

Discrete Element Modeling of Soils as Granular Materials

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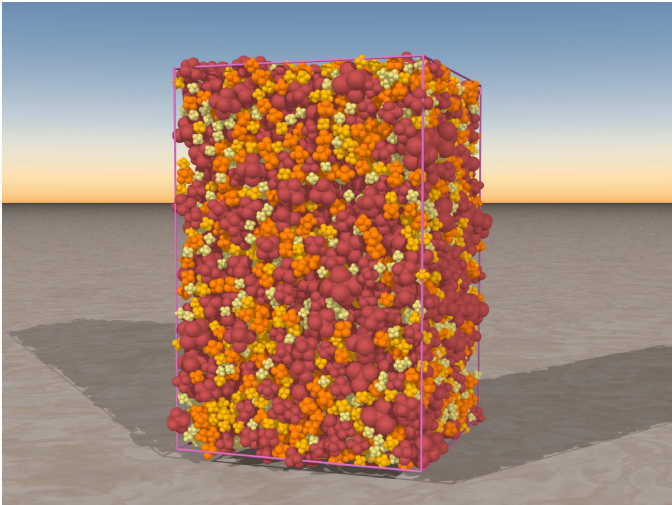
Outline

- 1 Discrete Element Method (DEM)
- 2 Soil simulations — liquefaction
- 3 Granular materials: Are they simple?

Contents

- 1 Discrete Element Method (DEM)
 - Background of DEM
 - Coding and challenges
 - Alternatives to DEM
- 2 Soil simulations — liquefaction
- 3 Granular materials: Are they simple?

DEM assembly of 6400 particles



Discrete Element Method (DEM)

Background

- Granular media are modeled with individual particles
- Peter A. Cundall (1971)
1979 *Geotechnique* paper

DEM Algorithm

A finite difference (time-stepping) algorithm, in which
Elemental particles (spheres, polyhedra, etc.)

Interact in a pair-wise manner (contacts), such that the

Imbalances in the forces on each particle

Impel the particles to new positions

With each time step

Via Newton's equations of motion

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DEM Software

Commercial software

- PFC3D — Itasca, Inc.
- EDEM — DEM Solutions

Open source software

- Yade — yet another dynamics engine
- ESyS
- LAMMPS
- OVAL

Coding Challenges

- Particle shapes
 - Spheres = Easy
 - Other shapes = Difficult
- Contact detection: an N^2 problem
- Contact force models
 - Linear springs = Easy
 - Hertz-Mindlin springs = Difficult
 - Real (soil) particle interactions = Not yet attempted
- Problem types
 - “Element” tests
 - Field problems (realistic boundaries)

Modeling real soil problems = Very difficult

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DEM modeling — disadvantages

Shortcomings of DEM simulations:

- Realistic particle shapes and arrangements are difficult to create and to calibrate.
- Relative density is difficult to surmise.
- Roughness, texture, and sharp edges of particles are not modeled.
- Idealized contact models (Hertz-Mindlin, etc.)
- Particle breakage or chipping is (usually) disallowed.

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Alternatives

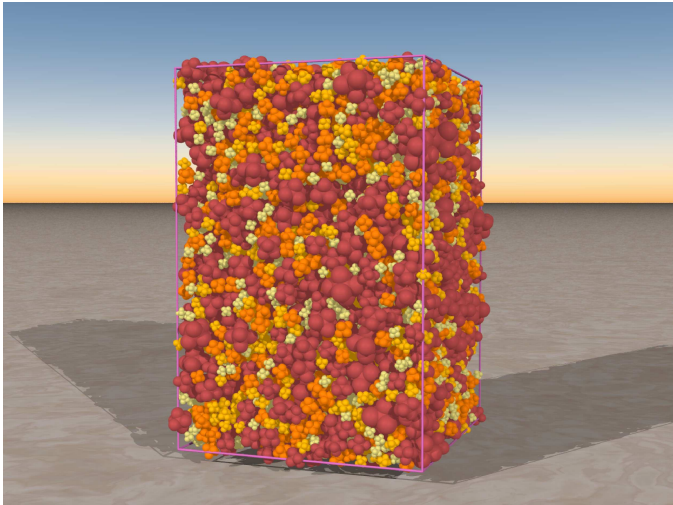
Alternative computational methods:

- Discontinuous Deformation Analysis (DDA)
G.-H. Shi and Y. Kishino
 $[K][u] + [C][\dot{u}] + [M][\ddot{u}] = 0$
- Contact Dynamics
M. Jean and J.-J. Moreau
Inequality constraints, non-linear programming
- Event-driven Models
Instantaneous collision-based

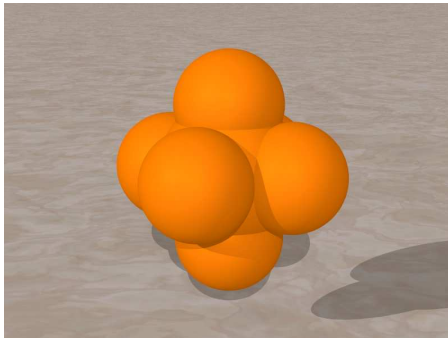
Contents

- 1 Discrete Element Method (DEM)
- 2 Soil simulations — liquefaction
 - Undrained loading & static liquefaction
 - Cyclic liquefaction
 - Severity measures
- 3 Granular materials: Are they simple?

DEM assembly of 6400 particles



Particle shape



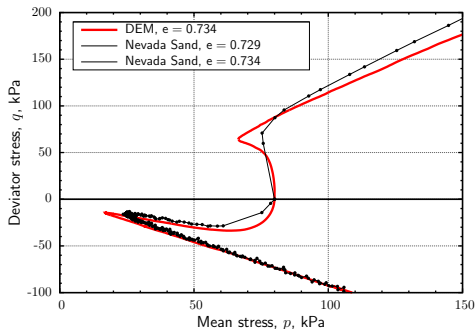
DEM model

Contact properties:

- Hertz-Mindlin (elastic-frictional) contact model
- $E = 29 \text{ GPa}$, $\nu = 0.15$
- $\mu = 0.60$ friction coefficient

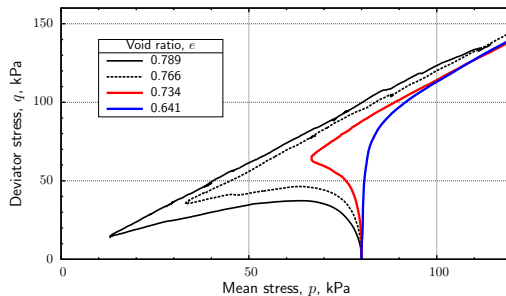
Verifying the DEM model

Undrained triaxial compression and extension tests



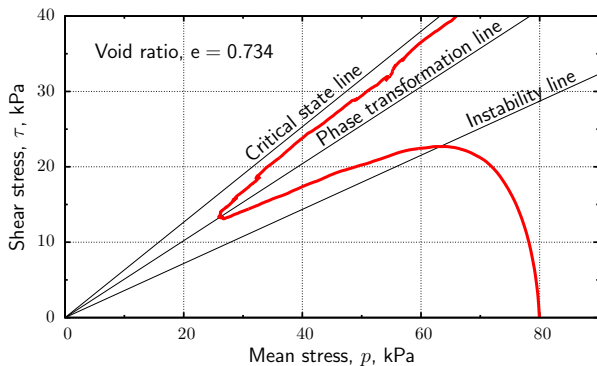
Verifying the DEM model

Undrained triaxial compression tests — range of densities



Undrained simple-shear results

Undrained simple-shear:



DEM modeling — advantages

Modeling soil behavior with DEM “element” simulations:

- Experiments can be initiated (or restarted) from the same assembly.
- Full stress and strain tensors can be measured.
- Arbitrary control of 6 stress or strain increments.
- Behavior simulated in the absence of shear bands.

