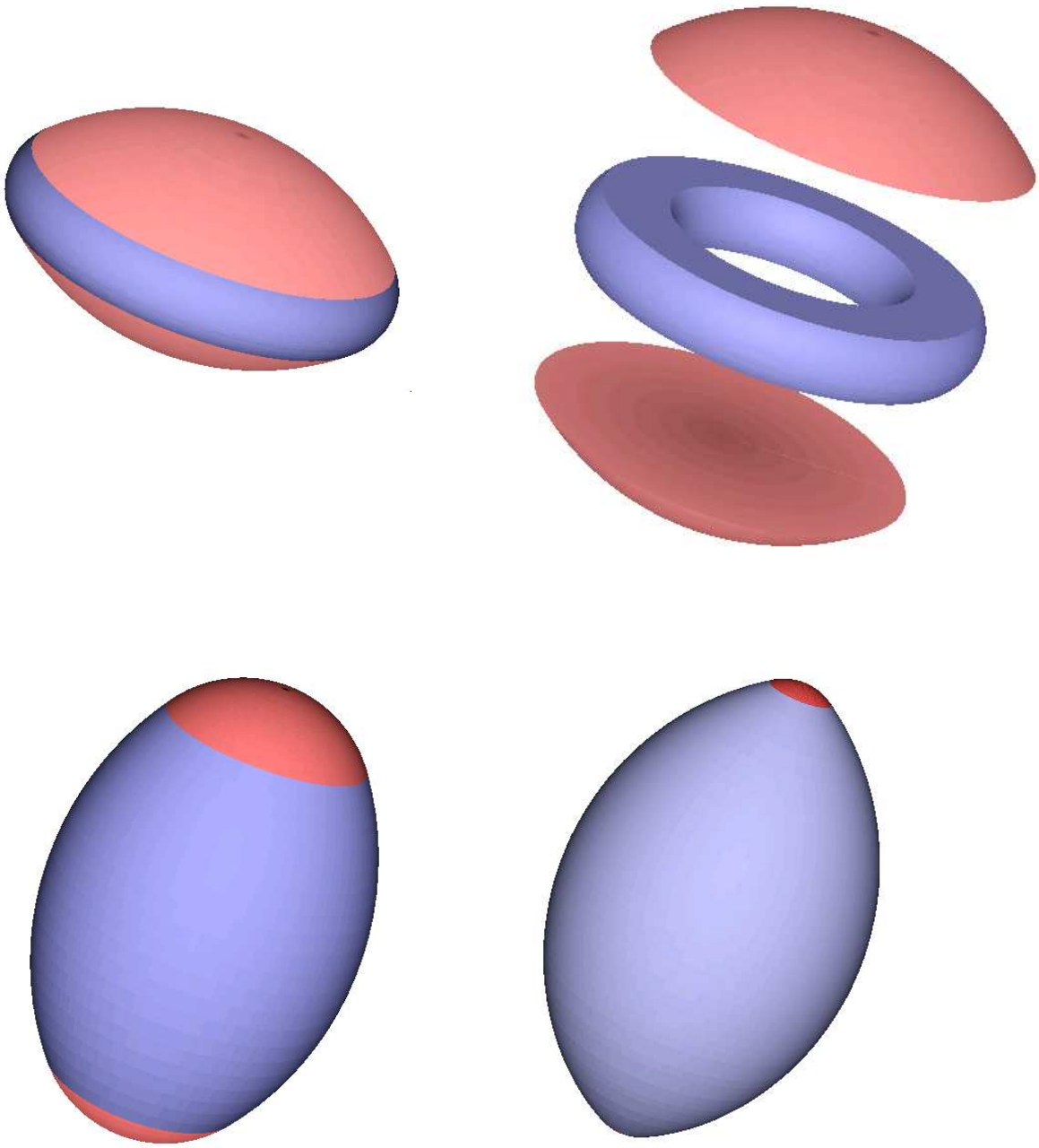
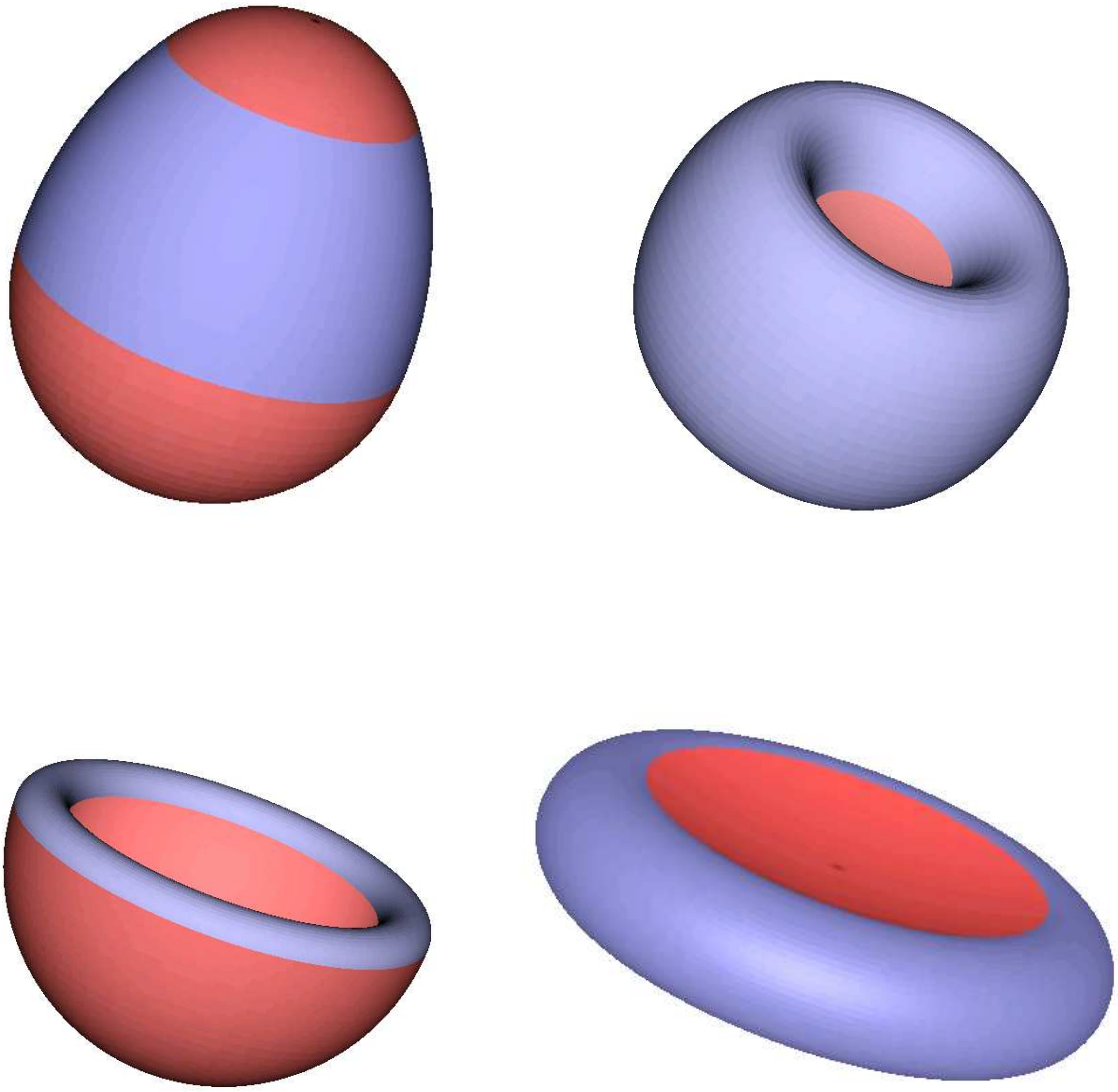
# A Torus Primitive For Particle Shapes With the Discrete Element Method