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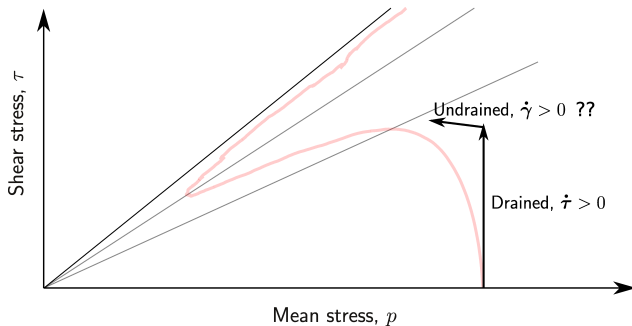
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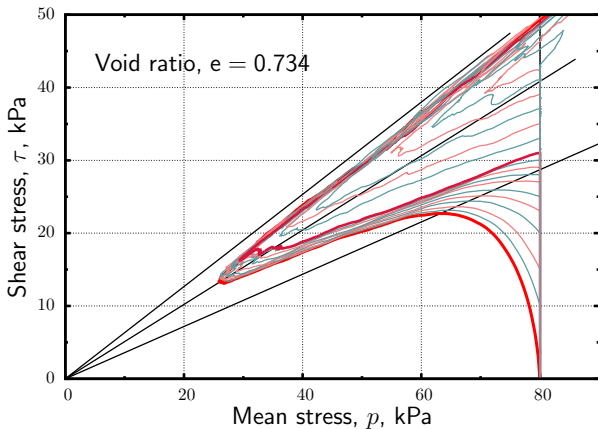
Static liquefaction

Stress path for inducing static liquefaction



Static liquefaction

Drained shearing followed by undrained shearing:

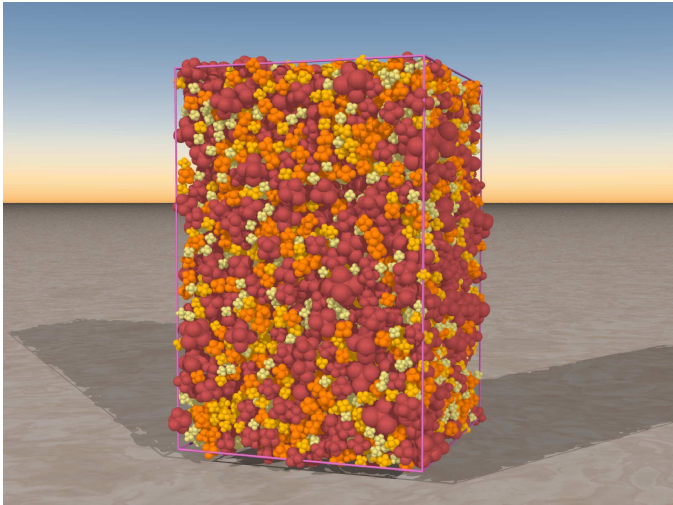


Cyclic liquefaction

Cyclic liquefaction simulations:

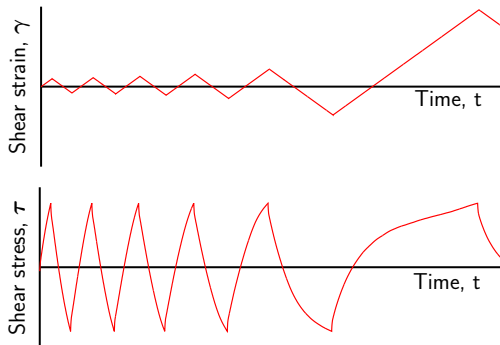
- Two loading cases
 - **Case I** Uniform amplitude cyclic shearing
 - **Case II** Erratic, seismic shearing
- “Severity Measure” for predicting initial liquefaction

DEM assembly of 6400 particles



Case I: Uniform cyclic shearing

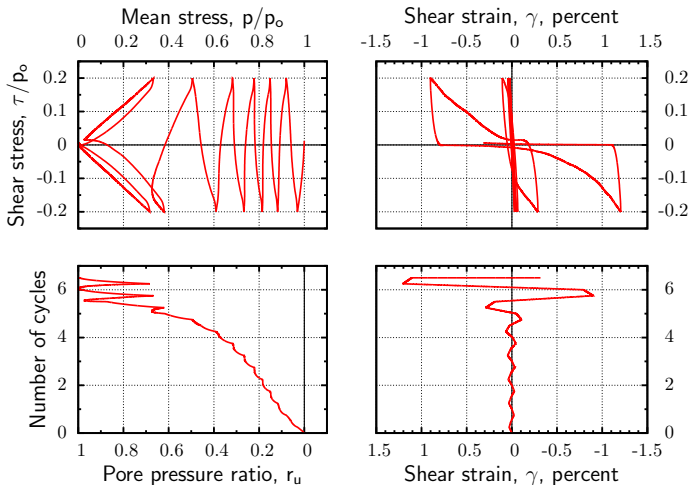
Uniform shearing amplitude:



Control strain rate $\dot{\gamma}$ in a sawtooth pattern until the targeted shear stress τ is attained.

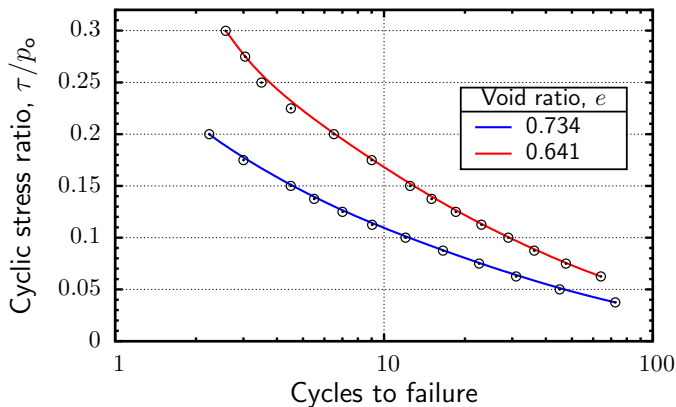
Case I: Uniform cyclic shearing

Conditions: $\tau = \pm 16$ kPa, $p_o = 80$ kPa



Case I: Uniform cyclic shearing

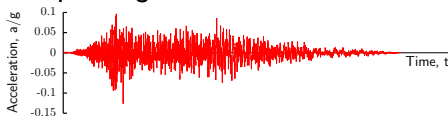
Liquefaction curves



Case II: Seismic shearing

Select 24 sequences of seismic loading (Dr. Steven L. Kramer)

- 1 Earthquake ground accelerations from PEER data base



Landers, 1992

M = 7.3

MCF000

- 2 Create CSR, cyclic shear record (Dr. Kramer)



SHAK91

- 3 Scale the CSR to prolong pre-liquefaction

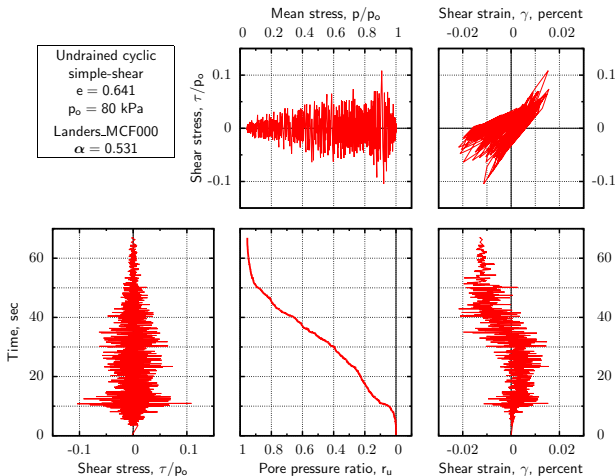


Scale factor:

$\alpha = 0.531$

Case II: Seismic shearing

Landers 1992 CSR record, scaling factor $\alpha = 0.531$



Severity Measures for cyclic loading

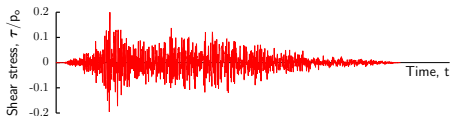
Ranking the severities, $1 / \alpha$,
of 24 stress records,
as surmised from DEM simulations

CHICHI_CHY088-N_h2	2.398
KOCAEL_CNA000_h2	2.392
CAPEMEND_SHL090	2.262
CHICHI_TCU107-A_h2	1.965
ITALY_A-BRZ000	1.923
LANDERS_MCF000	1.883
COYOTELK_G06320	1.876
WHITTIER_A-CAM009	1.859
WHITTIER_A-116360	1.783
WHITTIER_A-WHD152	1.754
GREECE_E-PLK-NS	1.58
LOMAP_TIB290	1.577
COYOTELK_G04360	1.543
MAMMOTH_L-FIS090	1.517
LOMAP_AG2043	1.499
WHITTIER_A-RO3000	1.42
COALINGA_H-COH090	1.416
BIGBEAR_HOS180	1.34
WHITTIER_A-ALT090	1.261
COALINGA_D-PVP360	1.227
PALMSPR_MVH135	1.124
HECTOR_12543090	0.864
NORTHR_VEN090	0.773
MAMMOTH_H-XMC207	0.553