Matthew R. Kuhn
University of Portland, U.S.A.

# Tori and Spheres



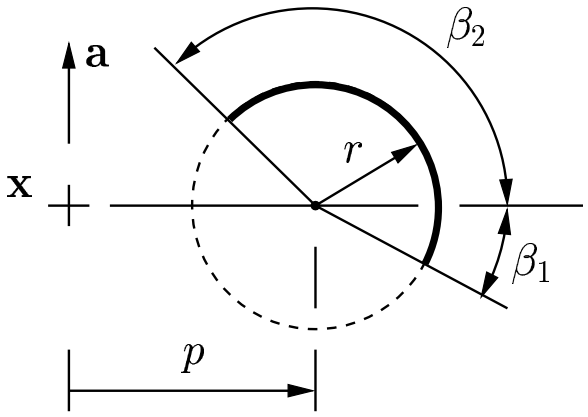
# Tori and Spheres

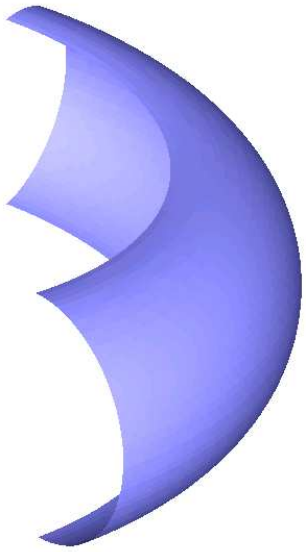


# Torus Characteristics

1. **Compact geometric representation**
2. Smooth
3. Strictly convex patches
4. A transition between convex and concave patches
5. Simple surface characteristics:
   - principal curvatures
   - directions of principal curvatures
6. Fast contact detection

# Geometric Representation



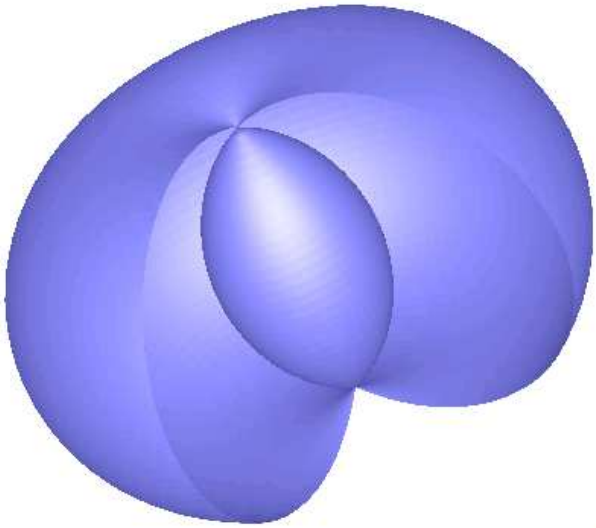
$\mathbf{x}$: Position

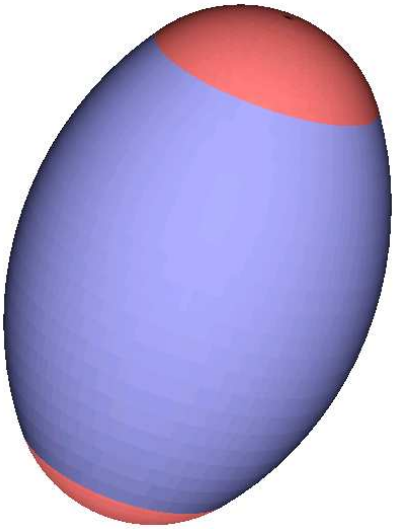
$\mathbf{a}$: Orientation

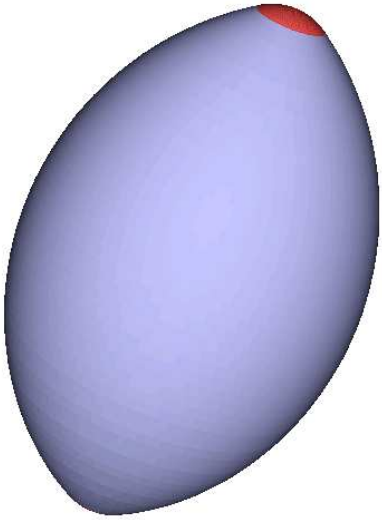
$p$, $r$: Radii

$\beta_1$, $\beta_2$: Latitudes

$\alpha_1$, $\alpha_2$: Longitudes







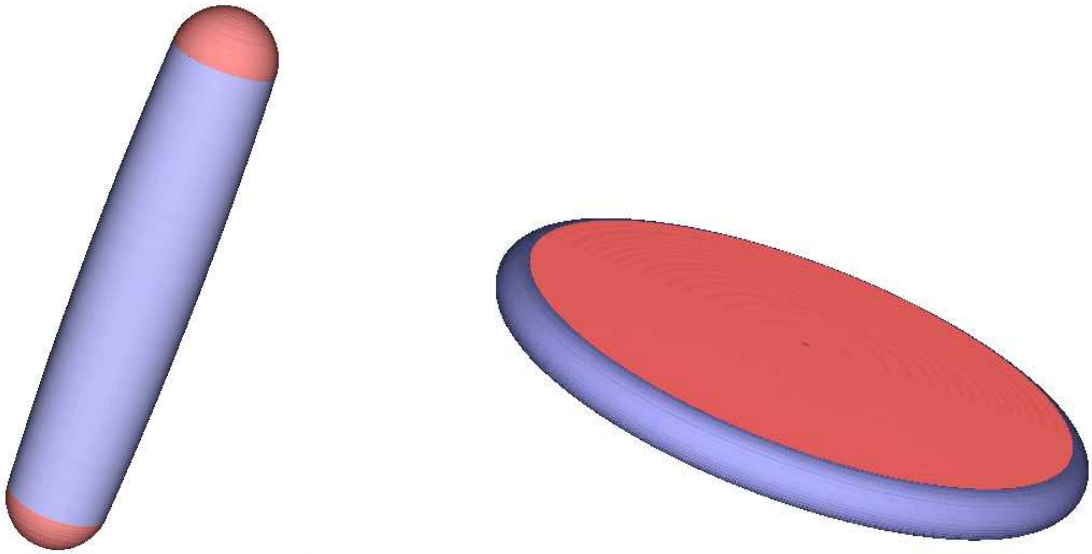
# Torus Characteristics

1. Compact geometric representation
2. **Smooth**
3. Strictly convex patches
4. A transition between convex and concave patches
5. Simple surface characteristics:
   - principal curvatures
   - directions of principal curvatures
6. Fast contact detection

# Torus Characteristics

1. Compact geometric representation
2. Smooth
3. **Strictly convex patches**
4. A transition between convex and concave patches
5. Simple surface characteristics:
   - principal curvatures
   - directions of principal curvatures
6. Fast contact detection

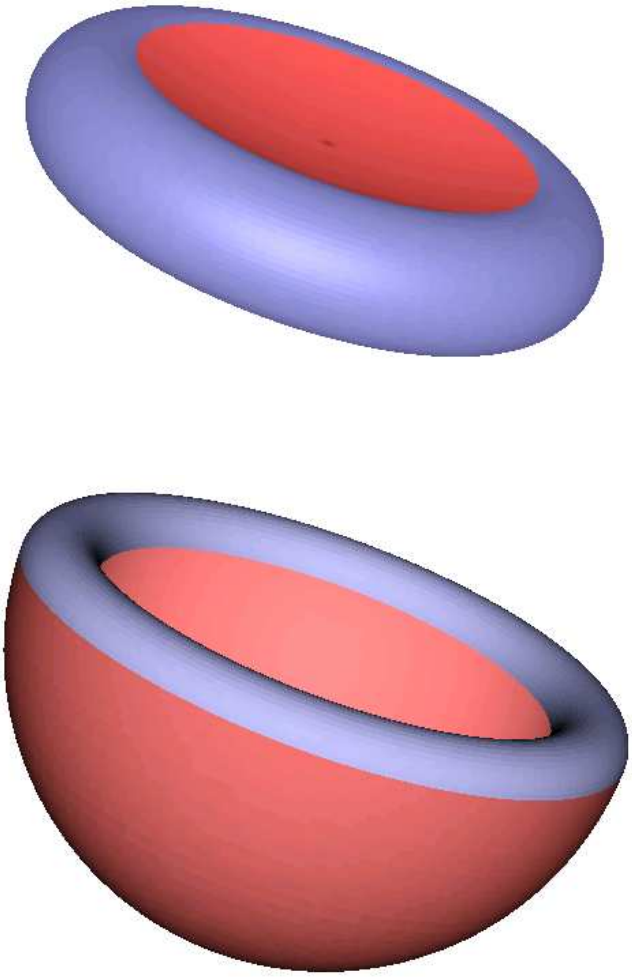
# Strictly Convex Patches



# Torus Characteristics

1. Compact geometric representation
2. Smooth
3. Strictly convex patches
4. **A transition between convex and concave patches**
5. Simple surface characteristics:
   - principal curvatures
   - directions of principal curvatures
6. Fast contact detection