Severity Measures

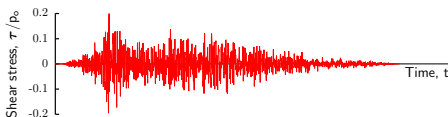
“Severity Measure”:

- a scalar predictor of **initial liquefaction**
- computed from a cyclic stress (or strain) record



Scalar value at
initial liquefaction

Severity Measures



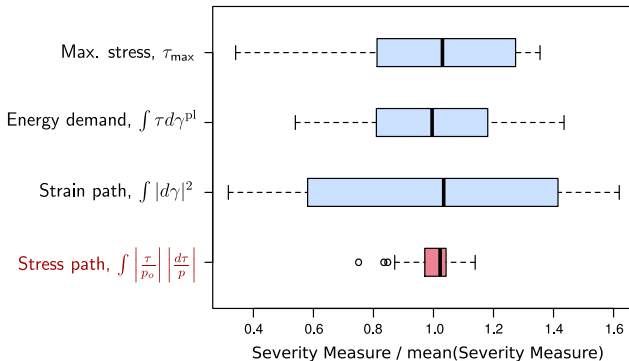
Possible Severity Measures for the 24 stress records:

- Maximum shear stress, $|\tau/p_0|_{\max}$
- Energy demand, $\int \tau d\varepsilon^{\text{plastic}}$
- Strain path, $\int |d\varepsilon|^2$
- Stress path, $\int \left| \frac{\tau}{p_0} \right| \left| \frac{d\tau}{p} \right|$

Use DEM results to test the **efficiency** and **sufficiency** of each Severity Measure.

Severity Measures

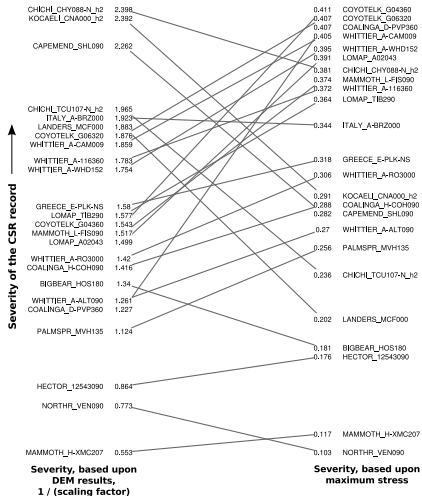
Efficiencies of four Severity Measures: 24 cyclic stress records



Severity Measures

Sufficiency of the
 Maximum Shear Stress
 as a Severity Measure:

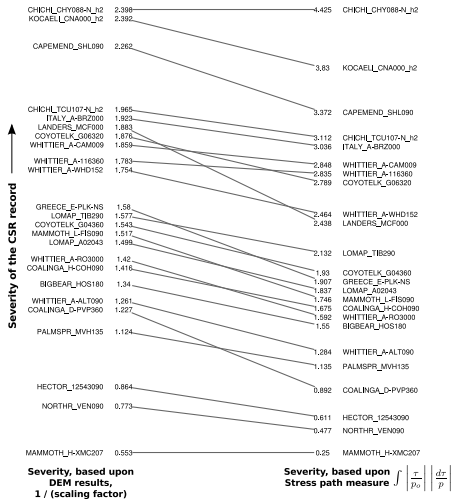
$$|\tau / p_o|_{\max}$$



Severity Measures

Sufficiency of a stress path scalar as a Severity Measure:

$$\int \left| \frac{\tau}{p_0} \right| \left| \frac{d\tau}{p} \right|$$



Contents

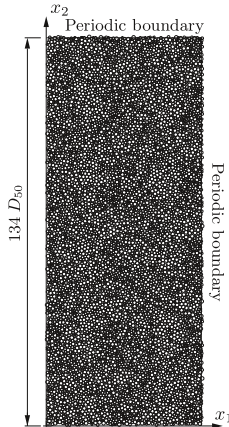
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- 3 Granular materials: Are they simple?
 - Shear bands and non-classical continua
 - Strain gradient-dependent materials
 - DEM measurement of strain gradient effects

Granular Mechanics

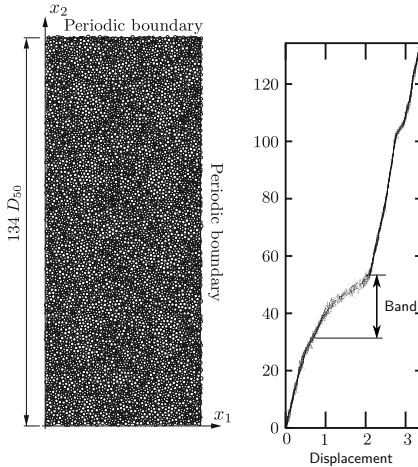
Shear bands and non-classical continua

- Shear bands have a characteristic thickness
- Granular materials have an “inherent length scale”
- This scale is not accessible via classical continuum mechanics

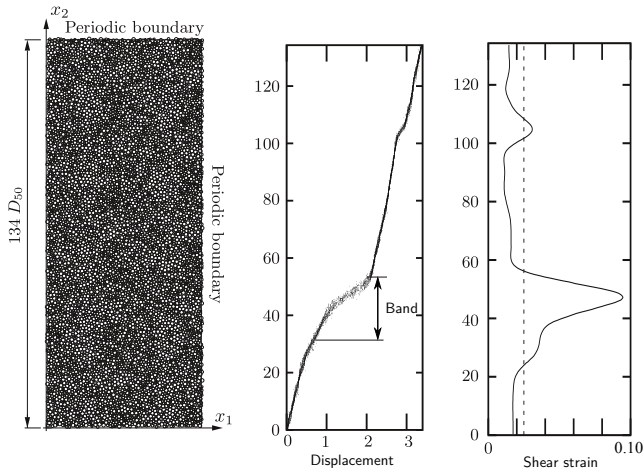
Shear bands in DEM simulation — free deformation



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Shear bands in DEM simulation — free deformation



Non-classical continuum models

Continuum models with inherent length scale:

- 1 Cosserat / micropolar continua
- 2 Strain gradient-dependent material models
- 3 Non-local material material models

Non-classical continuum models

Continuum models with inherent length scale:

1 Cosserat / micropolar continua

2 Strain gradient-dependent material models

$$\boldsymbol{\tau} = f(\boldsymbol{\epsilon}, \dot{\boldsymbol{\epsilon}})$$

Simple material

$$\boldsymbol{\tau} = f(\boldsymbol{\epsilon}, \dot{\boldsymbol{\epsilon}}, \nabla \boldsymbol{\epsilon}, \nabla(\nabla \boldsymbol{\epsilon}), \dots)$$

Gradient-dependent material

3 Non-local material material models

Non-classical continuum models

Continuum models with inherent length scale:

1 Cosserat / micropolar continua

2 Strain gradient-dependent material models

$$\boldsymbol{\tau} = f(\boldsymbol{\epsilon}, \dot{\boldsymbol{\epsilon}}) \quad \text{Simple material}$$

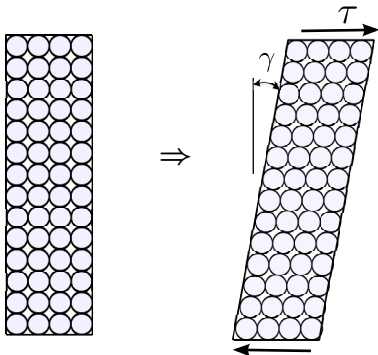
$$\boldsymbol{\tau} = f(\boldsymbol{\epsilon}, \dot{\boldsymbol{\epsilon}}, \nabla \boldsymbol{\epsilon}, \nabla(\nabla \boldsymbol{\epsilon}), \dots) \quad \text{Gradient-dependent material}$$

Does stress really depend upon the spatial gradients of strain?

3 Non-local material material models

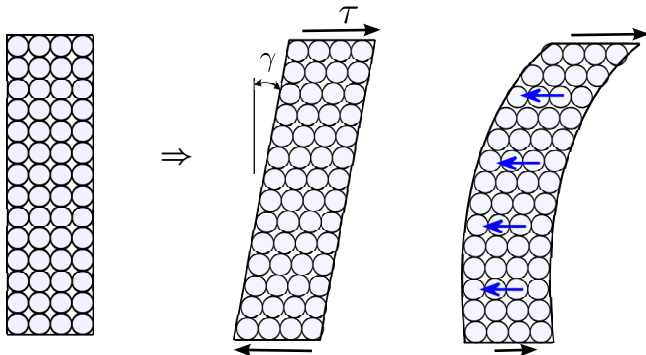
Strain gradient-dependent materials

Does stress depend upon the spatial gradients of strain?