# Transitions Between Concave and Convex



# Torus Characteristics

1. Compact geometric representation
2. Smooth
3. Strictly convex patches
4. A transition between convex and concave patches
5. **Simple surface characteristics:**
   - **principal curvatures**
   - **directions of principal curvatures**
6. Fast contact detection

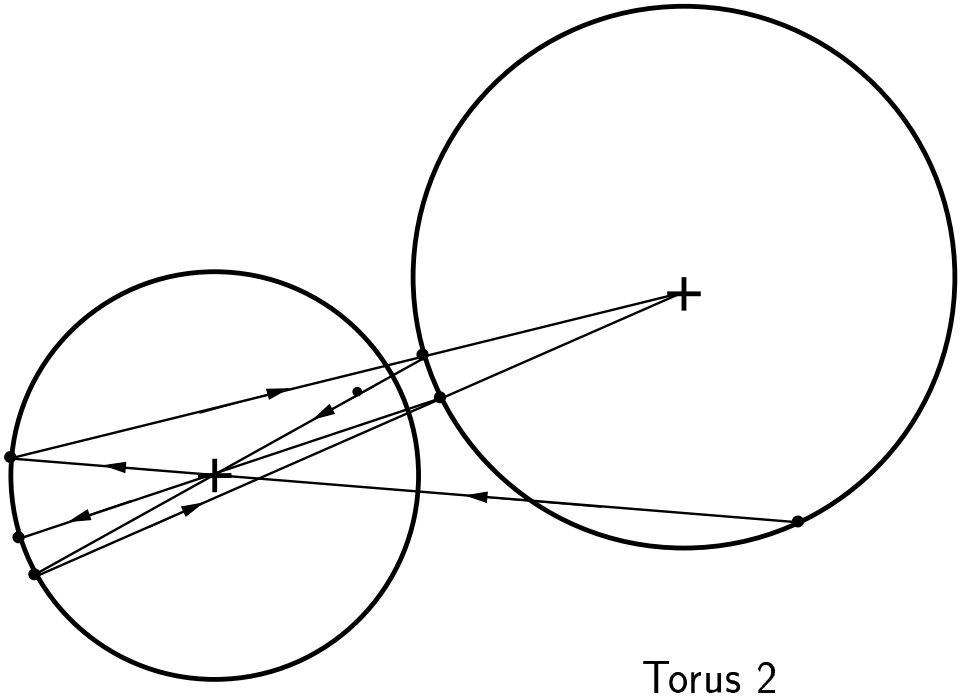
# Local Surface Curvatures

1. Rolling contact mechanisms
2. Contact prediction
3. Rolling (non-holonomic) constraints—hard contacts

# Torus Characteristics

1. Compact geometric representation
2. Smooth
3. Strictly convex patches
4. A transition between convex and concave patches
5. Simple surface characteristics:
   - principal curvatures
   - directions of principal curvatures
6. **Fast contact detection**

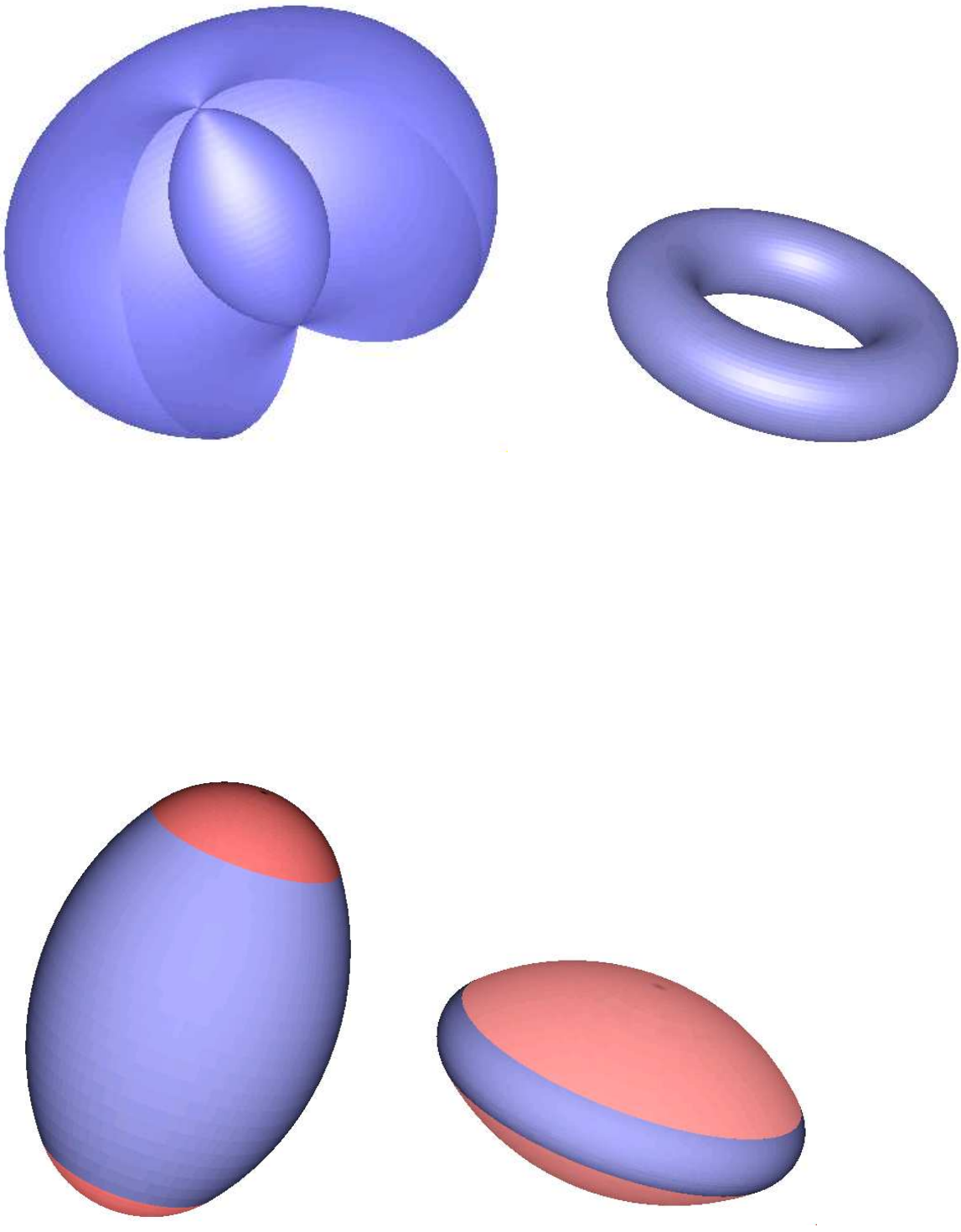
# Iteration for Contact Inquiry



Torus 2

Torus 1

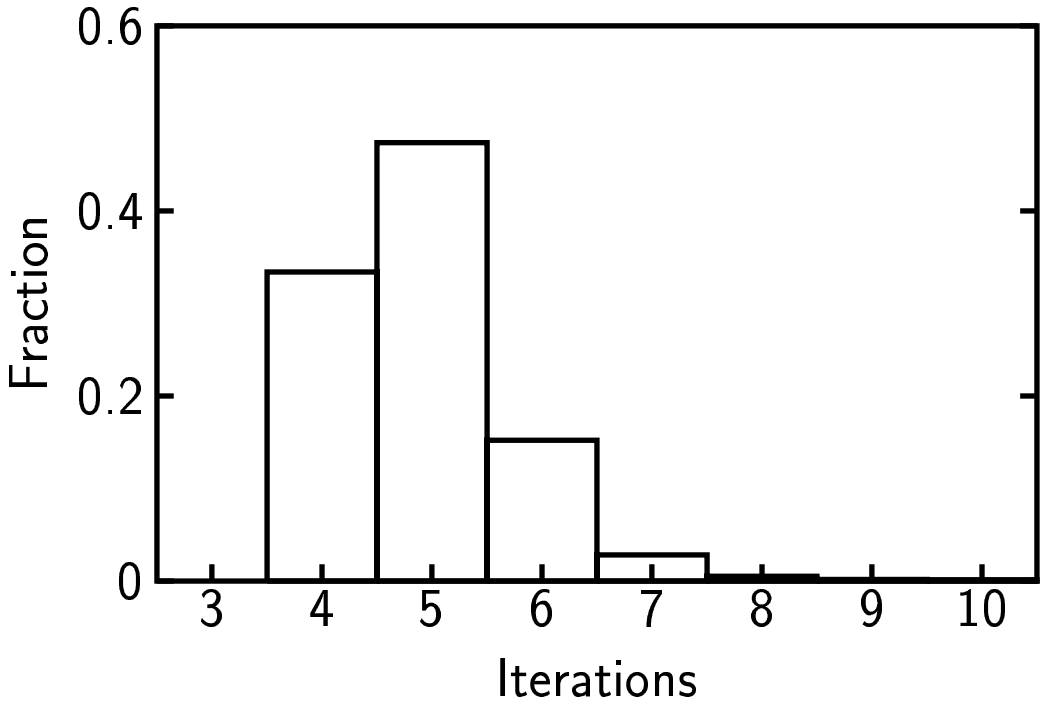
# Extremal Distances



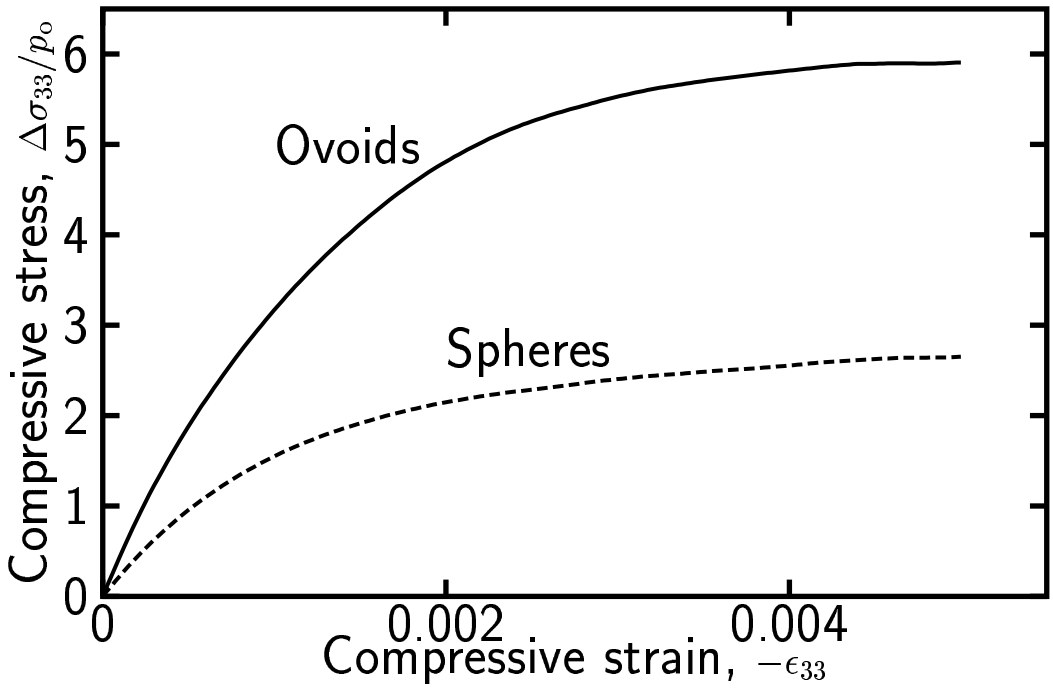
# Aitken's Acceleration

## Convergence

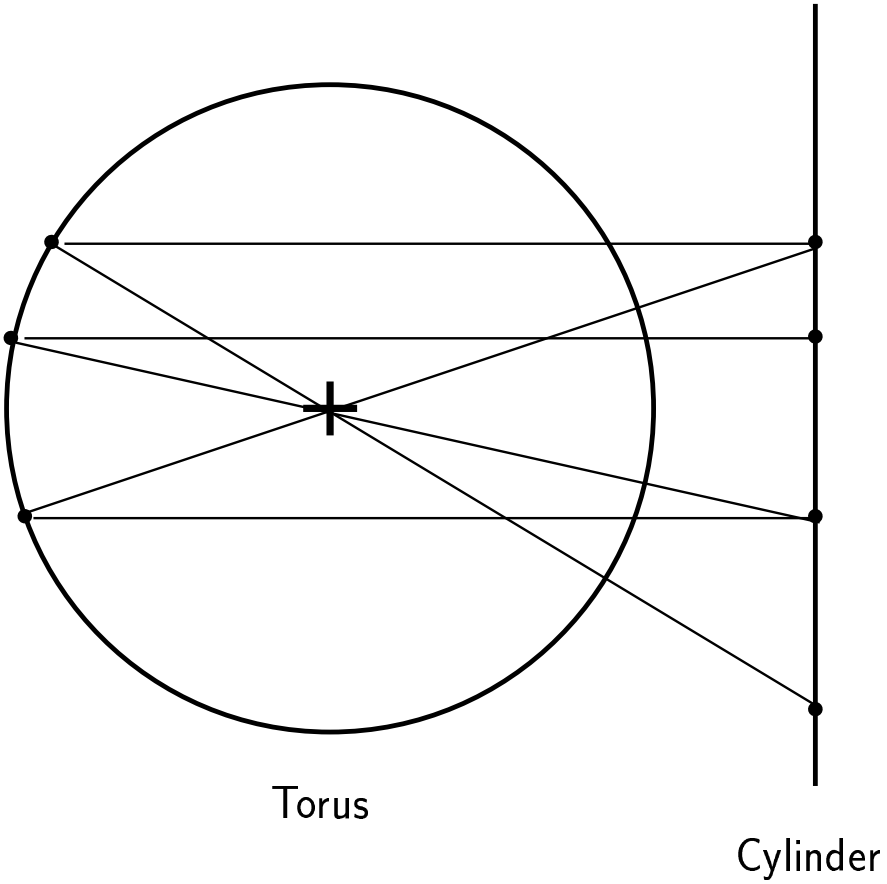