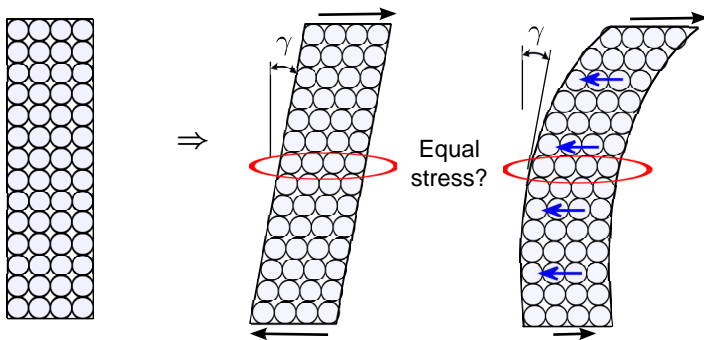
Strain gradient-dependent materials

Does stress depend upon the spatial gradients of strain?

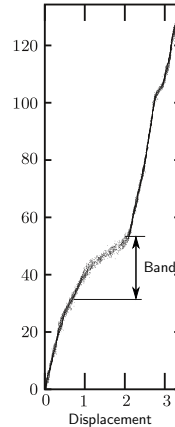
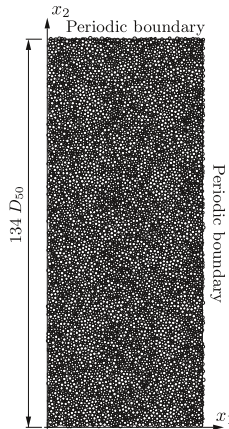


Strain gradient-dependent materials

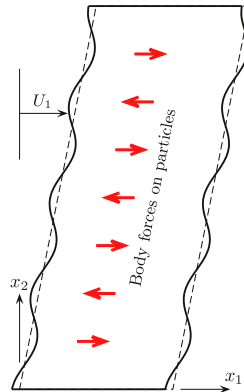
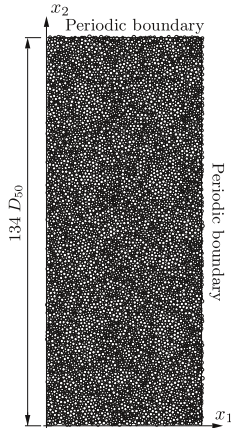
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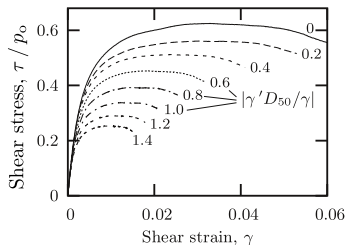
Shear bands in DEM simulation — free deformation



Constrained deformation using body forces

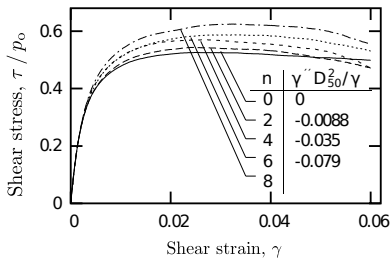


Effect of the first strain gradient



An increasing first gradient, γ' , has a softening effect.

Effect of the second strain gradient



An increasing second gradient, γ'' , has a hardening effect.

Shear bands

Shear bands and gradient-dependent behavior:

- Persistent bands develop near the peak stress state
- Shear strain γ is non-uniform within a shear band
- Shear stress depends upon γ , γ' , and γ''
- In incremental form, $d\tau = f(d\gamma, d\gamma', d\gamma'')$
- Shear stress is constant within a shear band: $d\tau/dx_2 = 0$
- Can an incremental model explain the profile of strain within a shear band?

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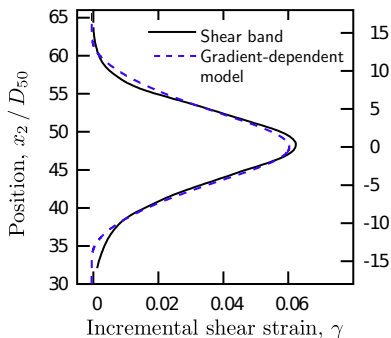
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Solution of the incremental model:



Questions?