$\epsilon = 1 \times 10^{-18}$



# Biaxial Compression: Spheres and Ovoids

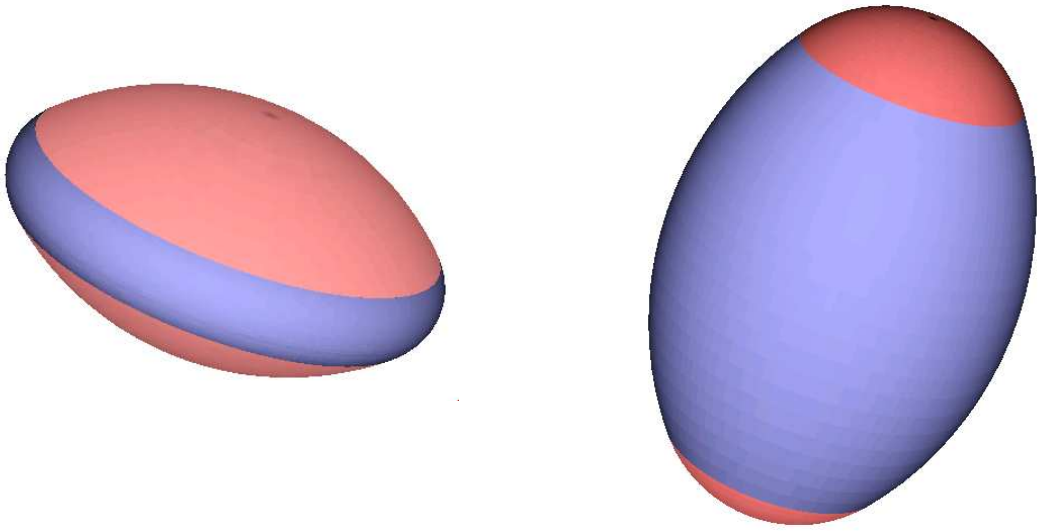


# Torus–Cylinder Iteration



# Run-times: Ovoids and Spheres

| | Ovoids | Spheres | Ovoids / Spheres |
|---|---|---|---|
| Contact inquiry calls | $5.2\times10^8$ | $2.5\times10^8$ | 2.1 |
| Contact inquiry time, s | 528 | 73 | 7.2 |
| Total runtime, s | 1430 | 720 | 2.0 |

1800 particles
30,000 time steps
1.05 MHz single Athlon processor

# Oblate and Prolate Ovoids